

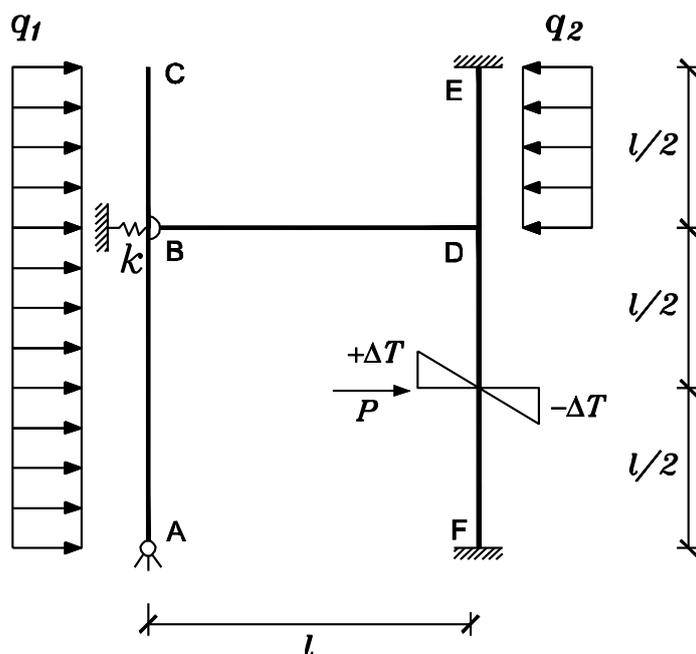
# TECNICA DELLE COSTRUZIONI

ALLIEVI CIVILI E EDILI

TEMA ESAME DEL 13 GENNAIO 2014

DOCENTI: PROF. ING. GIOVANNI A. PLIZZARI ESERCITATORI: ING. LUCA FACCONI  
ING. FAUSTO MINELLI ING. FABIO MACOBATTI

## Esercizio



$$q_1 = \frac{8}{9} q_2$$

$$\frac{\alpha \Delta T}{t} = \frac{14 q l^2}{3 EJ}$$

$$q_2 = 16q$$

$$k = 18 \frac{EJ}{l^3}$$

$$P = 16ql$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

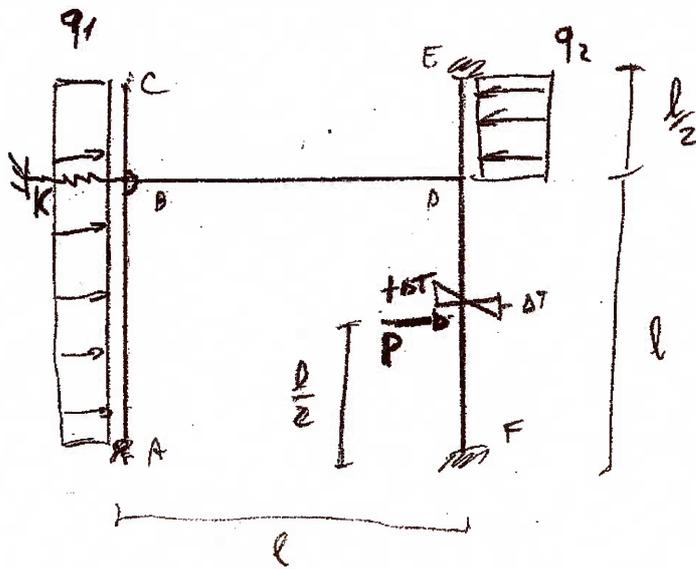
$$q_1 = \frac{8}{9} q_2 = \frac{128}{9} q$$

$$q_2 = 16q$$

$$P = 16ql$$

$$K = 15 \frac{EJ}{l^3}$$

$$\frac{d\Delta T}{t} = \frac{14}{3} \frac{qe_2}{EJ}$$



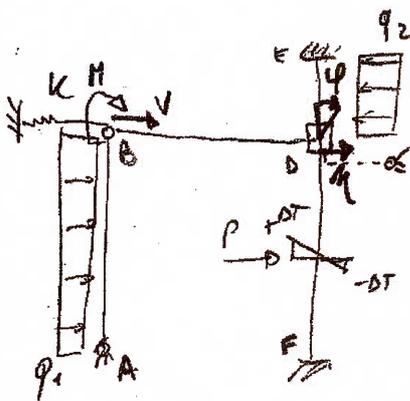
T.E. 01-13

BC APPENDICE I STATICA:  $V = q_1 \cdot \frac{l}{2} = \frac{64}{9} ql = 7,111 ql$

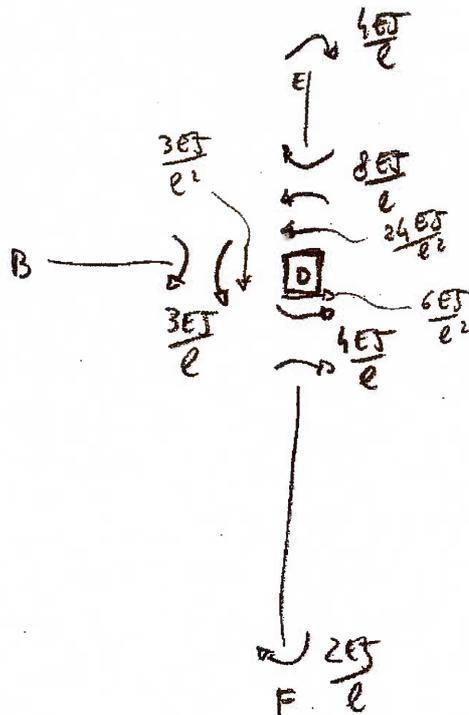
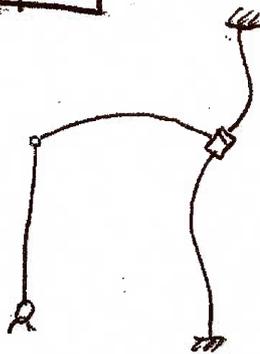
$$M = q_1 \cdot \frac{l^2}{8} = \frac{16}{9} ql^2 = 1,778 ql^2$$

$\oplus$  MOMENTO SU NODO

$\leftarrow +$  AZIONE SU NODO



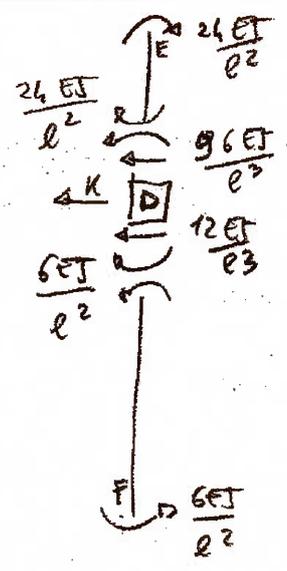
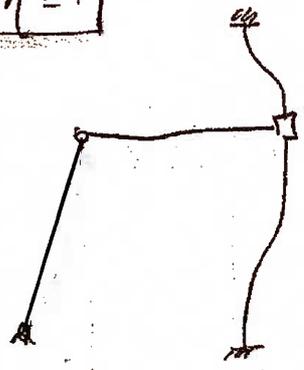
1)  $P=1$



$$M_{op} = \frac{8EJ}{l} + \frac{3EJ}{l} + \frac{4EJ}{l} = \frac{15EJ}{l}$$

$$h_{op} = \frac{24EJ}{l^2} - \frac{6EJ}{l^2} = \frac{18EJ}{l^2}$$

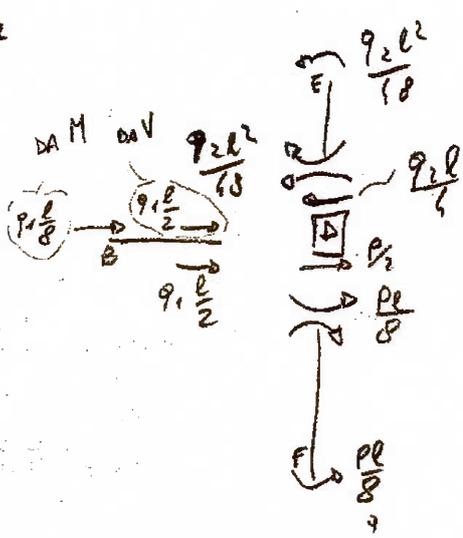
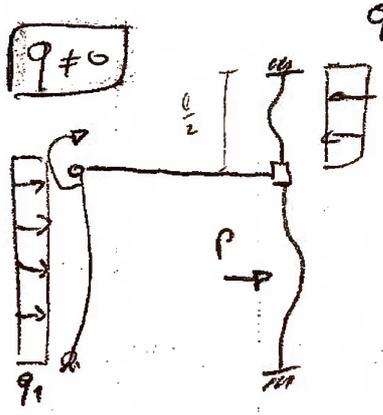
$\eta = 1$



$$M_{D, \eta} = \frac{24EI}{l^2} - \frac{6EI}{l^2} = \frac{18EI}{l^2}$$

$$h_{D, \eta} = \frac{96EI}{l^3} + \frac{12EI}{l^3} + K = 108 \frac{EI}{l^3} + K$$

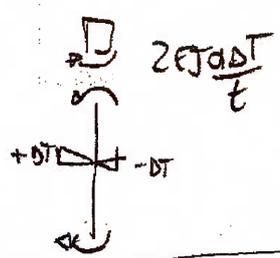
$q \neq 0$



$$M_{D, q} = \frac{pl}{8} + \frac{9_2 l^2}{68} = 29l^2 + \frac{1}{3} \frac{16}{48} q l = 29l^2 + \frac{1}{3} q l^2 = \frac{7}{3} q l^2$$

$$h_{D, q} = \frac{9_2 l}{4} - \frac{p}{2} - \frac{9}{8} q_1 l = \frac{1}{4} \cdot 16 - 8 + \frac{9}{8} \cdot \frac{128}{8} q l = 4 - 8 + 16 = -20 q l$$

$\Delta T \neq 0$



$$M_{D, \Delta T} = -2EJ \frac{\alpha \Delta T}{t} = -2 \cdot \frac{16}{3} \frac{q l^2}{EI} = -\frac{28}{3} q l^2$$

$$\left\{ \begin{aligned} \frac{15EI}{l} \varphi + \frac{18EI}{l^2} \eta &= -\frac{7}{3} q l^2 + \frac{28}{3} q l^2 = 7 q l^2 \\ \frac{18EI}{l^2} \varphi + 126 \frac{EI}{l^3} \eta &= 20 q l \end{aligned} \right. \Rightarrow$$

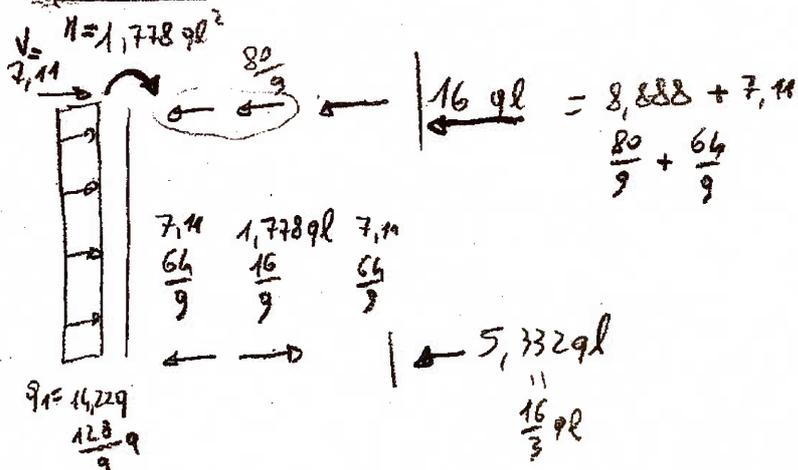
$$\varphi = \frac{1}{3} \frac{q l^3}{EI}$$

$$\eta = \frac{1}{9} \frac{q l^2}{EI}$$

SISTEMA RISOLVENTE

AZIONI SULLE ASTR

ASTA AB



ASTA BD



$$M = 5,332 ql \cdot x - 14,22 q \frac{x^2}{2} = 0$$

$$x(-7,11x + 5,332l) = 0$$

$$x = \frac{5,332l}{7,11} = 0,75l = \frac{3}{4}l$$

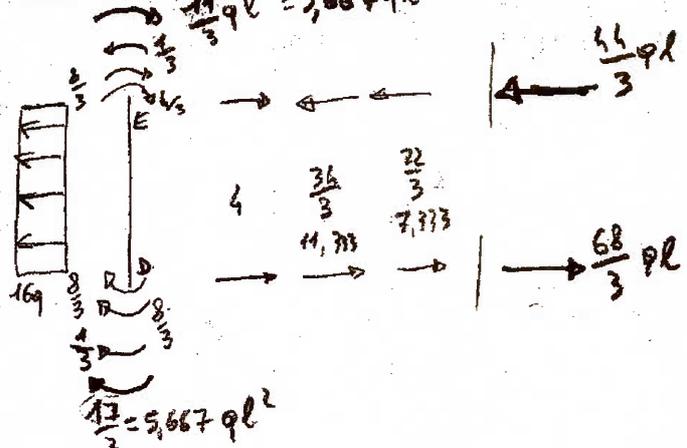
$$M_{max} - qx = 5,332l - 14,22qx = 0$$

$$x = \frac{5,332}{14,22} = 0,375l$$

$$M_{max} = 5,332l \cdot 0,375l - 14,22 \cdot \frac{0,375^2 l^2}{2} = 1,9395 - 0,9998 = ql^2$$

$$\frac{1}{48} \cdot 16 = \frac{1}{3}$$

ASTA DE



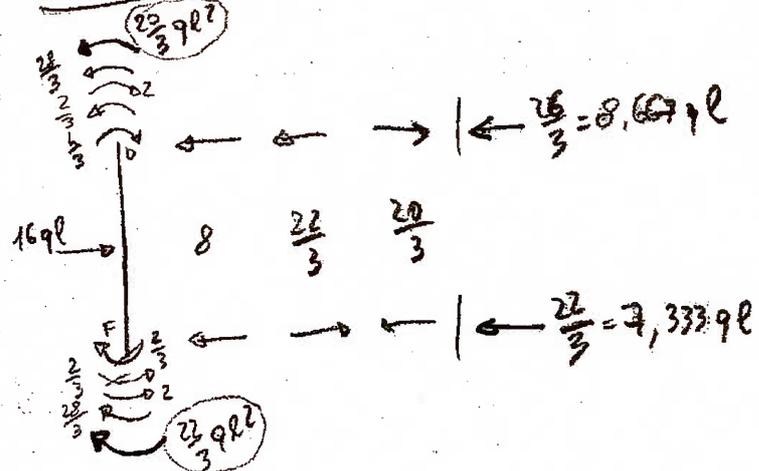
$$M_D = \frac{8}{3} + \frac{8}{3} + \frac{1}{3} = \frac{17}{3} ql^2 = 5,667 ql^2$$

$$M_E = \frac{4}{3} + \frac{8}{3} - \frac{1}{3} = \frac{11}{3} ql^2$$

$$V_E = \frac{-12 + 36 + 22}{3} = \frac{46}{3} ql = 15,333 ql$$

$$V_D = \frac{12 + 36 + 22}{3} = \frac{70}{3} ql = 23,333 ql$$

ASTA DF



$$M_D = \frac{4}{7} - \frac{2}{3} + 2 - \frac{28}{3} = \frac{4 - 2 + 6 - 28}{3} = -\frac{20}{3} ql^2$$

$$M_F = \frac{28}{3} - 2 = \frac{28 - 6}{3} = \frac{22}{3} ql^2$$

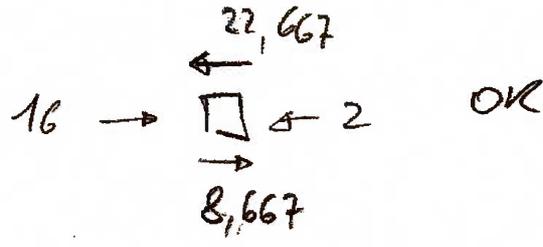
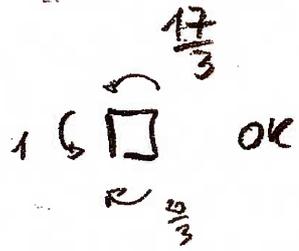
$$V_D = \frac{24 + 22 - 20}{3} = \frac{26}{3} ql = 8,667 ql$$

$$V_F = \frac{24 + 20 - 22}{3} = \frac{22}{3} ql = 7,333 ql$$

**EQUILIBRI, MODO D**

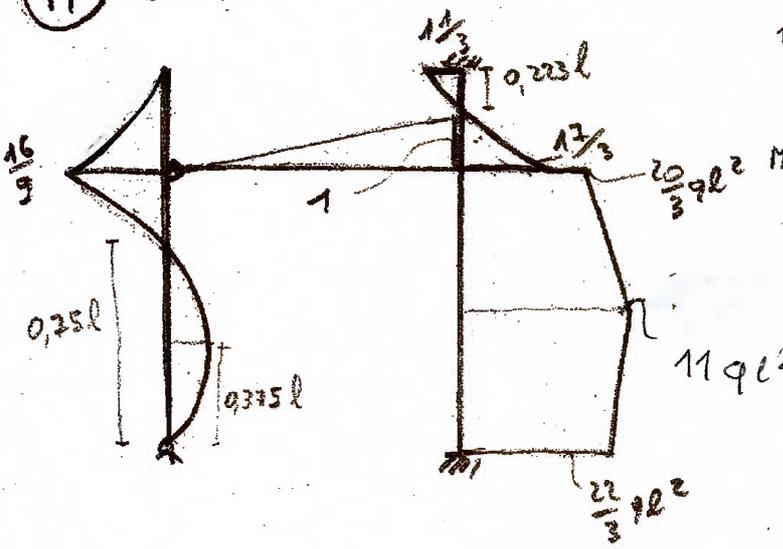
MOMENTO

TRASAZIONE

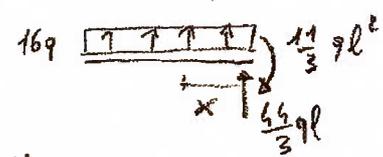


**DIAGRAMMI**

(M) [ql<sup>2</sup>]



ASTA DE



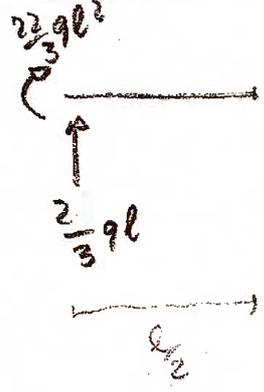
$M = 0$   
 $M = \frac{16q}{2}x^2 + \frac{44}{3}qlx - \frac{11}{3}ql^2 = 0$

$24x^2 + 44qlx - 11ql^2 = 0$

$x_{1,2} = \frac{-44 \pm \sqrt{1936 + 1056}}{48} = \frac{-44 \pm 54,7}{48}$

$= 0,223l$

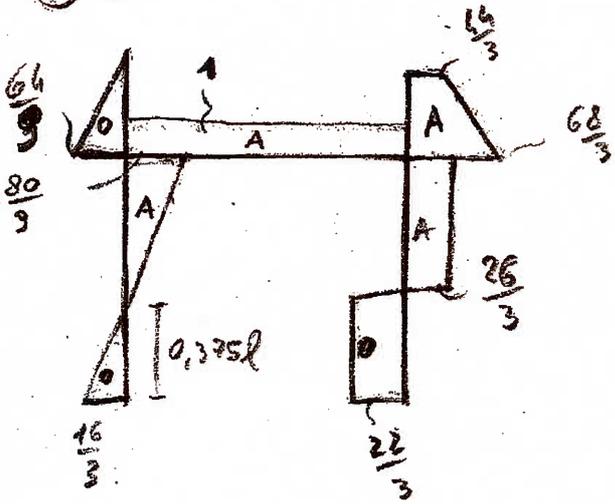
ASTA DE



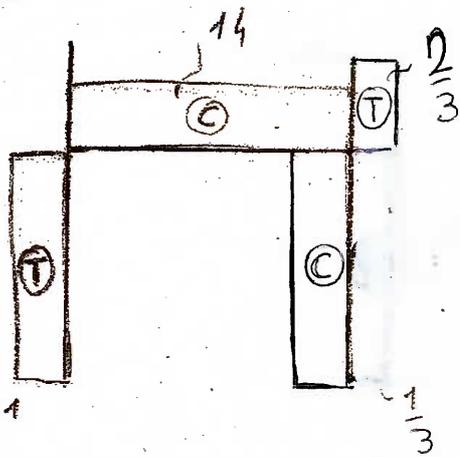
$M_{max \text{ ASTA DE}} = \frac{22}{3}ql^2 \cdot \frac{l}{2} + \frac{22}{3}ql^2 = \frac{33}{3}ql^2 = 11ql^2$

4 OK

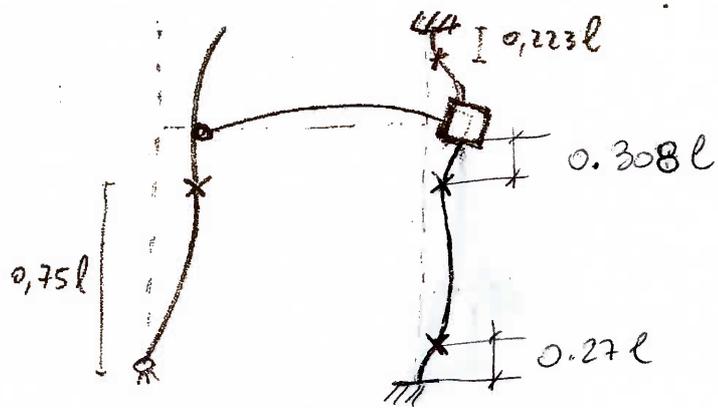
⑤ [ql]



⑥ [ql]



DEFORMATA



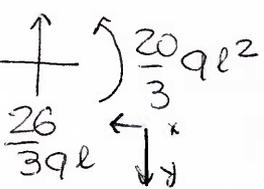
EFFETTO DT SU ASTA DF  
CONVENZIONI CURVATURA



$$y'' = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{t}$$

BISOGNA CONSIDERARE 2 CAMPI DI INTEGRAZIONE PER LA PRESENZA DEL CARICO CONCENTRATO  $P=16ql$

• DA NODO 'D'



$$M(x) = \frac{20}{3} ql^2 + \frac{26}{3} qlx$$

$$y'' = \frac{1}{EJ} \left( \frac{26}{3} qlx - \frac{20}{3} ql^2 \right) + \frac{28}{3} \frac{ql^2}{EJ}$$

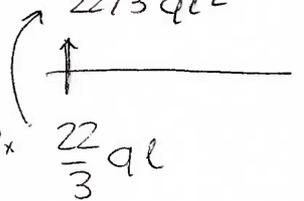
$y''=0 \rightarrow$  TROVO FLESSO

$$\frac{26}{3} qlx - \frac{20}{3} ql^2 + \frac{28}{3} ql^2 = 0 \rightarrow x = \frac{8}{26} l = \frac{4}{13} l = 0,308l$$

$$y'' > 0 \rightarrow x < \frac{4}{13} l$$

• DA NODO "F"

$\frac{22}{3} ql^2$



$M(x) = \frac{22}{3} ql^2 + \frac{22}{3} qlx$

$y'' = -\frac{1}{EI} \left[ \frac{22}{3} ql^2 + \frac{22}{3} qlx \right] + \frac{28}{3} \frac{ql^2}{EI}$

$y'' = 0 \rightarrow$  TROVO FUOCO

$$-\frac{22}{3} ql^2 - \frac{22}{3} qlx = -\frac{28}{3} ql^2$$

$$+22qlx = +6ql^2$$

$$x = \frac{6l}{22} = \frac{3}{11} l = \boxed{0.27l}$$

$y'' > 0 \rightarrow \boxed{x < \frac{3}{11} l}$