

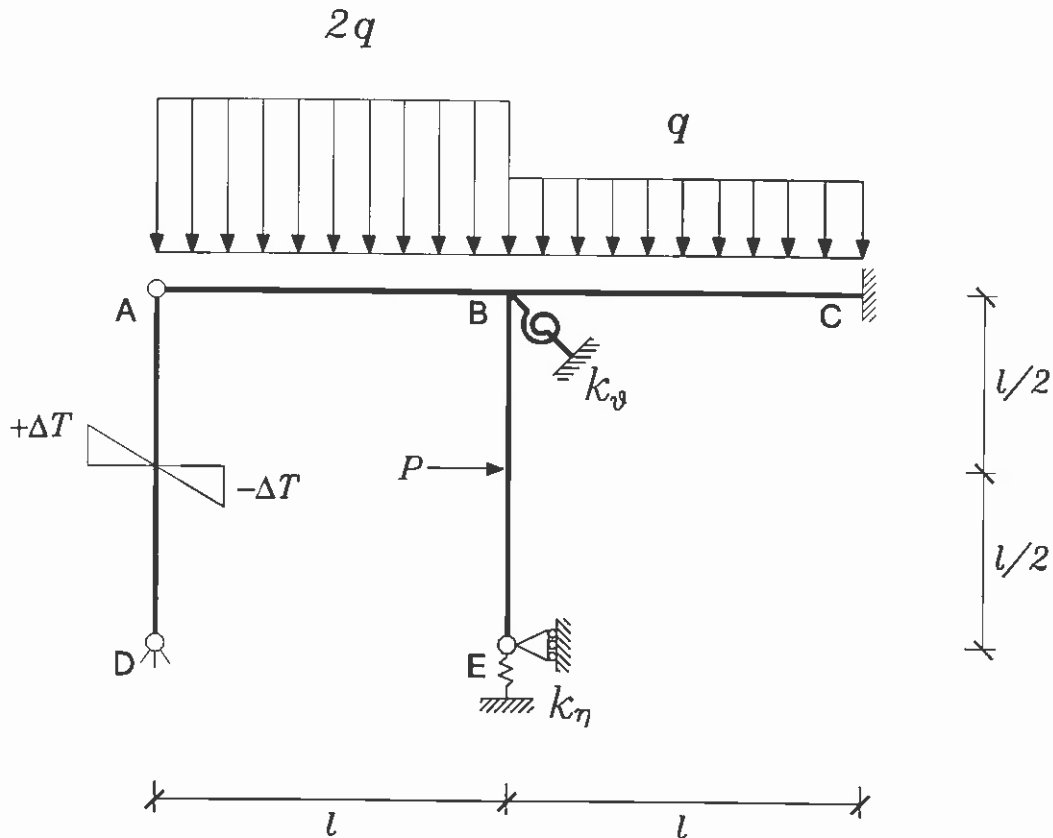
TECNICA DELLE COSTRUZIONI

ALLIEVI CIVILI

TEST INTERMEDIO DEL 24 APRILE 2014

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Esercizio



$$k_v = \frac{1}{10} \frac{EJ}{l}$$

$$k_n = 5 \frac{EJ}{l^3}$$

$$P = 2ql$$

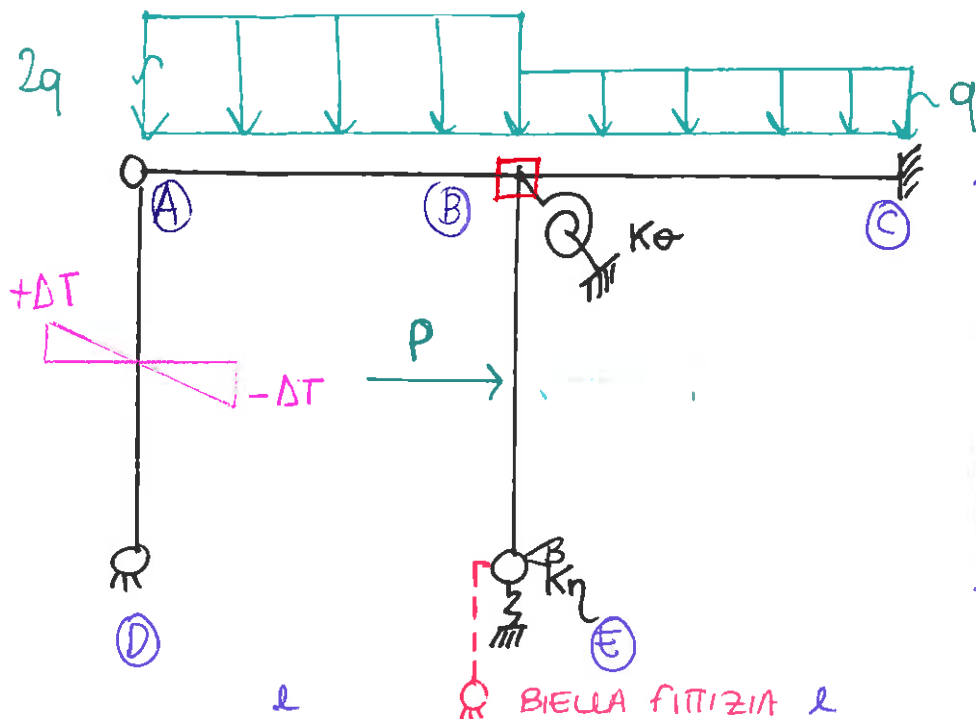
$$\frac{\alpha \Delta T}{l} = 42 \frac{ql^2}{EJ}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME - CIVILI 24 APRILE 2014



$$K_{\theta} = \frac{1}{10} \frac{EJ}{l}$$

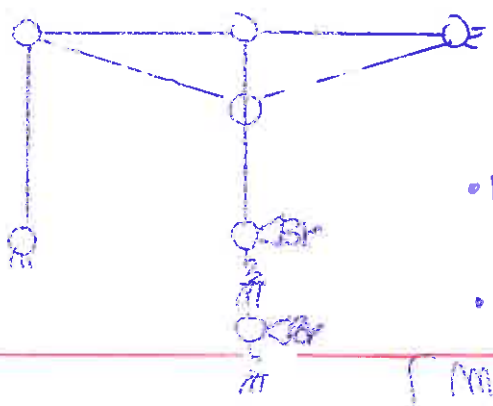
$$l/2 \quad K_{\eta} = \frac{5}{7} \frac{EJ}{l^3}$$

$$l/2 \quad \frac{\Delta T}{l} = 42 \frac{ql^2}{EJ}$$

$$P = 2ql$$

NB) AD → APPENDICE IOSTATICA!
 IL CARICO TERMICO NON INFLUENZA LE AZIONI MA SOLO LA DEFORMATA AD POSSO CONSIDERARLA UN CARRELLIO.

NODI FISSI O SPOSTABILI

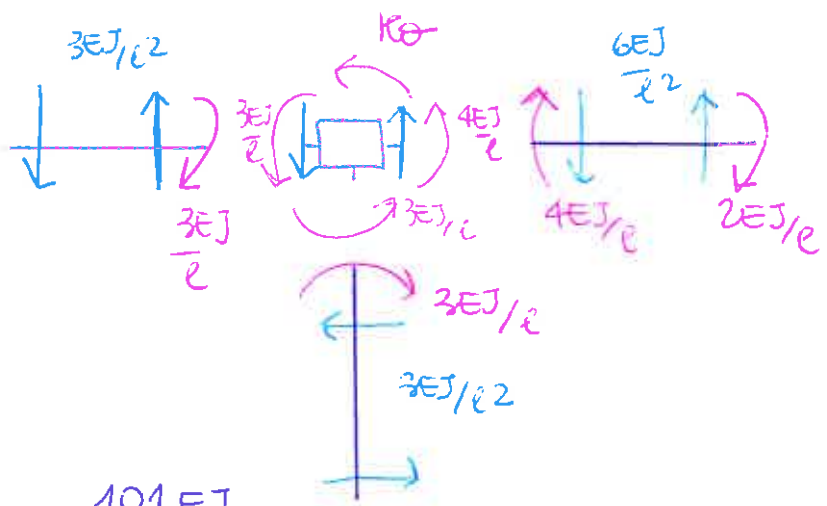
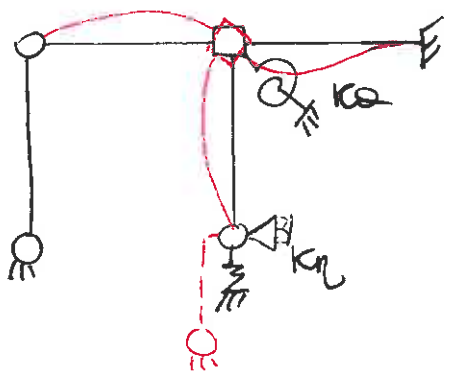


- NODI SPOSTABILI IN y → AGGIUNGO BIELLA FITTIZIA IN (E)
- CONVENZIONI: → ⊕ ← BIELLA

SISTEMA RISOLVENTE

$$\begin{cases} M_{BB} \psi_B + M_{B\eta} \eta + M_{B\theta} \theta = 0 \\ H_{\eta B} \psi_B + H_{\eta\eta} \eta + H_{\eta\theta} \theta = 0 \end{cases}$$

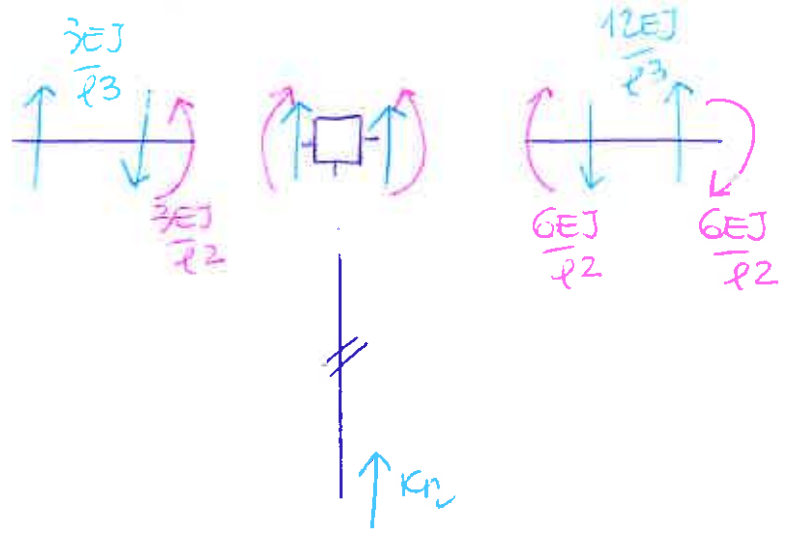
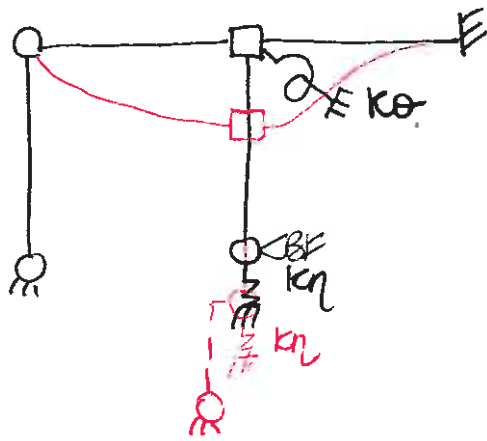
CASO 1: $\psi_B = 1$



$$M_{BB} = \frac{3EJ}{l} + \frac{4EJ}{l} + \frac{3EJ}{l} + K_{\theta} = \frac{101}{10} \frac{EJ}{l}$$

$$H_{\eta B} = \frac{3EJ}{l^2} - \frac{6EJ}{l^2} = -\frac{3EJ}{l^2}$$

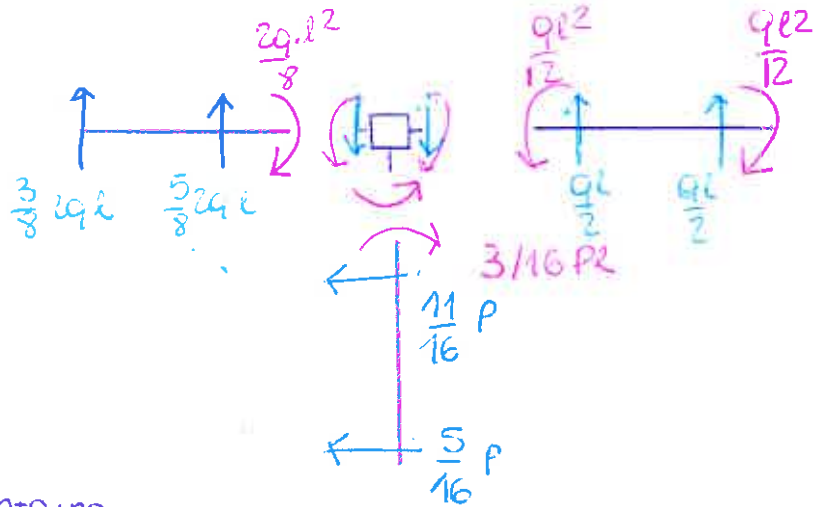
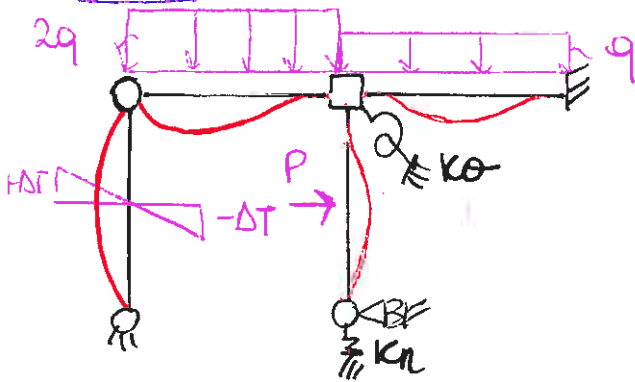
• CASO 2: $\eta = 1$



$$M_{B\eta} = \frac{6\eta EJ}{l^2} - \frac{3\eta EJ}{l^2} = \frac{3\eta EJ}{l^2}$$

$$H_{\eta\eta} = -\frac{3\eta EJ}{l^3} - \frac{12\eta EJ}{l^3} - K\eta = -20 \frac{EJ}{l^3}$$

• CASO 3: CARICHI $\neq 0$



ΔT non contribuisce alle azioni interne
ma solo alla deformata

$$M_{B\phi} = \frac{ql^2}{4} + \frac{3}{16} Pl - \frac{ql^2}{12} = \frac{1}{4} ql^2 + \frac{6}{16} ql^2 - \frac{ql^2}{12} = \frac{13}{24} ql^2$$

$$H_{\eta\phi} = \frac{5}{4} ql + \frac{ql}{2} = \frac{7}{4} ql$$

$$\begin{cases} m_{BB} \varphi_B + m_{B\eta} \eta + m_{B\emptyset} = 0 \\ H_{\eta B} \varphi_B + H_{\eta\eta} \eta + H_{\eta\emptyset} = 0 \end{cases}$$

$$\begin{cases} \left(\frac{101 EJ}{10 l} \right) \varphi_B + \left(\frac{3 EJ}{l^2} \right) \eta = - \frac{13}{24} q l^2 \\ \left(- \frac{3 EJ}{l^2} \right) \varphi_B + \left(- \frac{20 EJ}{l^3} \right) \eta = - \frac{7}{4} q l^2 \end{cases}$$

$$\begin{cases} \frac{101}{10} \varphi_B l + 3 \eta = - \frac{13}{24} \frac{q l^4}{EJ} \\ + 3 \varphi_B l + 20 \eta = + \frac{7}{4} \frac{q l^4}{EJ} \cdot \left(\frac{3}{20} \right) \end{cases}$$

$$\begin{cases} \frac{101}{10} \varphi_B l + 3 \eta = - \frac{13}{24} \frac{q l^4}{EJ} \\ \frac{9}{20} \varphi_B l + 3 \eta = + \frac{21}{80} \frac{q l^4}{EJ} \end{cases}$$

$$\begin{aligned} \left(\frac{101}{10} - \frac{9}{20} \right) \varphi_B l &= \left(- \frac{13}{24} - \frac{21}{80} \right) \frac{q l^4}{EJ} \\ \frac{202-9}{20} \varphi_B l &= \frac{-130-63}{240} \frac{q l^4}{EJ} \end{aligned}$$

$$\varphi_B = \frac{1}{l} \cdot \left(- \frac{193}{240} \right) \frac{q l^4}{EJ} \cdot \frac{20}{193} \quad \rightsquigarrow \quad \boxed{\varphi_B = - \frac{1}{12} \frac{q l^3}{EJ}}$$

Sostituisco φ_B .

$$3 \left(- \frac{1}{12} \frac{q l^3}{EJ} \right) l + 20 \eta = \frac{7}{4} \frac{q l^4}{EJ}$$

$$- \frac{1}{4} \frac{q l^4}{EJ} + 20 \eta = \frac{7}{4} \frac{q l^4}{EJ} \quad \rightsquigarrow \quad \boxed{\eta = \frac{1}{10} \frac{q l^4}{EJ}}$$

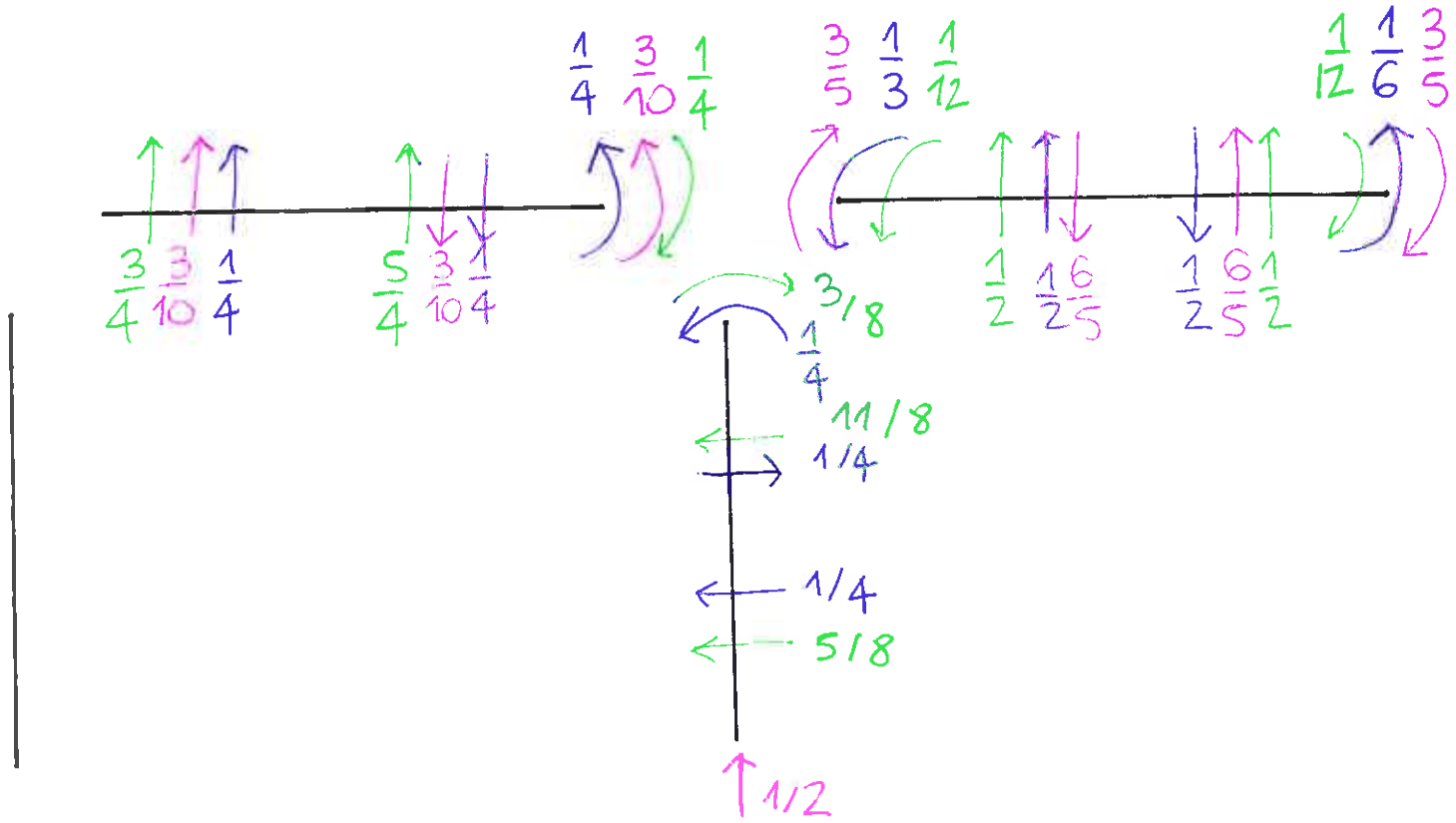
• MOMENTO MOLLA TORSIONALE:

$$\frac{1}{10} \frac{EJ}{l} \cdot \frac{1}{12} \frac{q l^3}{EJ} = \boxed{\frac{1}{120} q l^2} \quad \text{IN SENSO ORARIO} \quad \curvearrowright$$

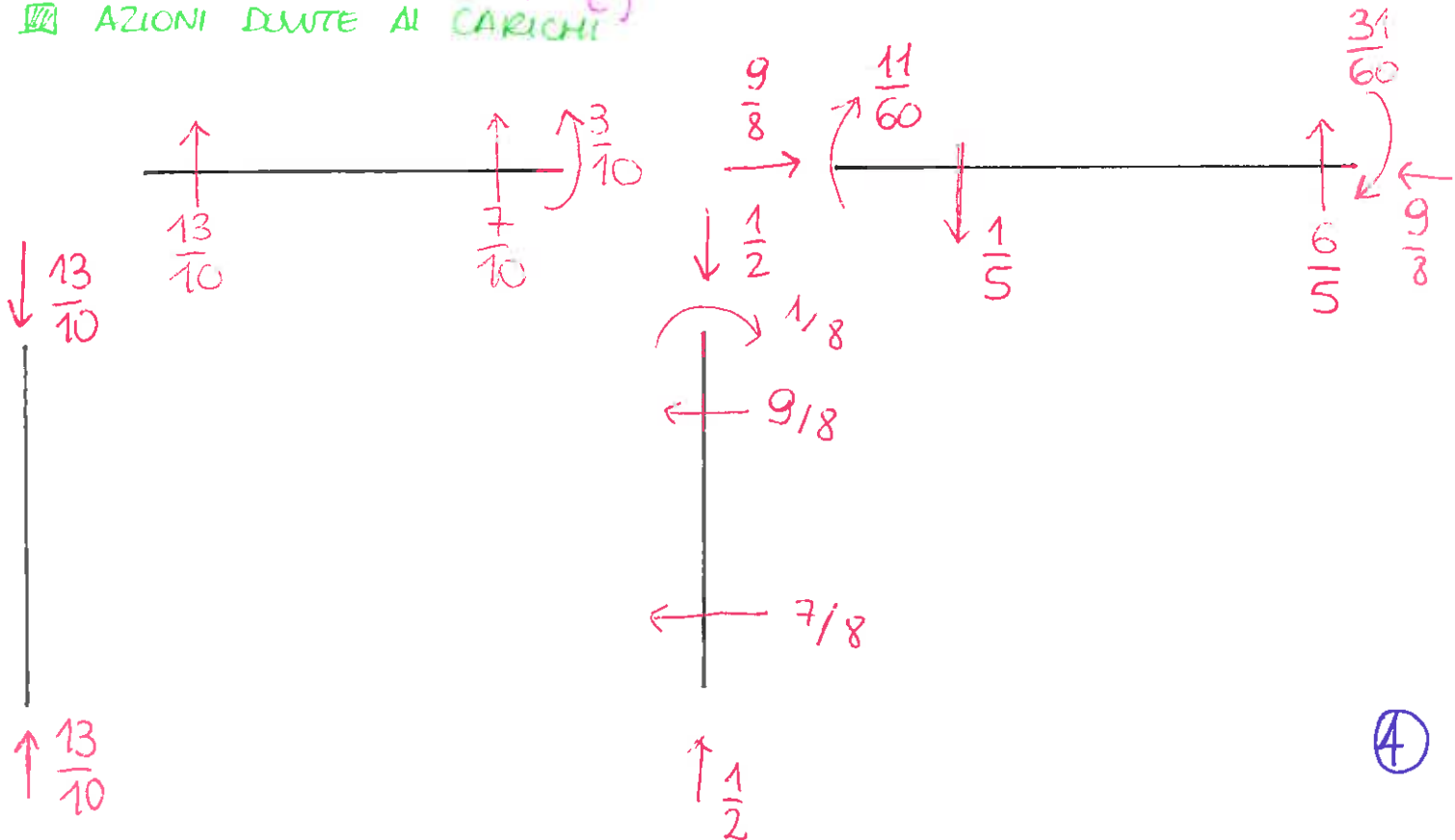
• FORZA MOLLA ASSIALE

$$\frac{5 EJ}{l^3} \cdot \frac{1}{10} \frac{q l^4}{EJ} = \boxed{\frac{1}{2} q l} \quad \text{VERSO L'ALTO} \quad \uparrow$$

$M [ql^2]$
 $V [ql]$
 $N [ql]$

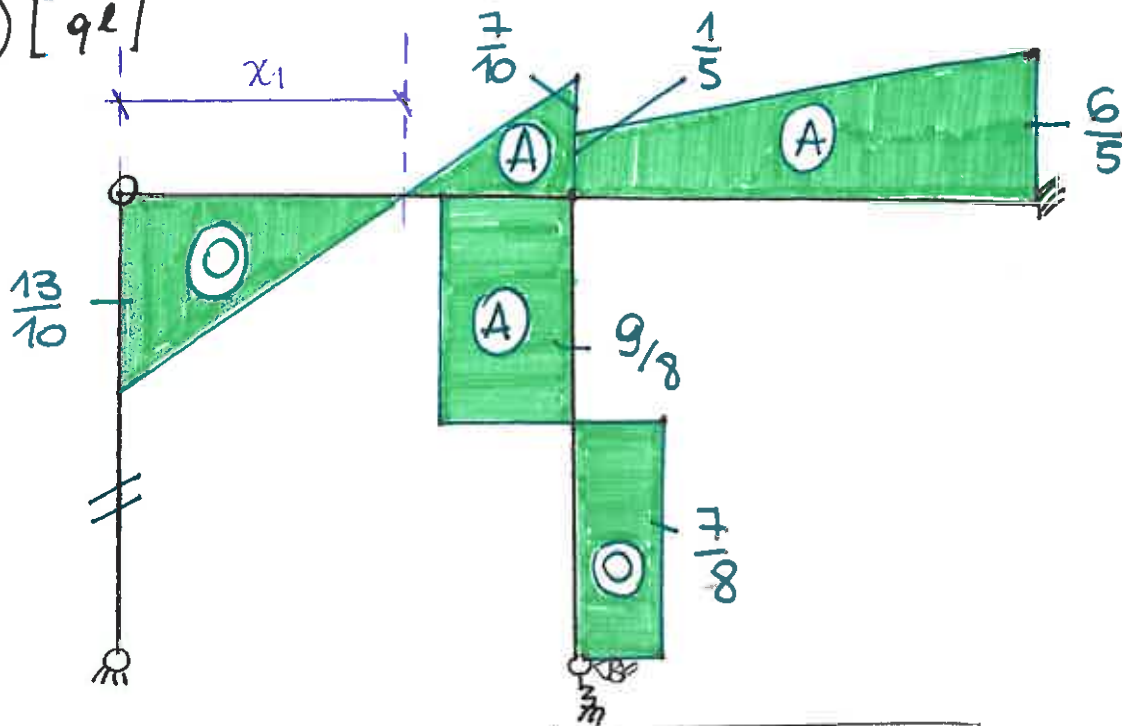


- AZIONI DOWTE A $\varphi_B = -\frac{1}{12} \frac{ql^3}{EJ}$
- AZIONI DOWTE A $\delta = \frac{1}{10} \frac{ql^4}{EJ}$
- AZIONI DOWTE AI CARICHI



DIAGRAMMI AZIONI INTERNE

Ⓟ [ql]

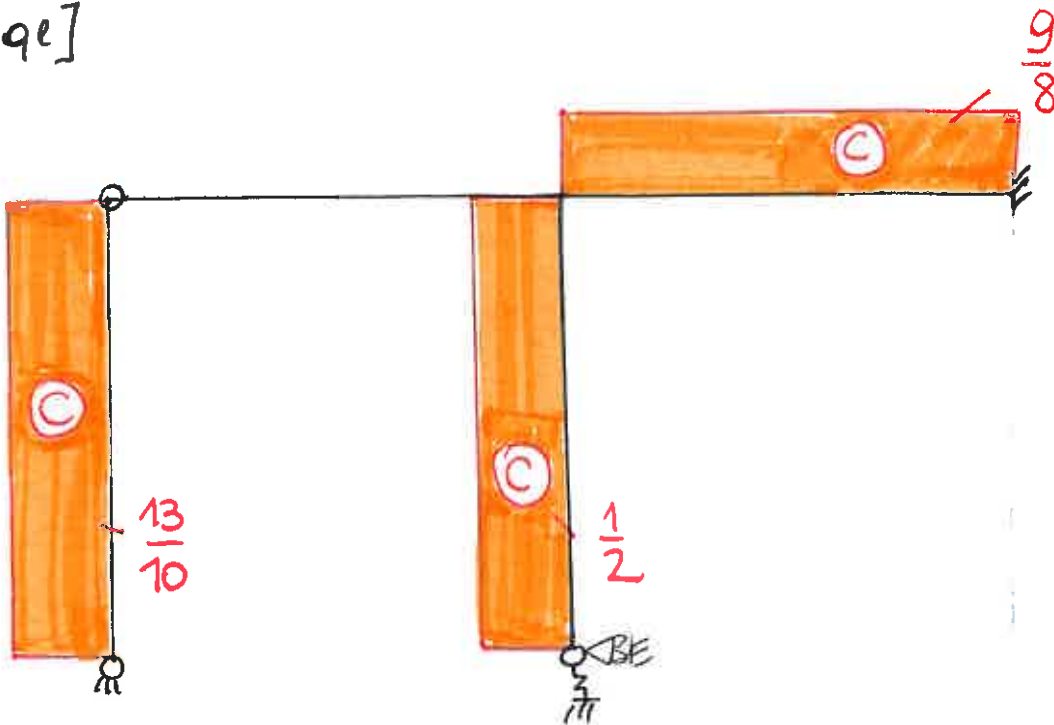


$x_1) \frac{13}{10} : x_1 = \left(\frac{7}{10} + \frac{13}{10}\right) : l \rightsquigarrow$

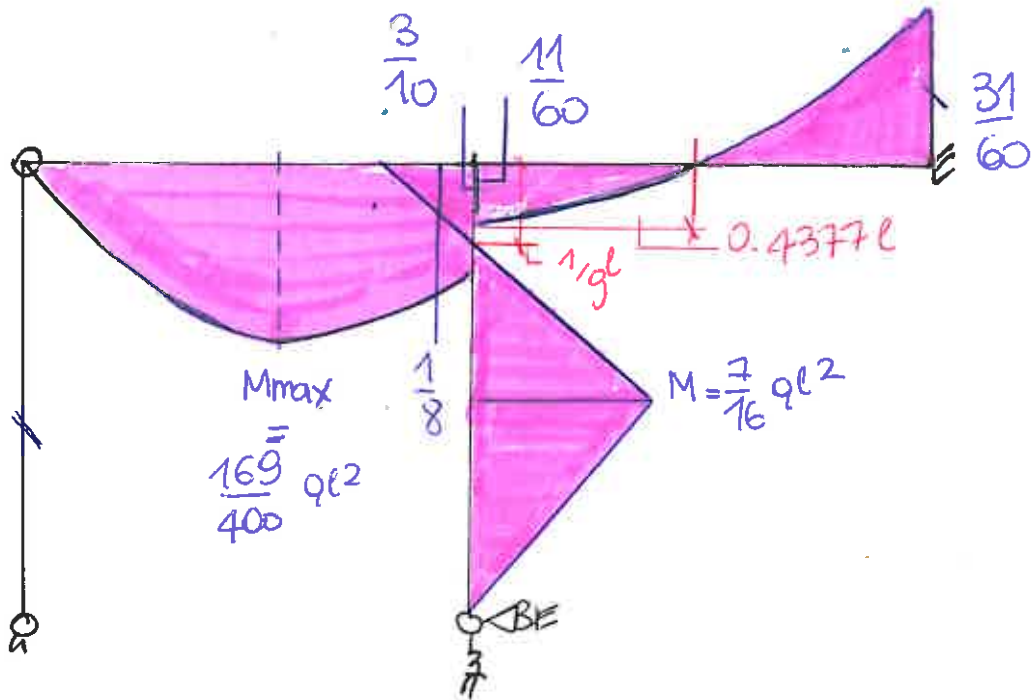
$x_1 = \frac{13 \cdot 10}{20} = \frac{13}{2} l$

PUNTO IN CUI
HO Mmax SU ASTA
AB

Ⓝ [ql]

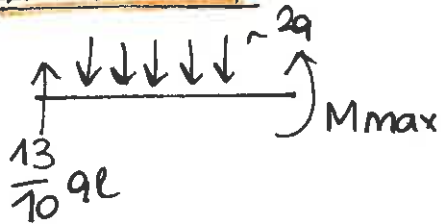


Ⓜ [ql²]



ASTA AB

CALCOLO M_{max}

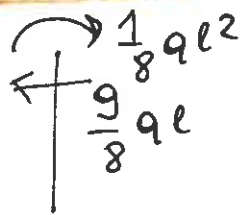


$$\frac{13}{10} ql \cdot \left(\frac{13x}{20}\right) - qx \left(\frac{13x}{20}\right) \left(\frac{13}{40}\right) l - M_{max} = 0$$

$$M_{max} = \frac{169}{200} ql^2 - \frac{169}{400} ql^2$$

$$M_{max} = \left(\frac{338 - 169}{400}\right) ql^2 = \frac{169}{400} ql^2 = 0.4225 ql^2$$

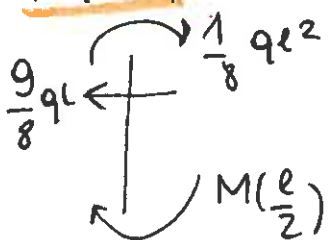
ASTA BE



$$\frac{1}{8} ql^2 - \frac{9}{8} qlx = 0$$

$$\frac{9}{8} x = \frac{1}{8} l \quad \boxed{x = \frac{1}{9} l} \rightarrow \text{PUNTO DI FLESSO}$$

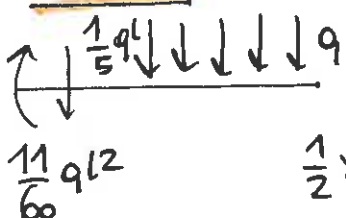
M(l/2)



$$\frac{1}{8} ql^2 + M\left(\frac{l}{2}\right) - \frac{9}{8} ql \cdot \frac{l}{2} = 0$$

$$M\left(\frac{l}{2}\right) = \frac{9}{16} ql^2 - \frac{1}{8} ql^2 = \frac{7}{16} ql^2$$

ASTA BC



$$\frac{11}{60} ql^2 - \frac{1}{5} qlx - \frac{qx^2}{2} = 0$$

$$\frac{1}{2} x^2 + \frac{1}{5} lx - \frac{11}{60} l^2 = 0$$

$$30x^2 + 12lx - 11l^2 = 0$$

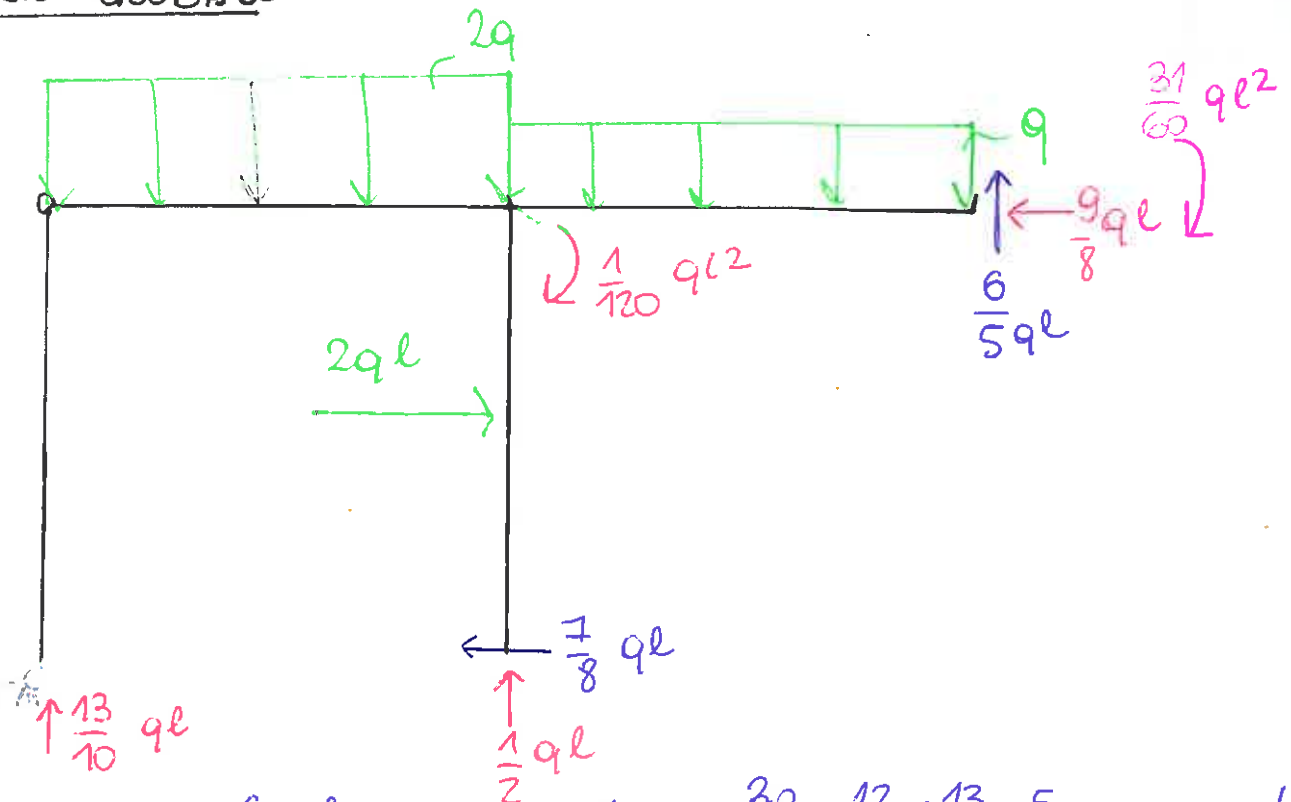
$$x_{1/2} =$$

$$\frac{-12l \pm \sqrt{144 - 4(30)(-11)}}{60}$$

$$\boxed{x = 0.4377 l \rightarrow \text{P.T.O DI FLESSO}}$$

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EQUILIBRIO GLOBAL

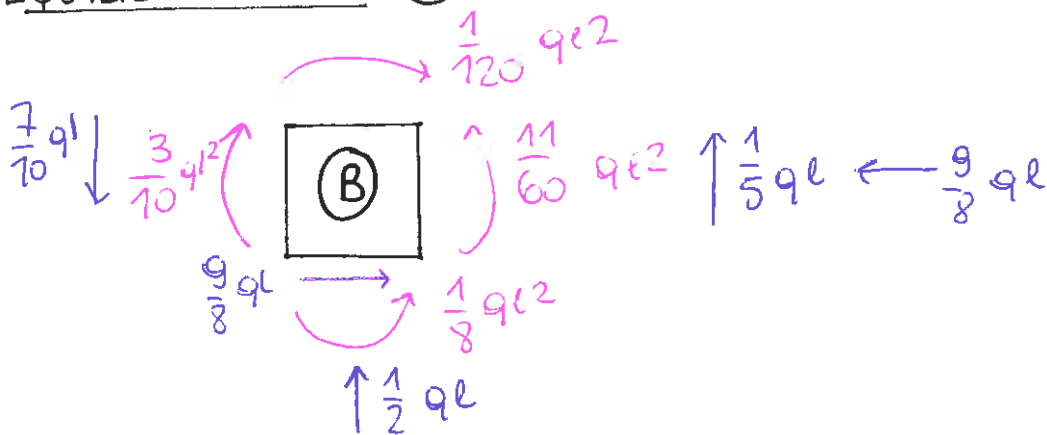


$$\Sigma F_v = 0) \quad 2ql + ql - \frac{6}{5}ql - \frac{13}{10}ql - \frac{1}{2}ql = \frac{20 - 12 - 13 - 5}{10}ql = 0 \quad \text{ok!}$$

$$\Sigma F_h = 0) \quad 2ql - \frac{9}{8}ql - \frac{7}{8}ql = 0 \quad \text{ok!}$$

$$\Sigma M_{(E)} = 0) \quad \left(+ \frac{13}{10}ql^2 - (2ql)\frac{l}{2} + ql^2 + \frac{1}{120}ql^2 - \frac{6}{5}ql^2 - \frac{9}{8}ql^2 + \frac{31}{60}ql^2 \right. \\ \left. + \frac{2ql^2}{2} = 0 \quad \text{ok.} \right.$$

EQUILIBRIO NODO (B)



$$\Sigma F_v = 0) \quad \frac{7}{10}ql - \frac{1}{5}ql - \frac{1}{2}ql = 0 \quad \text{ok!}$$

$$\Sigma F_h = 0) \quad \frac{9}{8}ql - \frac{9}{8}ql = 0 \quad \text{ok!}$$

$$\Sigma M = 0) \quad \frac{3}{10}ql^2 + \frac{1}{120}ql^2 - \frac{11}{60}ql^2 - \frac{1}{8}ql^2 = 0 \quad \text{ok!}$$

