

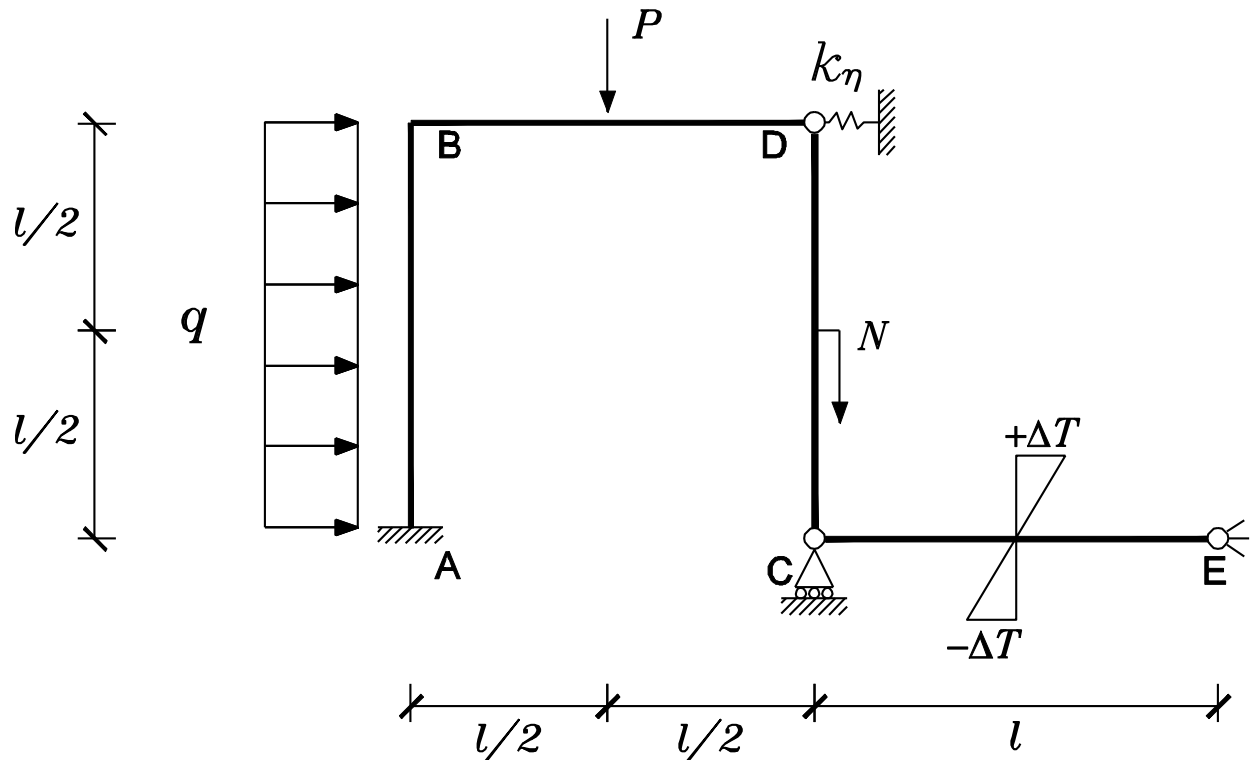
# TECNICA DELLE COSTRUZIONI

TEST INTERMEDIO DEL 25 NOVEMBRE 2014

DOCENTE: ING. FAUSTO MINELLI

ESERCITATORE: ING. ANTONIO CONFORTI

Esercizio: FILA 1



$$P = 84ql$$

$$N = 7ql$$

$$k_n = 21 \frac{EJ}{l^3}$$

$$\frac{\alpha \Delta T}{h} = \frac{30}{7} \frac{ql^2}{EJ}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

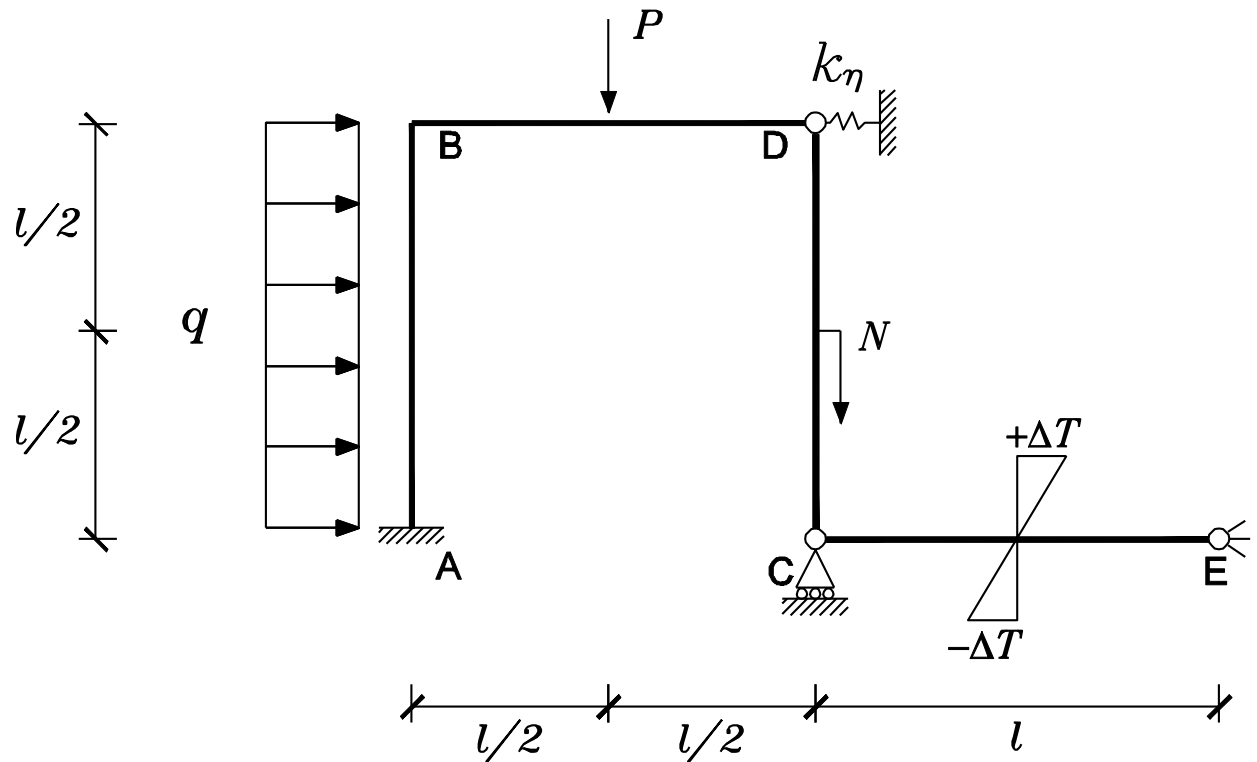
# TECNICA DELLE COSTRUZIONI

TEST INTERMEDIO DEL 25 NOVEMBRE 2014

DOCENTE: ING. FAUSTO MINELLI

ESERCITATORE: ING. ANTONIO CONFORTI

Esercizio: FILA 2



$$P = 20ql$$

$$N = 5ql$$

$$k_n = 15 \frac{EJ}{l^3}$$

$$\frac{\alpha \Delta T}{h} = \frac{1}{2} \frac{ql^2}{EJ}$$

Dato il telaio in figura

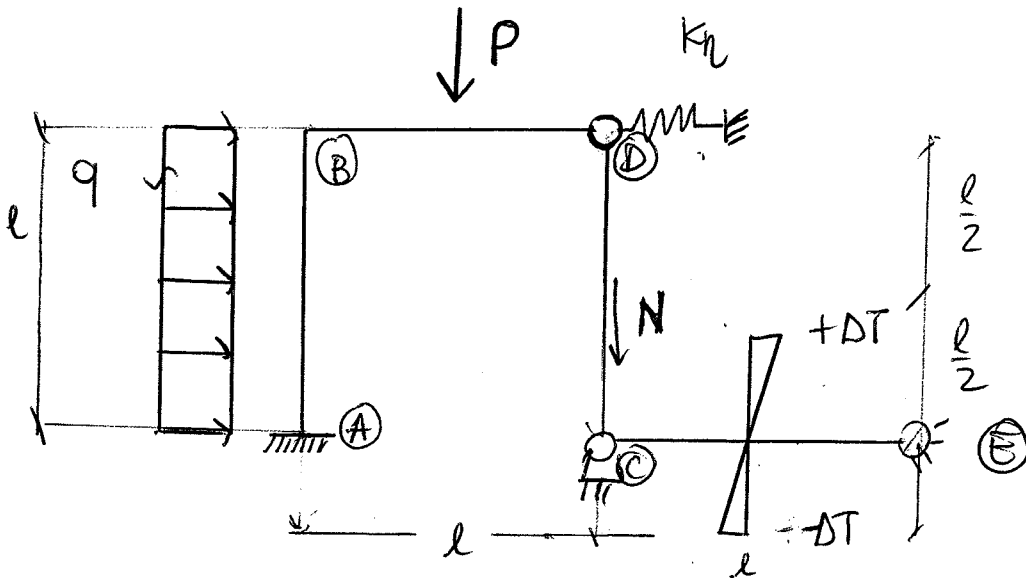
Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

# TEMA ESAME - TEST INTERMEDIO

25/11/14

FILA 1



$$K_\eta = 21 \frac{EJ}{l^3}$$

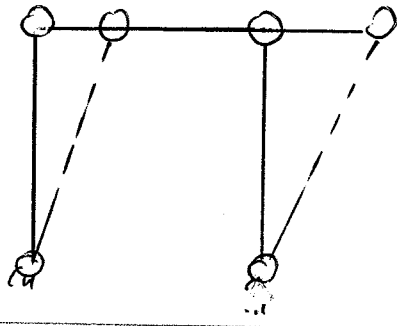
$$P = 84 ql$$

$$N = 7ql$$

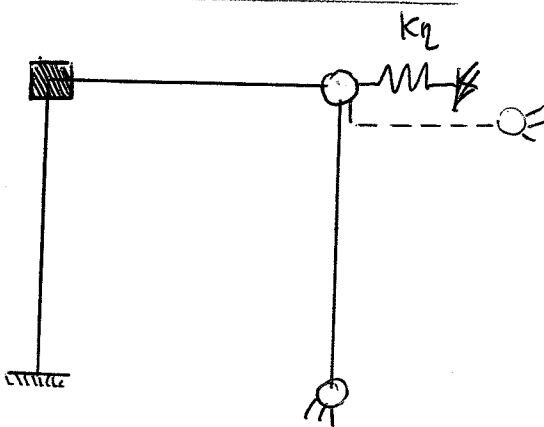
$$\frac{\alpha \Delta T}{t} = \frac{30}{7} \frac{ql^2}{EJ}$$

CE →  
APPENDICE  
ISOSTATICA

VALUTO SE IL SISTEMA È A NODI FISSI O SPOSTABILI



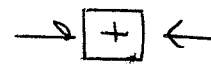
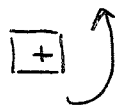
NODI SPOSTABILI IN X  
L'AGGIUNGO BIELLA FIRTZIA



SISTEMA RISOLVENTE.

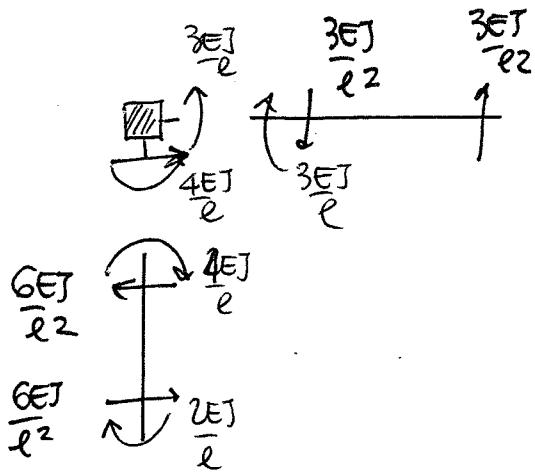
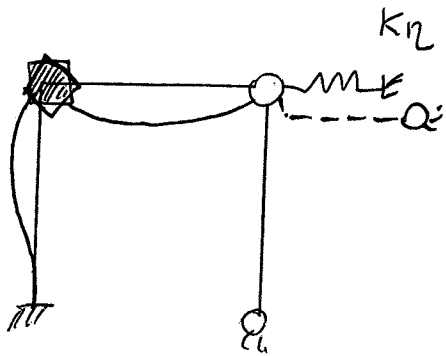
$$\begin{cases} M_{BB} \varphi_B + M_{B\eta} \eta + M_{B\phi} \phi = 0 \\ H_{\eta B} \varphi_B + H_{\eta\eta} \eta + H_{\eta\phi} \phi = 0 \end{cases}$$

CONVENZIONI DI SEGNO



MOMENTO  
SUL NODO

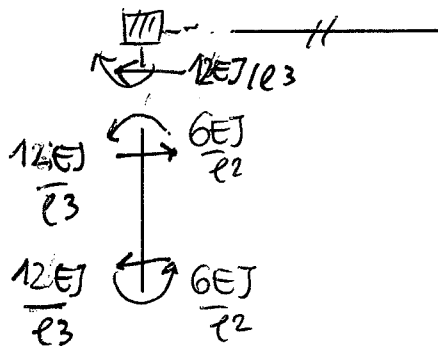
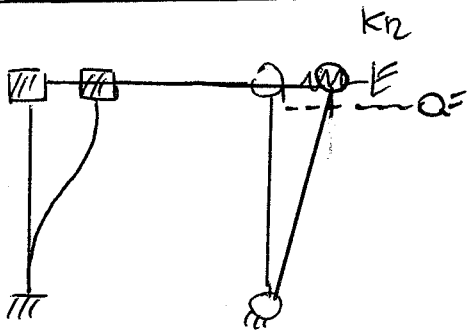
CASO 1 :  $\psi_B = 1, \eta = 0$



$$M_{BB} = \frac{4EI}{l} + \frac{3EI}{l} = \frac{7EI}{l}$$

$$H_{\eta B} = \frac{6EI}{l^2}$$

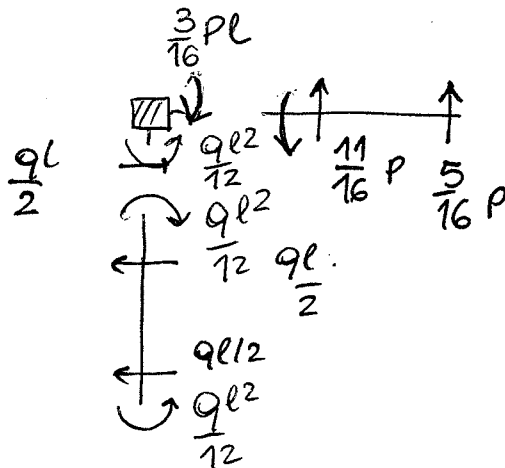
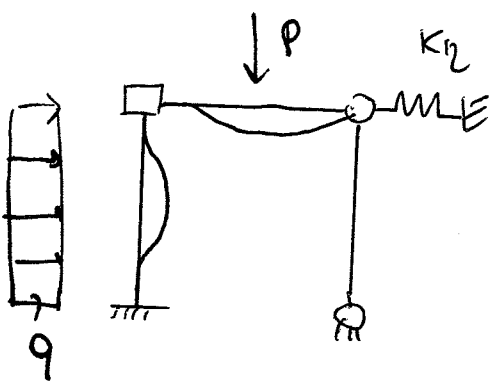
CASO 2 :  $\psi_B = 0, \eta = 1$



$$M_{B\eta} = -\frac{6EI}{l^2}$$

$$H_{\eta\eta} = -\frac{12EI}{l^3} - k\eta$$

CASO 3 : CARICHI  $\neq 0$



$$M_{B\phi} = \frac{9EI}{l^2} - \frac{3}{16}Pl$$

$$H_{\eta\phi} = \frac{9EI}{l}$$

# SISTEMA RISOLVENTE

$$\begin{cases} \sum M_{BB} \varphi_B + M_{B\eta} \eta + M_{B\theta} = 0 \\ H_{\eta B} \varphi_B + H_{\eta\eta} \eta + H_{\eta\theta} = 0 \end{cases}$$

$$\begin{cases} \frac{7EJ}{l} \varphi_B - \frac{6EJ}{l^2} \eta + \left( \frac{ql^2}{12} - \frac{3}{16} pl \right) = 0 \\ \frac{6EJ}{l^2} \varphi_B + \left( -\frac{12EJ}{l^3} - K_{\eta} \right) \eta + \frac{ql}{2} = 0 \end{cases}$$

$$\begin{cases} \frac{7EJ}{l} \varphi_B - \frac{6EJ}{l^2} \eta + \left( \frac{ql^2}{12} - \frac{3}{16} \cdot 84 ql^2 \right) = 0 \\ \frac{6EJ}{l^2} \varphi_B + \left( -\frac{12EJ}{l^3} - 21 \frac{EJ}{l^3} \right) \eta + \frac{ql}{2} = 0 \end{cases}$$

$$\begin{cases} \frac{7EJ}{l} \varphi_B - \frac{6EJ}{l^2} \eta + \left( \frac{ql^2}{12} - \frac{63}{4} ql^2 \right) = 0 \\ \frac{6EJ}{l^2} \varphi_B - 33 \frac{EJ}{l^3} \eta + \frac{ql}{2} = 0 \end{cases}$$

$$\begin{cases} 7\varphi_B l - 6\eta - \frac{47}{3} \frac{ql^4}{EJ} = 0 \quad \cdot 6 \\ 6\varphi_B l - 33\eta + \frac{ql^4}{2EJ} = 0 \quad \cdot 7 \end{cases}$$

$$\begin{cases} 42\varphi_B l - 36\eta - 94 \frac{ql^4}{EJ} = 0 \\ 42\varphi_B l - 231\eta + \frac{7}{2} \frac{ql^4}{EJ} = 0 \end{cases} \quad \ominus$$

$$+195\eta - \frac{195}{2} \frac{ql^4}{EJ} = 0$$

$$195\eta = \frac{195}{2} \frac{ql^4}{EJ}$$

$$\begin{aligned} \eta &= \frac{1}{2} \frac{ql^4}{EJ} \\ \varphi_B &= \frac{8}{3} \frac{ql^3}{EJ} \end{aligned}$$

Ricavo  $\varphi_B$

$$7\varphi_B l - 6 \cdot \frac{1}{2} \frac{ql^4}{EJ} - \frac{47}{3} \frac{ql^4}{EJ} = 0$$

$$7\varphi_B l - 3 \frac{ql^4}{EJ} - \frac{47}{3} \frac{ql^4}{EJ} = 0$$

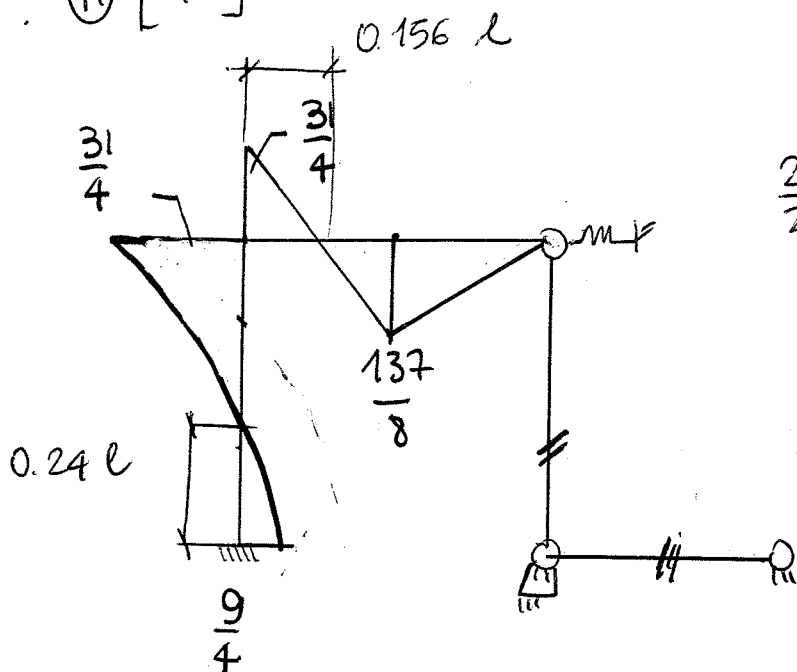
$$7\varphi_B l = \frac{56}{3} \frac{ql^4}{EJ}$$

$$\varphi_B = \frac{8}{3} \frac{ql^3}{EJ}$$

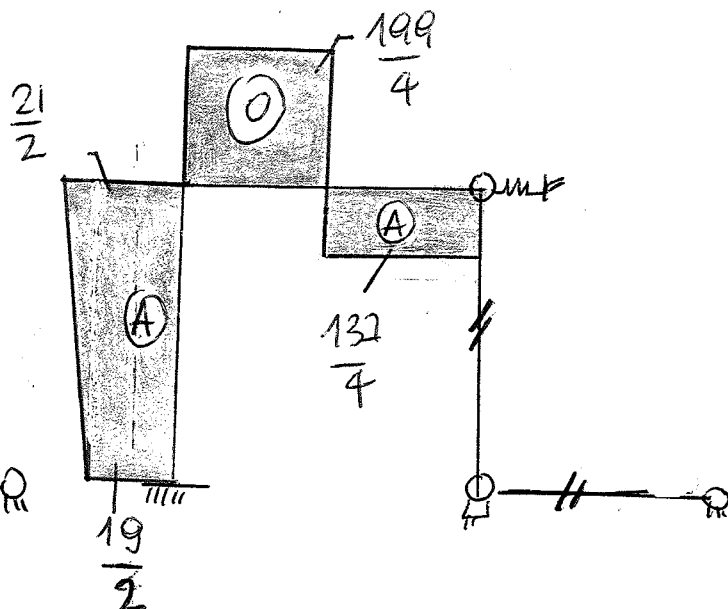
REAZIONE DELLA MOCCA

$$\frac{1}{2} \frac{ql^4}{EJ} \cdot \frac{21 EJ}{l^3} = \frac{21}{2} ql \quad \leftarrow$$

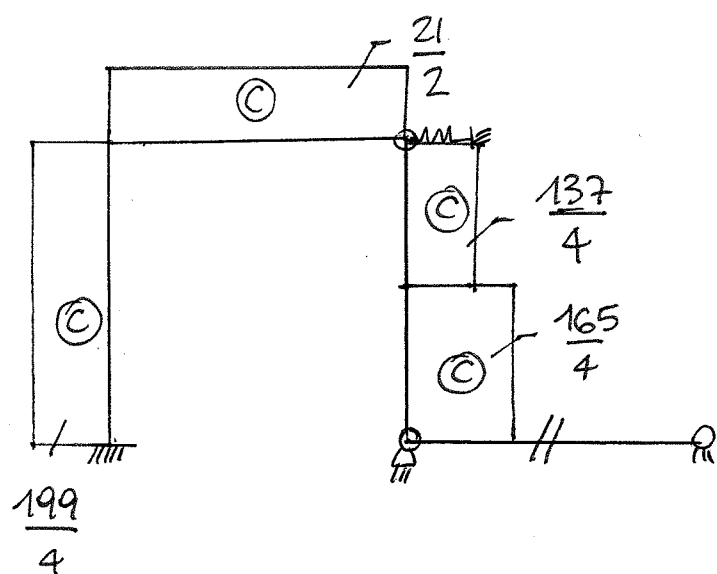
Ⓜ [ql<sup>2</sup>]



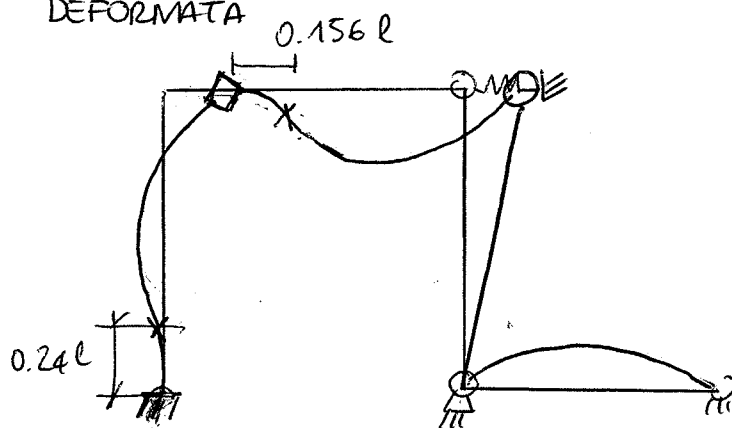
Ⓥ [ql]



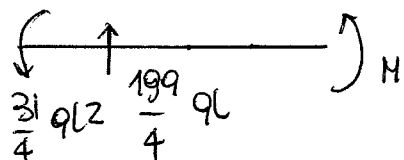
Ⓝ



DEFORMATA



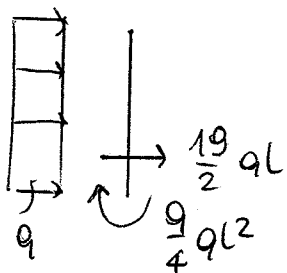
• MOMENTO IN MEZZERIA ASTA BD



$$M + \frac{31}{4} ql^2 - \frac{199}{4} ql \cdot \frac{l}{2} = 0$$

$$M = \frac{199}{8} ql^2 - \frac{31}{4} ql^2 = \frac{137}{8} ql^2$$

• CALCOLO DEL FLESSO ASTA AB



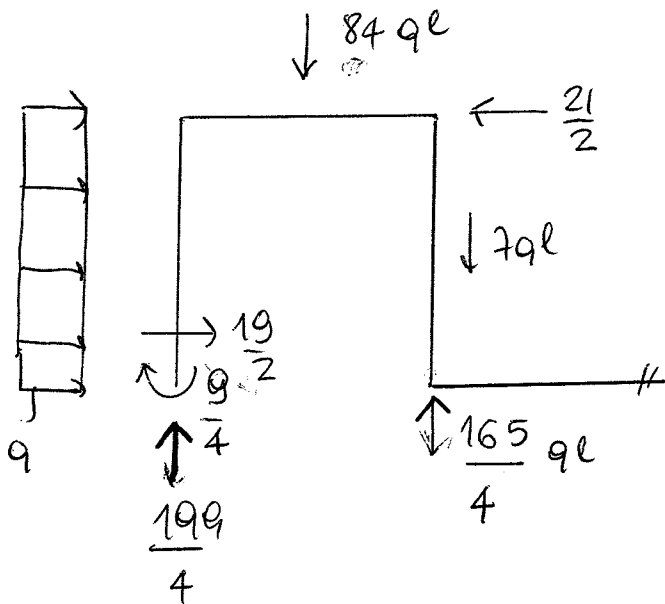
$$\frac{9}{4} ql^2 - \frac{19}{2} qlx + \frac{qx^2}{2} = 0$$

$$\frac{qx^2}{2} - \frac{19}{2} qlx + \frac{9}{4} ql^2 = 0$$

$$2x^2 - 38lx + 9l^2 = 0$$

$$\frac{+38 \pm \sqrt{(38)^2 - 4(2)(9)}}{4} \Rightarrow \boxed{0.24l} \rightarrow \text{Flesso}$$

# EQUILIBRIO GLOBALE

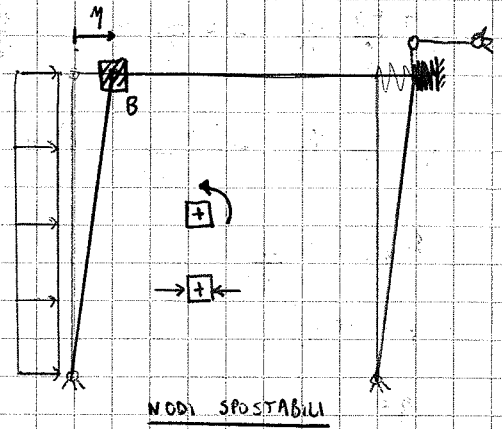
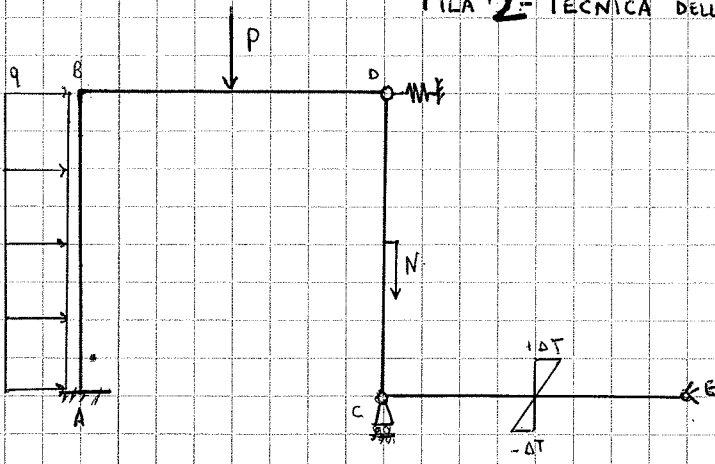


$$\sum F_v = 0 \quad \frac{199}{4} ql - 84 ql - 7 ql + \frac{165}{4} ql = 0 \quad \text{OK!}$$

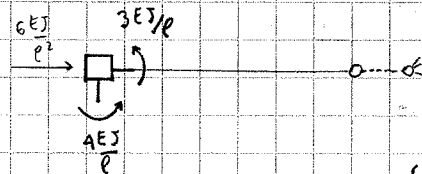
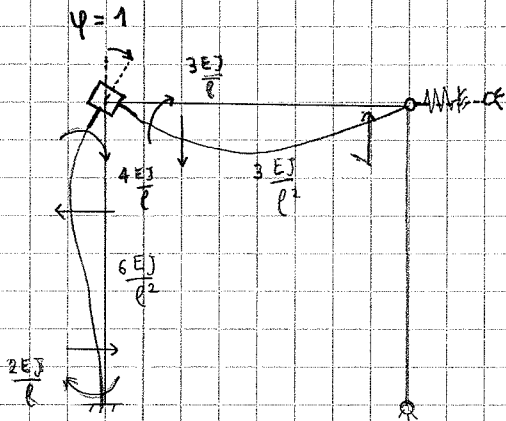
$$\sum F_h = 0 \quad ql + \frac{19}{2} ql - \frac{21}{2} ql = 0 \quad \text{OK!}$$

$$\begin{aligned} \sum M_{(A)} = 0 \quad & \frac{9}{4} ql^2 + \frac{ql^2}{2} + 84 ql \cdot \frac{l}{2} - \frac{21}{2} ql \cdot l + 7 ql \cdot l - \frac{165}{4} ql l = \\ & = 0 \quad \text{OK!} \end{aligned}$$

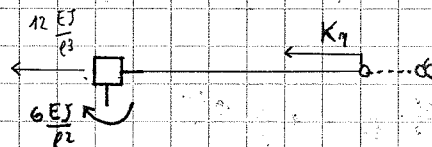
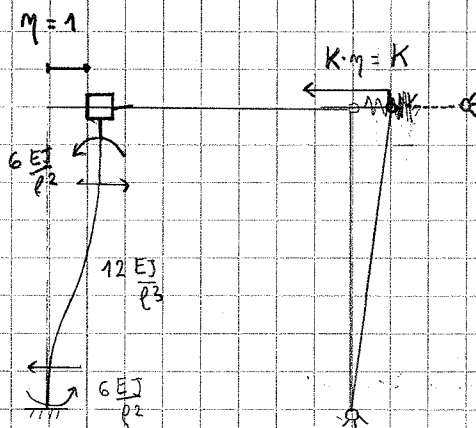
# FILA 2 - TECNICA DELLE COSTRUZIONI



$$\begin{cases} m_{B\varphi} \varphi + m_{B\eta} \eta + m_{B0} = 0 \\ R_{B\varphi} \varphi + R_{B\eta} \eta + R_{B0} = 0 \end{cases}$$

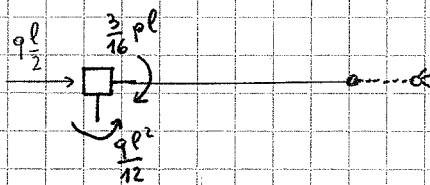
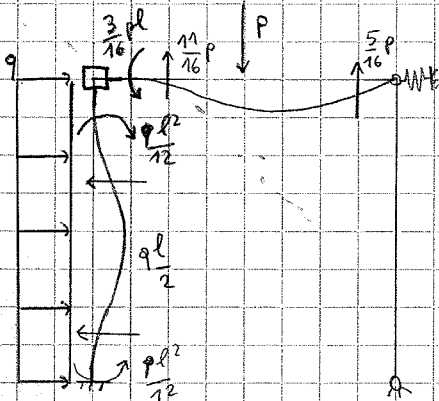


$$\begin{cases} m_{B\varphi} = 7EJ/l \\ R_{B\varphi} = 6EJ/l^2 \end{cases}$$



$$\begin{cases} m_{B\eta} = -6EJ/l^2 \\ R_{B\eta} = -12EJ/l^3 - K\eta \end{cases}$$

$P \neq 0; q \neq 0$



$$\begin{cases} m_{B0} = \frac{3}{16} Pl + \frac{q l^2}{12} \\ R_{B0} = q \frac{l}{2} \end{cases}$$



$$\begin{cases} 7 \frac{EJ}{l} \varphi_B - 6 \frac{EJ}{l^2} \eta - \frac{3}{16} pl + \frac{ql^2}{12} = 0 \\ 6 \frac{EJ}{l^2} \varphi_B - \left(12 \frac{EJ}{l^3} + K_\eta\right) \eta + ql = 0 \end{cases}$$

$$K_\eta = 15 \frac{EJ}{l^3}$$

$$p = 20 ql$$

$$\begin{cases} 7 \frac{EJ}{l} \varphi_B - 6 \frac{EJ}{l^2} \eta - \frac{3}{16} \cdot 20 ql^2 + \frac{ql^2}{12} = 0 \\ 6 \frac{EJ}{l^2} \varphi_B - \left(12 \frac{EJ}{l^3} + 15 \frac{EJ}{l^3}\right) \eta + ql = 0 \end{cases}$$

$$\begin{cases} 7 \frac{EJ}{l} \varphi_B - 6 \frac{EJ}{l^2} \eta - \frac{15}{4} ql^2 + \frac{ql^2}{12} = 0 \\ 6 \frac{EJ}{l^2} \varphi_B - 27 \frac{EJ}{l^3} \eta + ql = 0 \end{cases}$$

$$\begin{cases} 7 \frac{EJ}{l} \varphi_B - 6 \frac{EJ}{l^2} \eta - \frac{11}{3} ql^2 = 0 \\ 6 \frac{EJ}{l^2} \varphi_B - 27 \frac{EJ}{l^3} \eta + ql = 0 \end{cases}$$

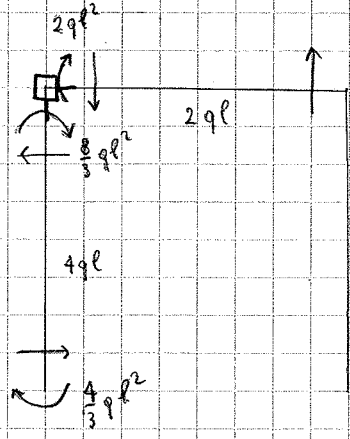
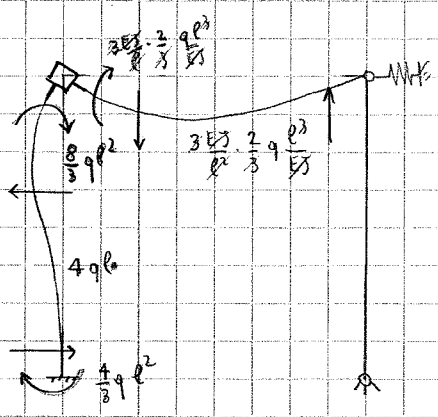
$$\begin{vmatrix} 7 & -6 & -\frac{11}{3} \\ 6 & -27 & \frac{1}{2} \end{vmatrix} \begin{matrix} [x-3] \\ = \\ [x \frac{2}{3}] \end{matrix} \begin{vmatrix} -21 & 18 & 11 \\ 4 & -18 & \frac{1}{3} \end{vmatrix} = \begin{vmatrix} -21 & 18 & 11 \\ -17 & 0 & \frac{34}{3} \end{vmatrix}$$

$$-17 \varphi = -\frac{34}{3}$$

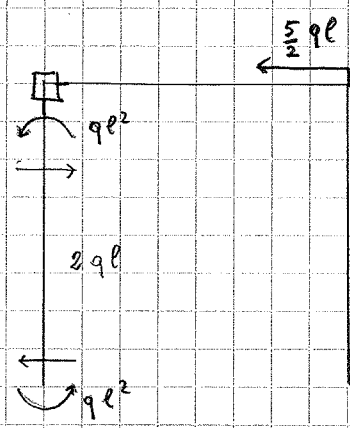
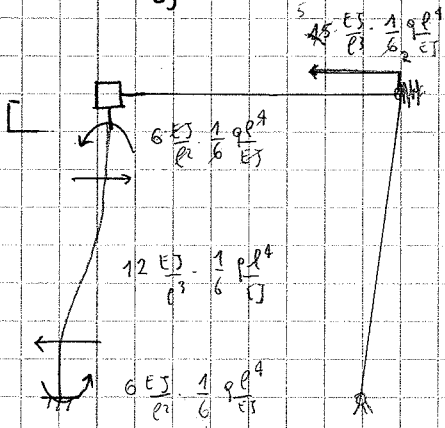
$$\rightarrow \varphi = + \frac{34}{3} \cdot \frac{1}{17} = \bullet \frac{2}{3} \frac{ql^3}{EJ}$$

$$\eta = \frac{1}{6} \frac{ql^4}{EJ}$$

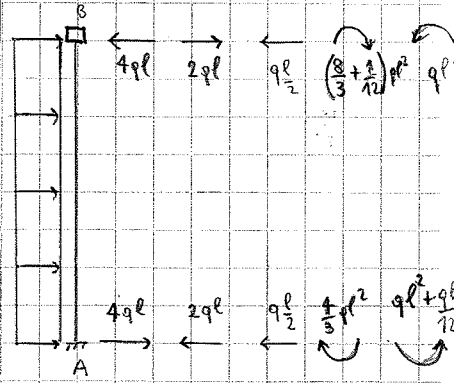
$$\psi = \frac{2}{3} \frac{q l^3}{EJ}$$



$$\eta = \frac{1}{6} \frac{q l^4}{EJ}$$



ASTA AB



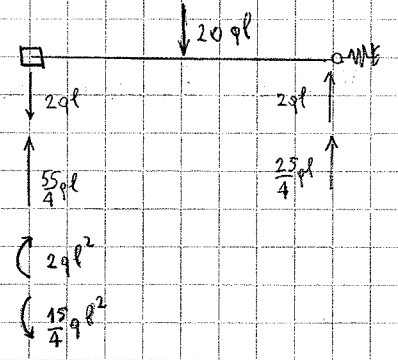
B)  $4ql - 2ql + \frac{ql}{2} = \frac{8-4+1}{2} = \frac{5}{2} ql$

$(\frac{8}{3} + \frac{1}{12} - 1) ql^2 = \frac{32+1-12}{12} = \frac{21}{12} ql^2$

A)  $-4ql + 2ql + \frac{ql}{2} = \frac{-8+4+1}{2} = -\frac{3}{2} ql$

$(\frac{4}{3} - 1 - \frac{1}{12}) ql^2 = \frac{16-12-1}{12} = \frac{1}{4} ql^2$

ASTA BD

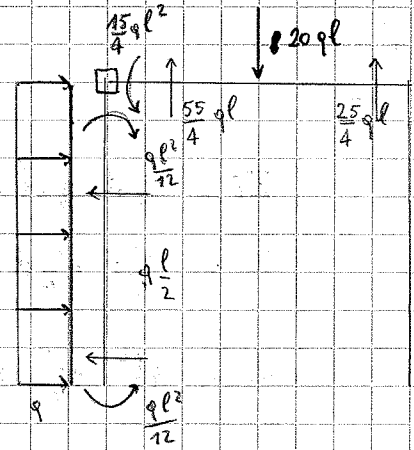
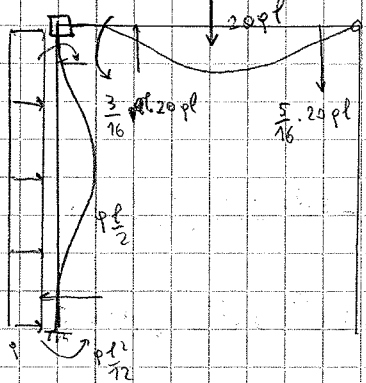


B)  $\frac{55}{4} ql - 2ql = \frac{55-8}{4} = \frac{47}{4} ql$

$\frac{15}{4} ql^2 - 2ql^2 = \frac{15-8}{4} = \frac{7}{4} ql^2$

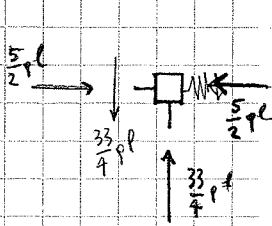
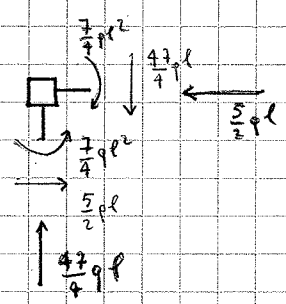
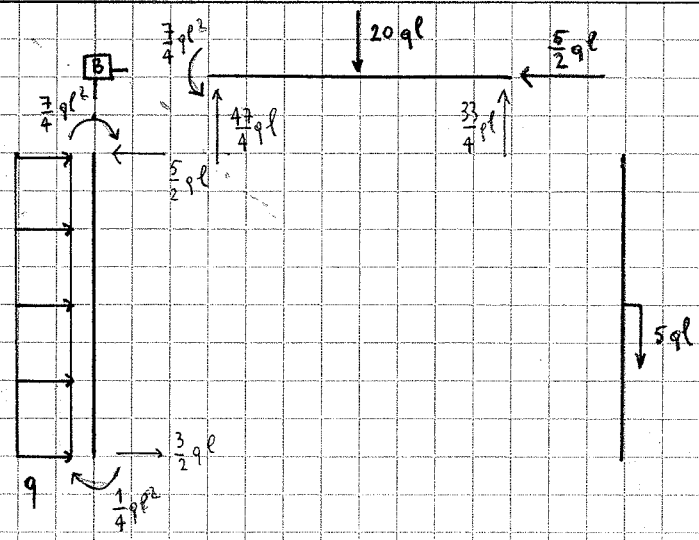
D)  $\frac{25}{4} ql + 2ql = \frac{25+8}{4} = \frac{33}{4} ql$

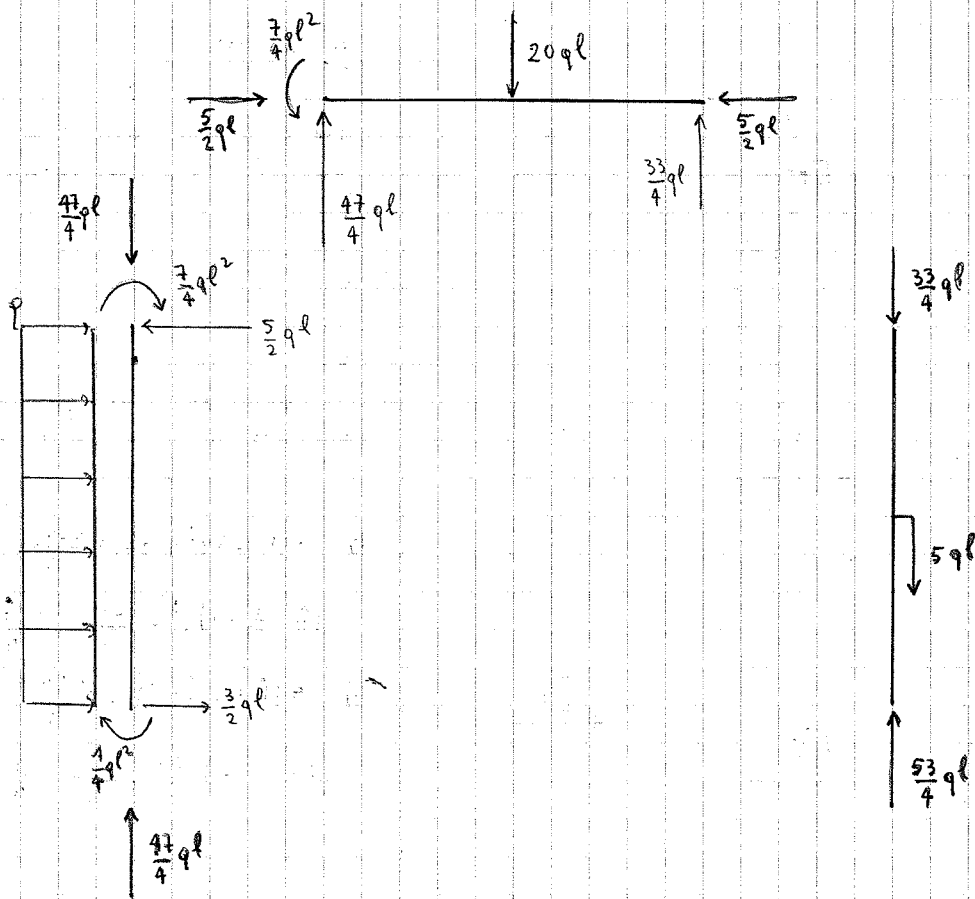
$p \neq 0; q \neq 0$



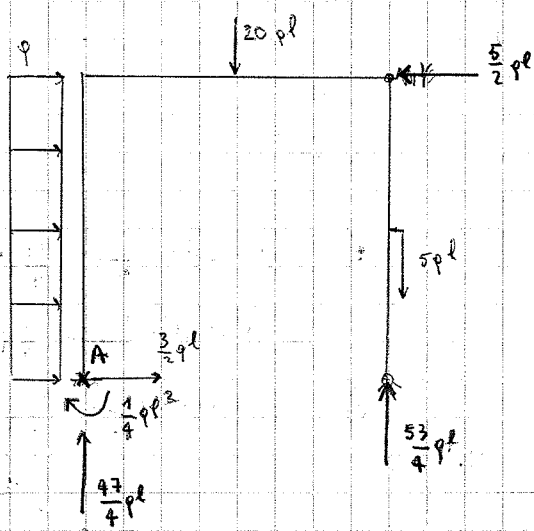
VERIFICA NODO B

NODO D



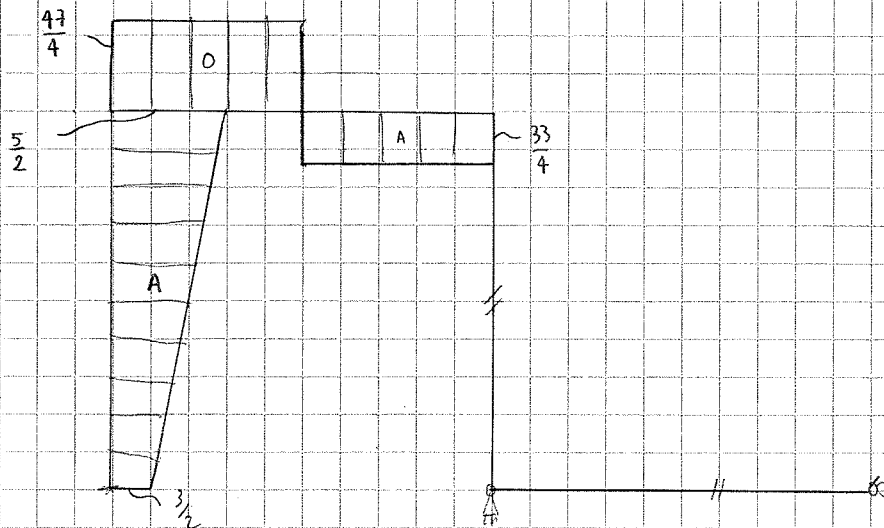


VERIFICA EQUILIBRIO GLOBALE

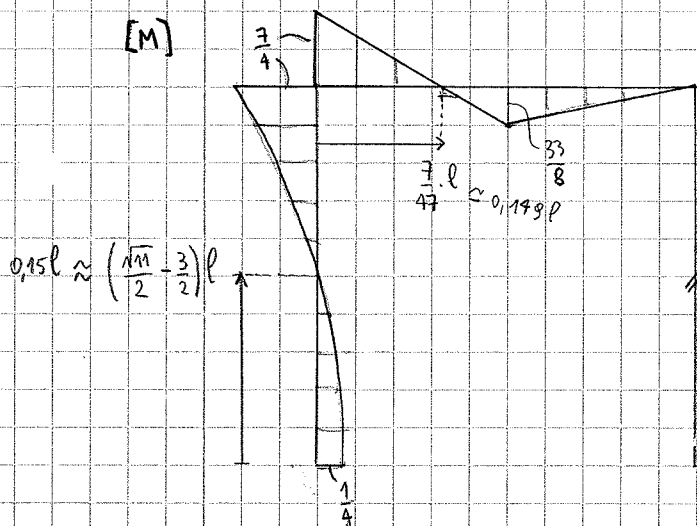


$$\left\{ \begin{aligned} \sum F_{\rightarrow} &= \frac{3}{2} pl - \frac{5}{2} pl + pl = 0 \\ \sum F_{\uparrow} &= \frac{47}{4} pl + \frac{53}{4} pl - 20 pl - 5 pl = \frac{47+53-80-20}{4} = 0 \\ \sum M_{(A)} &= pl^2 + pl^2 + 20 \frac{pl^2}{2} - \frac{5}{2} pl^2 + \frac{5}{2} pl^2 - \frac{53}{4} pl^2 = 0 \end{aligned} \right.$$

[T]



[M]



$$M^{OB}(0) \cdot \frac{47}{4} q l x - \frac{7}{4} q l^2 = 0$$

$$\frac{47}{4} q l x = \frac{7}{4} q l^2 \rightarrow x = \frac{7}{47} l$$

$$M_{max}^{OB}\left(\frac{l}{2}\right) = \frac{47}{4} q l \frac{l}{2} - \frac{7}{4} q l^2 = \frac{47}{8} q l^2 - \frac{7}{4} q l^2 = \frac{47-14}{8} q l^2 = \frac{33}{8} q l^2$$

$$M^{AB}(0) = \frac{1}{4} q l^2 - \frac{3}{2} q l x - q \frac{x^2}{2} = 0$$

$$q l^2 - 6 q l x - 2 q x^2 = 0$$

$$2 q x^2 + 6 q l x - q l^2 = 0$$

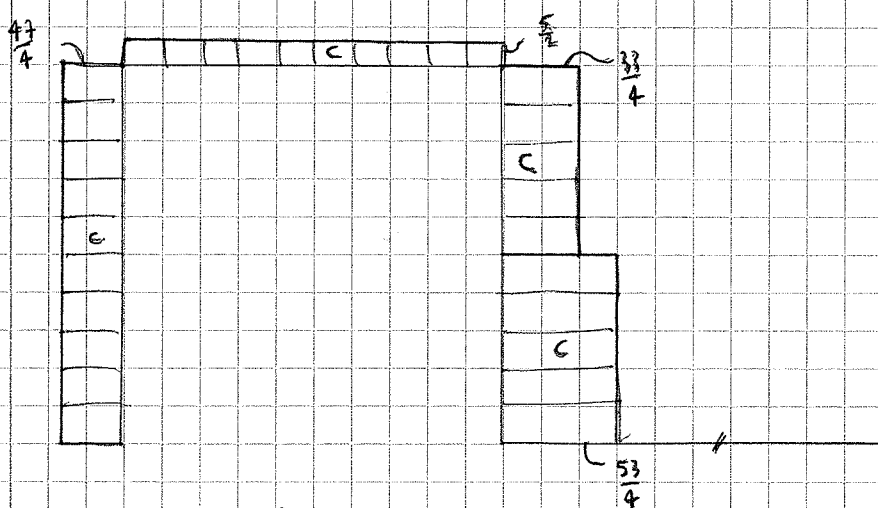
$$2 x^2 + 6 l x - l^2 = 0$$

$$x = \frac{-6l \pm \sqrt{36l^2 + 8l^2}}{4} = \frac{-6l \pm \sqrt{44l^2}}{4}$$

$$= \frac{-6 \pm \sqrt{44}}{4} l$$

$$= \frac{-6 + 2\sqrt{11}}{4} l = \frac{-3 + \sqrt{11}}{2} l (\approx 0,15l)$$

[N]



DEFORMATA

