

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 21 NOVEMBRE 2022

DOCENTI:

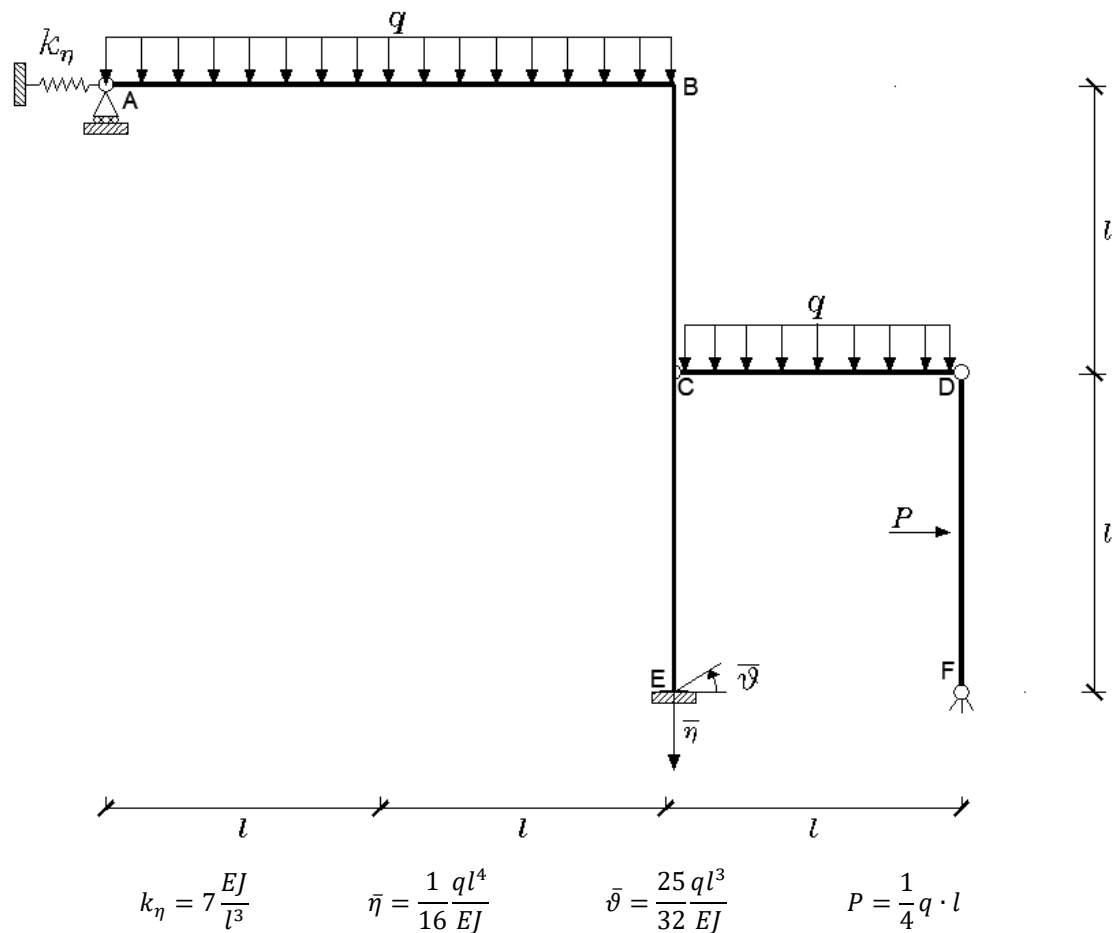
PROF. GIOVANNI A. PLIZZARI

PROF. FAUSTO MINELLI

PROF. GIUSEPPE TIBERTI

DURATA: 2 .30 ORE.

Esercizio

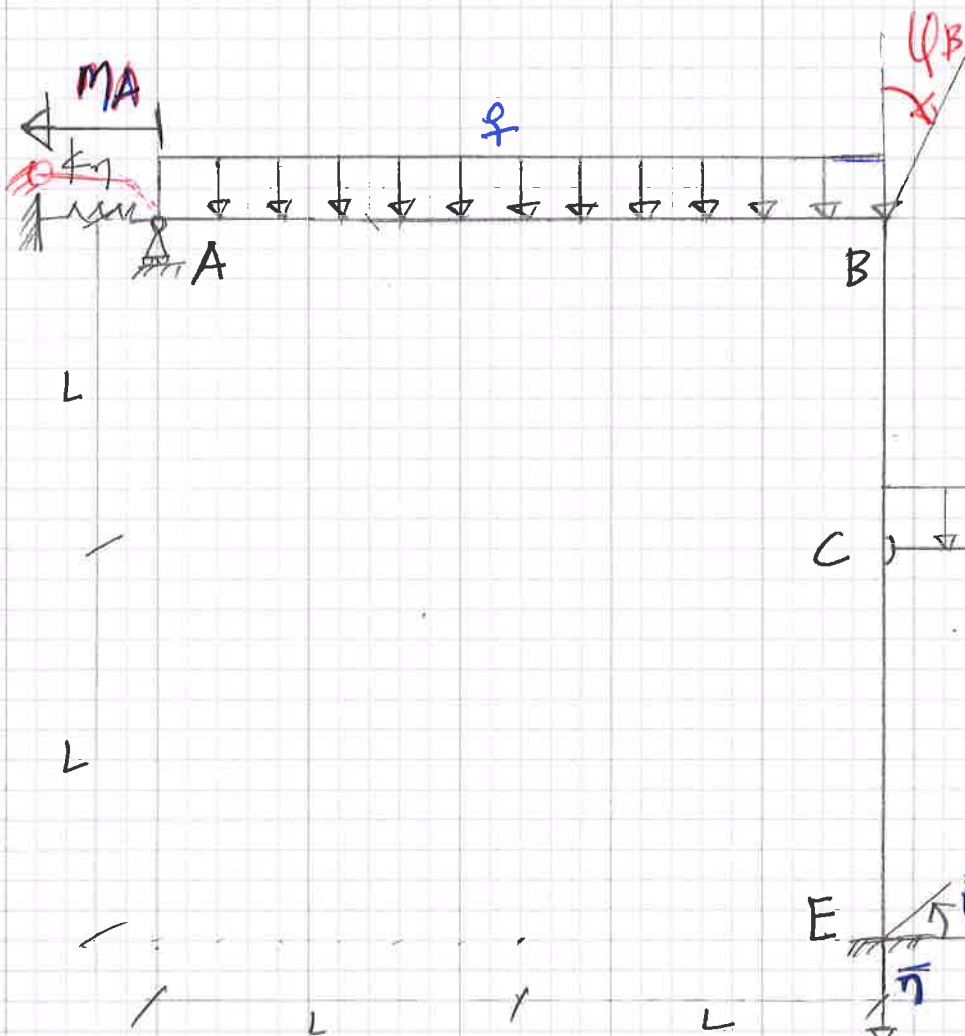


Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \rightarrow \infty$, $EJ = \text{costante}$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

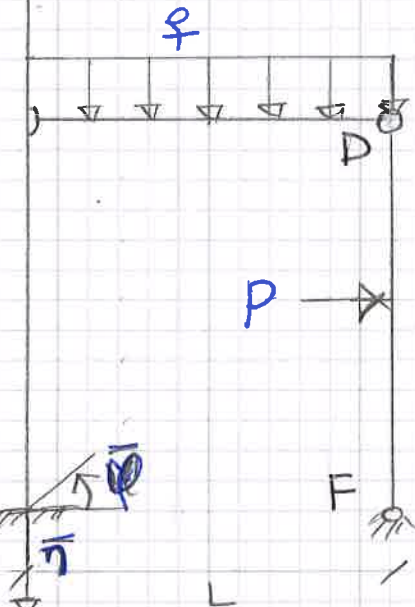


Given data

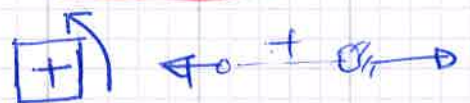
$$k_{\eta} = \frac{7EI}{L^3}, \quad P = \frac{qL}{4}$$

$$\bar{\varphi} = \frac{25}{32} \frac{qL^3}{EI}$$

$$\bar{\eta} = \frac{1}{16} \frac{qL^4}{EI}$$



Convention

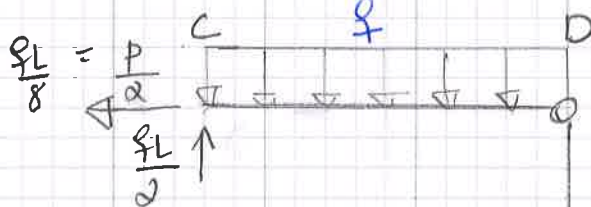


$$M_{BB}\varphi_B + M_{BA}\eta_A + M_{B\phi} = 0$$

$$H_{AB}\varphi_B + H_{AA}\eta_A + H_{A\phi} = 0$$

Appendice isostatica C D F

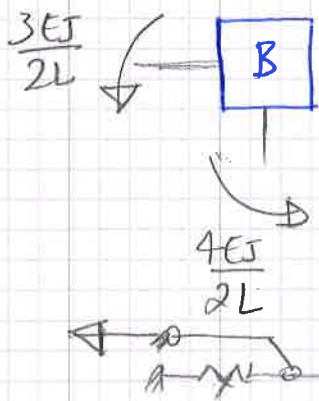
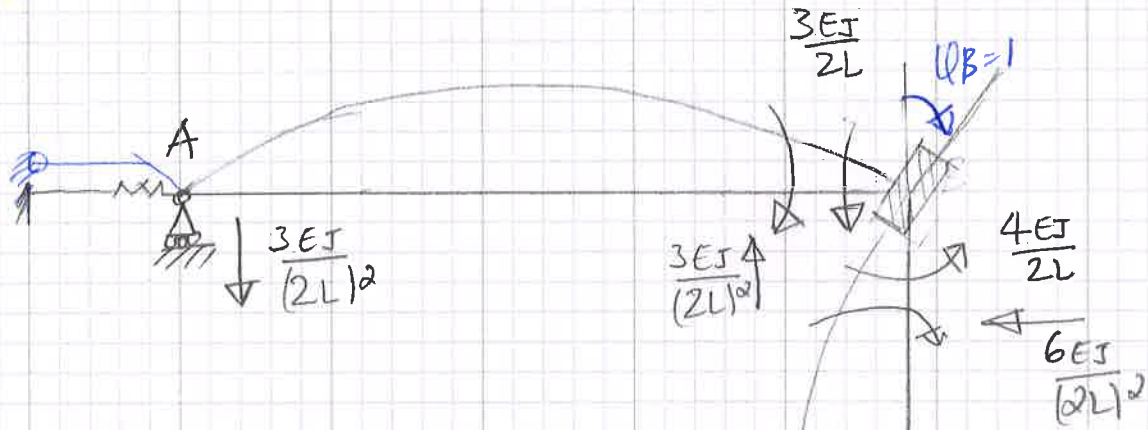
Solution



$$\frac{qL}{4} = P \rightarrow$$

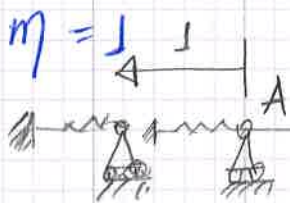
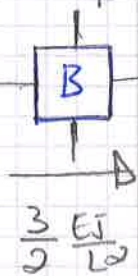
$$\frac{qL}{8} = \frac{P}{2} \rightarrow$$

$$\psi_B = 1$$

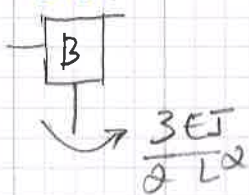


$$M_{BB} = \frac{7EI}{2L}$$

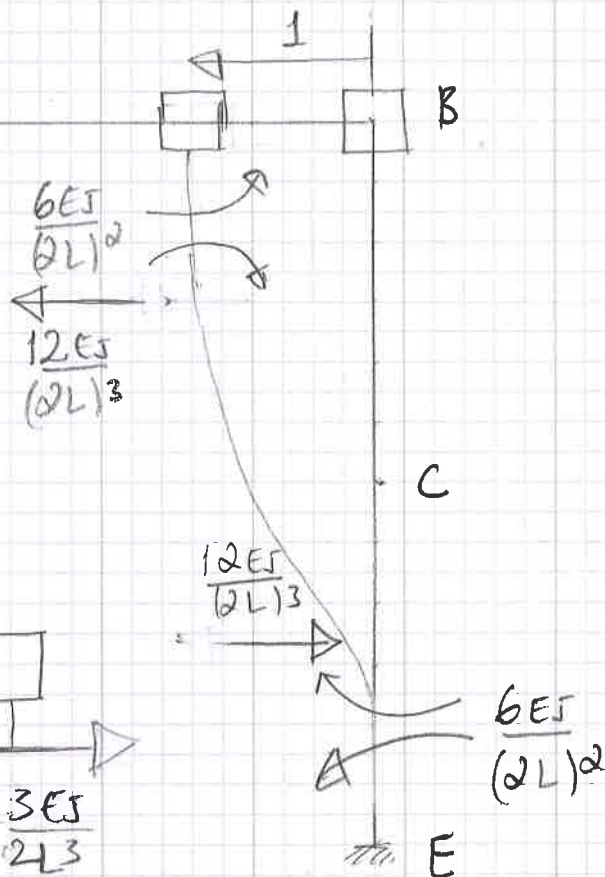
$$h_{AB} = \frac{3EI}{2L^2}$$



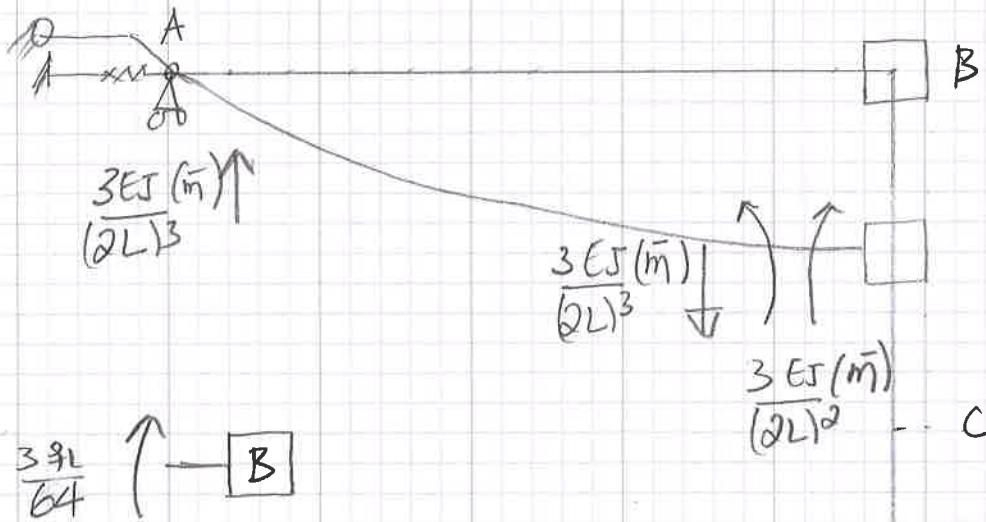
$$M_{BA} = \frac{3EI}{2L}$$



$$h_{AA} = \frac{3EI}{2L^3} + k_f$$



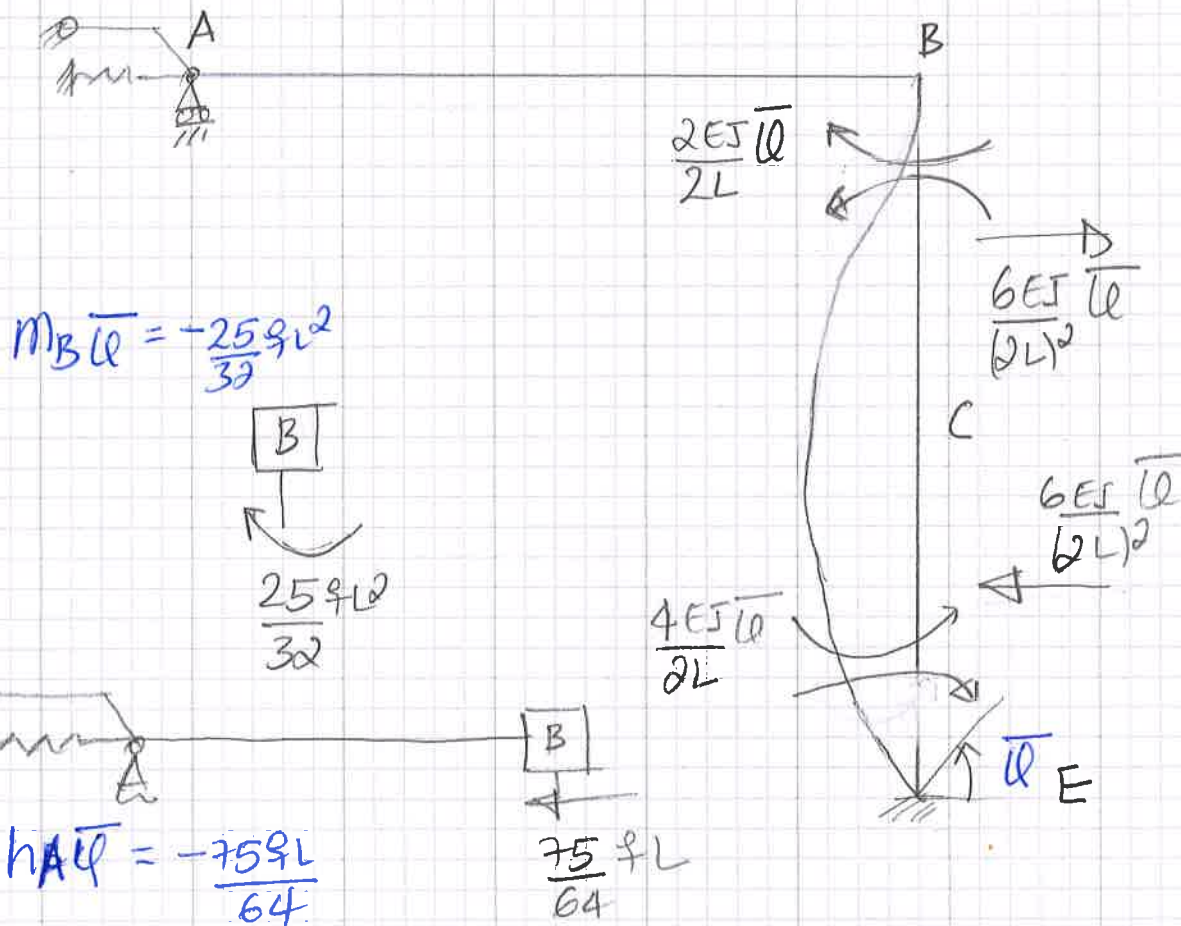
$$\bar{m} \neq 0$$



$$M_B \bar{m} = -\frac{3}{64} \gamma L$$



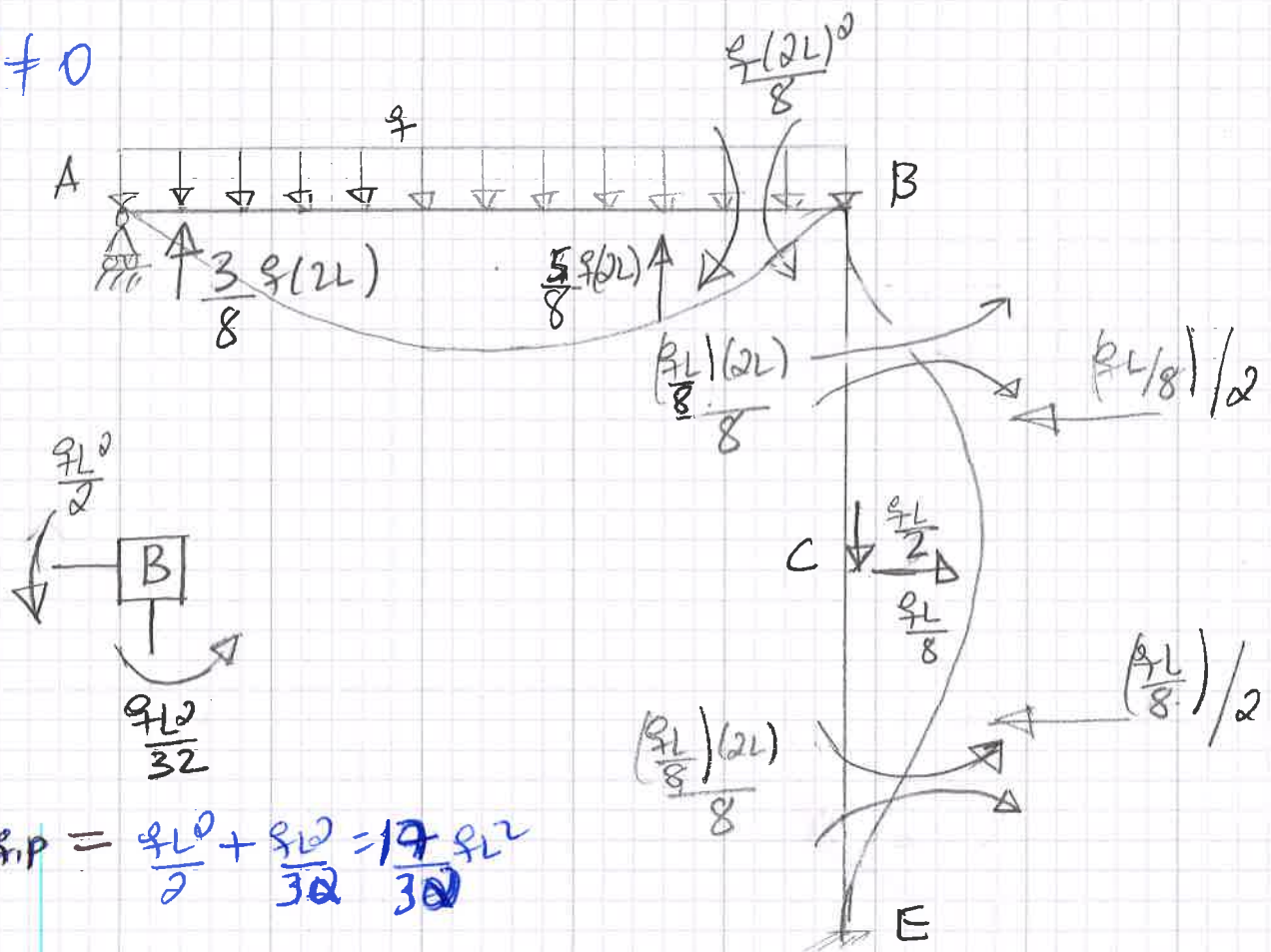
$$\bar{\varphi} \neq 0$$



$$M_B \bar{\varphi} = -\frac{25}{32} \gamma L^2$$

$$H_A \bar{\varphi} = -\frac{75}{64} \gamma L$$

$$q, P \neq 0$$



$$M_{B,q,p} = \frac{qL^2}{2} + \frac{qL^2}{32} = \frac{17}{32} qL^2$$



$$H_{A,q,p} = \frac{qL}{16}$$

$$M_{BB} \psi_B + M_{BA} \eta_A + M_{B\phi} = 0$$

$$H_{AB} \psi_B + H_{AA} \eta_A + H_{A\phi} = 0$$

$$\frac{7}{2} \frac{EI}{L} \psi_B + \frac{3}{2} \frac{EI}{L^2} \eta_A - \frac{25}{32} qL^2 - \frac{3}{64} qL^2 + \frac{17}{32} qL^2 = 0$$

$$\frac{3}{2} \frac{EI}{L^2} \psi_B + \left(\frac{3}{2} \frac{EI}{L^3} + \frac{7}{2} \frac{EI}{L^3} \right) \eta_A - \frac{75}{64} qL + \frac{qL}{16} = 0$$

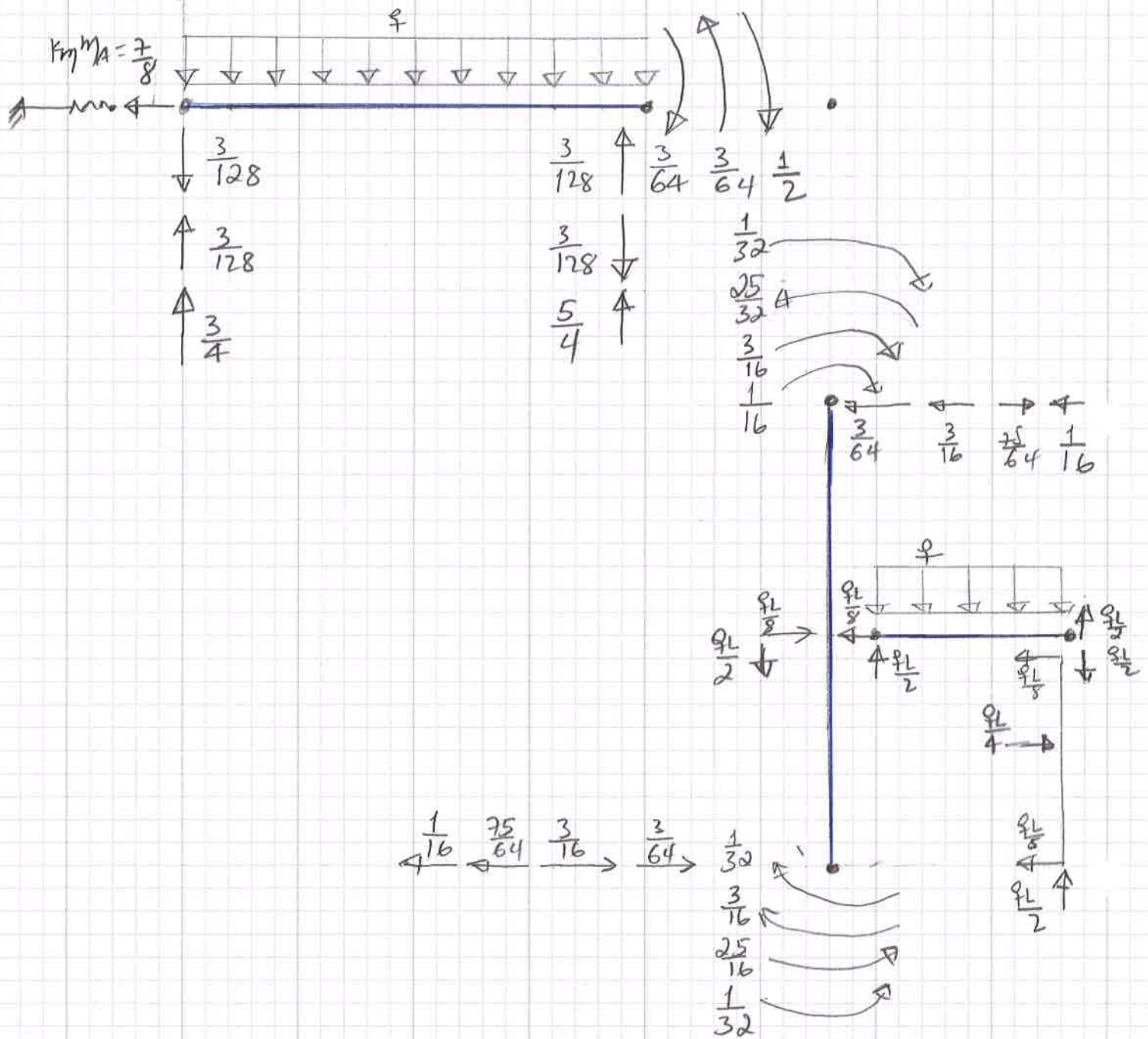
$$\frac{7}{2} \frac{EI}{L} \psi_B + \frac{3}{2} \frac{EI}{L^2} \eta_A - \frac{19}{64} qL^2 = 0$$

$$\frac{3}{2} \frac{EI}{L^2} \psi_B + \frac{17}{2} \frac{EI}{L^3} \eta_A - \frac{31}{64} qL = 0$$

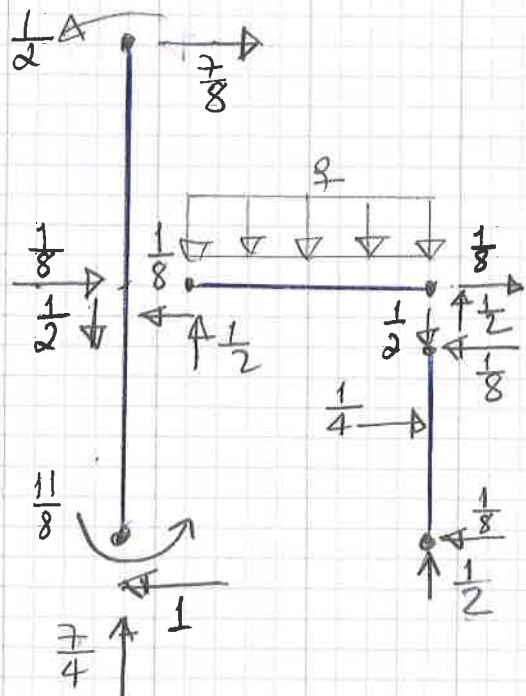
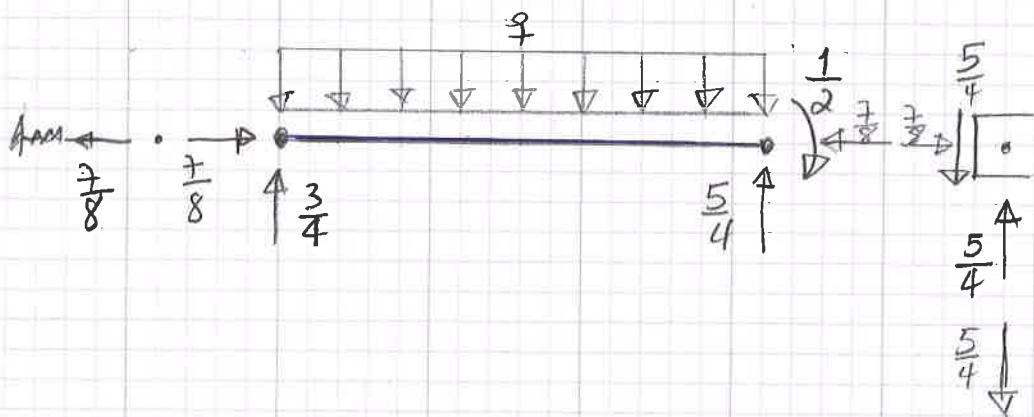
$$\psi_B = \frac{1}{32} \frac{qL^3}{EI}$$

$$\eta_A = \frac{1}{8} \frac{qL^4}{EI}$$

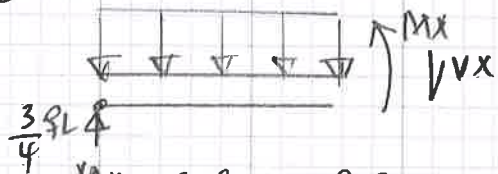
Internal Actions



Resultant internal actions



① AB



$$M_x = \frac{3}{4} q L x - \frac{q x^2}{2}$$

$$V_x = \frac{3}{4} q L - q x = 0, \quad \frac{3}{4} q L = x_1$$

$$x_1 = \frac{3}{4} L, \text{ where } M_{\max}$$

$$M_{\max} = \frac{3}{4} q L \left(\frac{3}{4} L \right) - \frac{q}{2} \left(\frac{3}{4} L \right)^2 = \frac{9}{32} q L^2$$

$$M_x = 0 \Rightarrow \frac{3}{4} q L x_2 - \frac{q x_2^2}{2} = 0 \Rightarrow \frac{3}{4} q L x_2 = \frac{q x_2^2}{2} \Rightarrow x_2 = \frac{3}{2} L$$

② BE



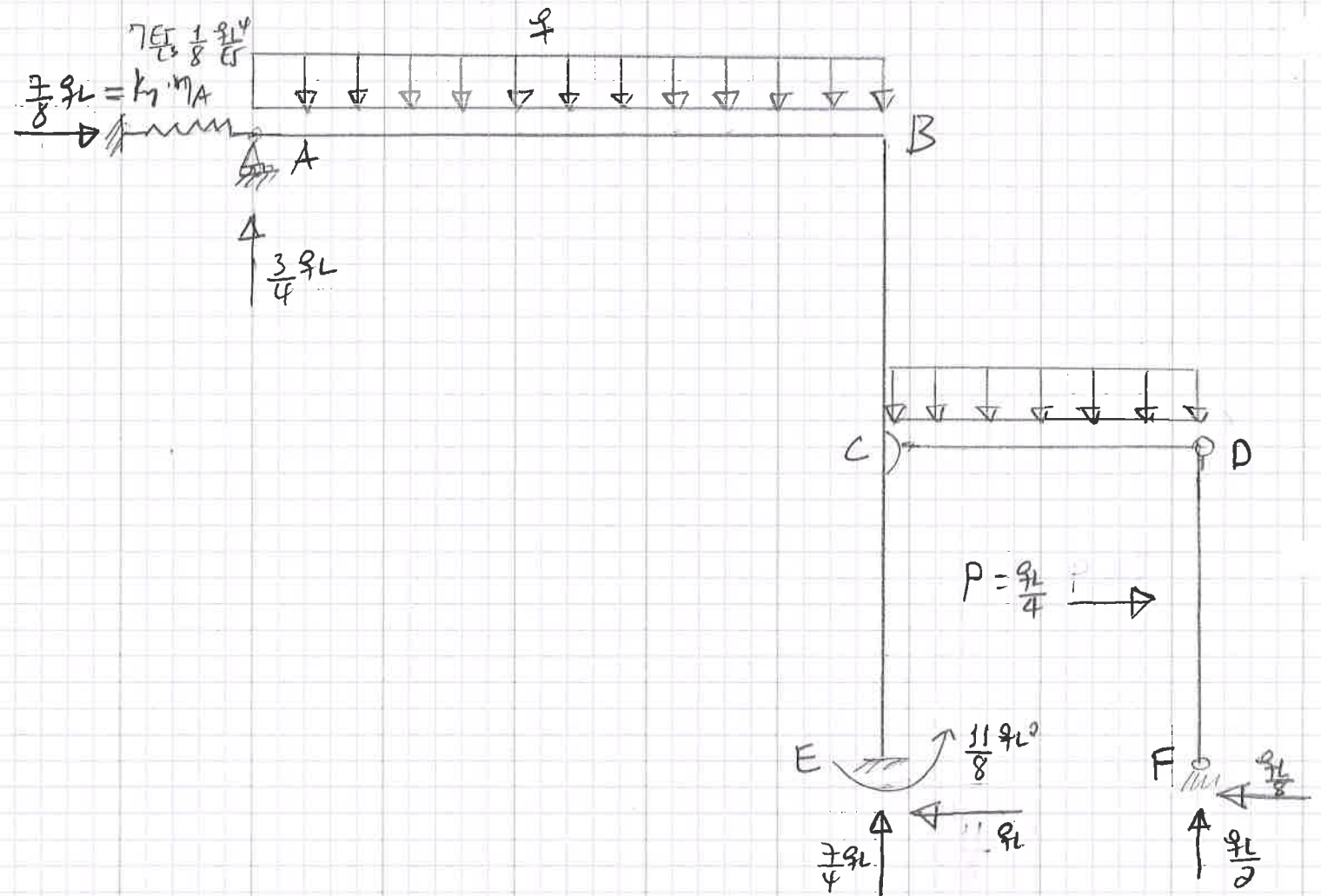
$$M_x = \frac{q L^2}{2} - \frac{7}{8} q L x_3 \Rightarrow M_x = 0 \Rightarrow \frac{q L^2}{2} - \frac{7}{8} q L x_3 = 0$$

$$\frac{q L^2}{2} = \frac{7}{8} q L x_3 \Rightarrow x_3 = \frac{4}{7} L$$

$$M(x=L) = \frac{q L^2}{2} - \frac{7}{8} q L x = \frac{3}{8} q L^2$$

$$x_4 = L$$

Globales Gleichgewicht



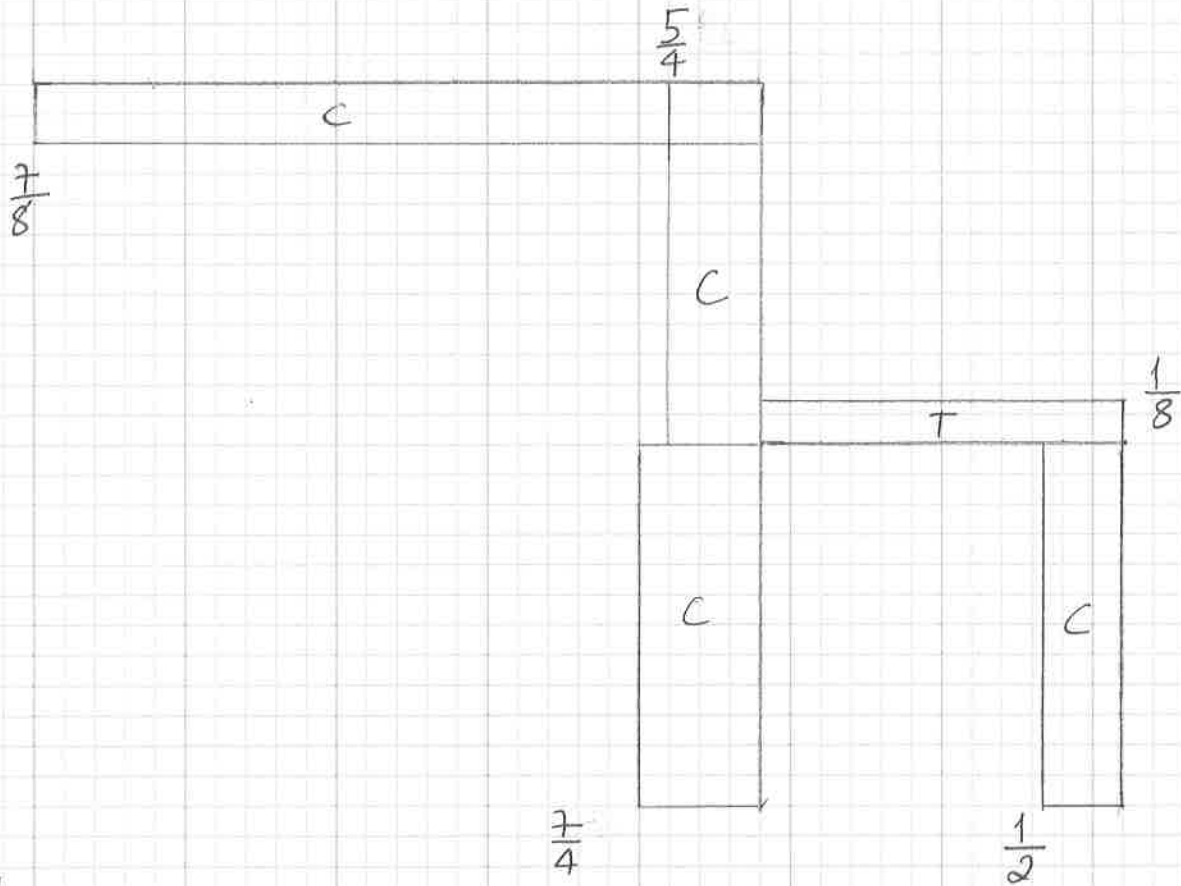
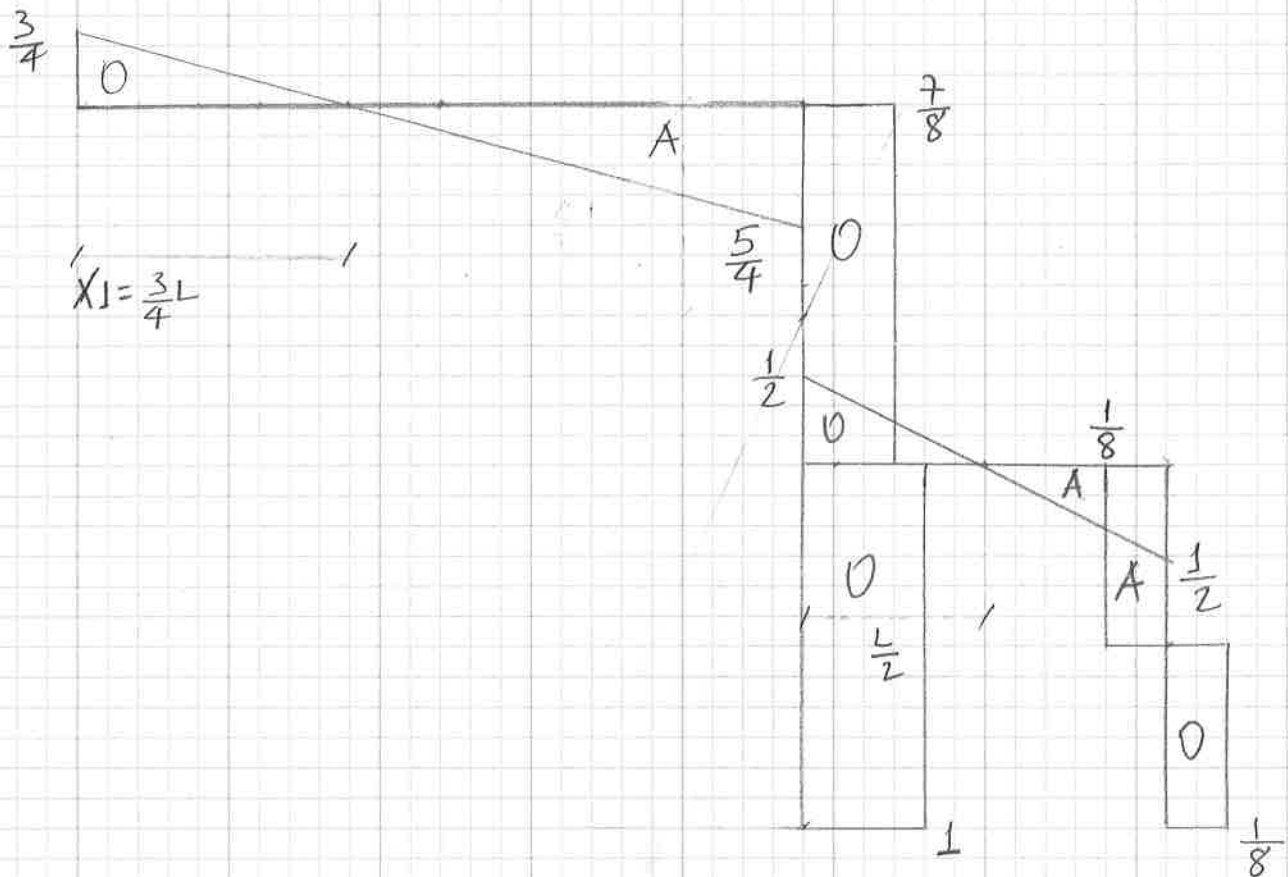
$$\sum F_H = \frac{7}{8}qL - qL - \frac{qL}{8} + \frac{qL}{4} = 0$$

$$\sum F_V = \frac{3}{4}qL - 2qL + \frac{7}{4}qL + \frac{qL}{2} - qL = 0$$

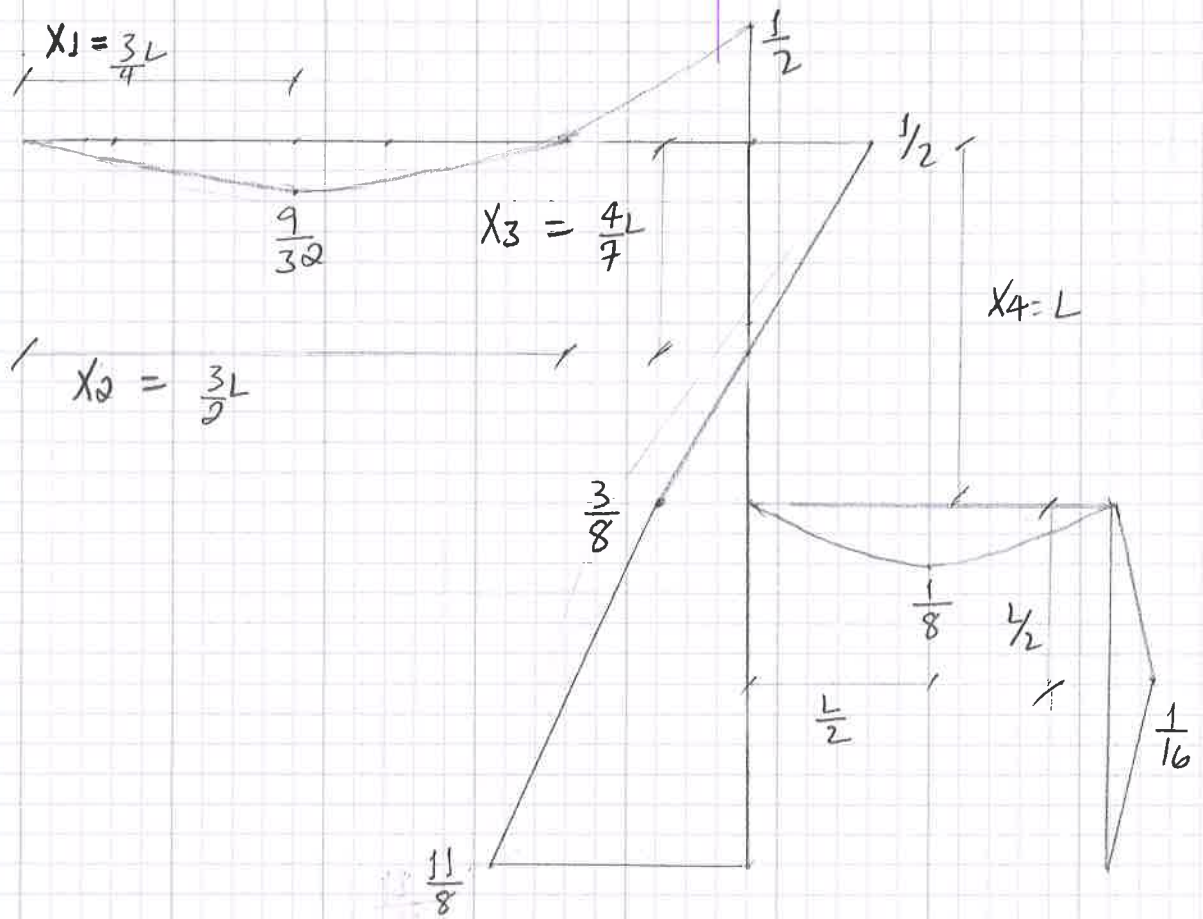
$$\sum M_B = -\left(\frac{3}{4}qL\right)(0L) + q(2L)(L) - (qL)\left(\frac{L}{2}\right) + \frac{11}{8}qL^2 - (qL)(2L) + \frac{qL}{2}L + \frac{qL}{8}(2L) = 0$$

$$-\frac{3}{2}qL^2 + 2qL^2 - \frac{qL^2}{2} + \frac{11}{8}qL^2 - 2qL^2 + \frac{qL^2}{2} - \frac{qL^2}{4} + \frac{3}{8}qL^2 = 0$$

Internal action diagram

$$N/g_L$$

$$V/RL$$


$$M/8L^2$$



Qualitative Deflected Shape

