

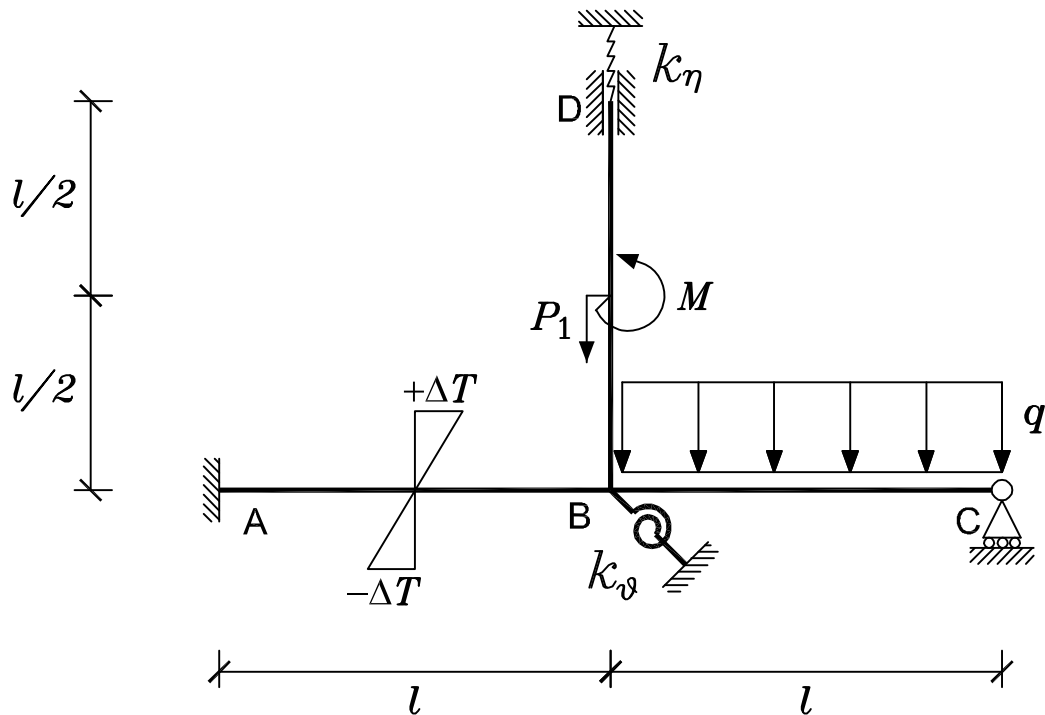
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 30 MARZO 2015

DOCENTE: ING. FAUSTO MINELLI

ESERCITATORE: ING. ANTONIO CONFORTI

Esercizio



$$P_1 = 4ql$$

$$M = 4ql^2$$

$$\frac{\alpha \Delta T}{h} = \frac{1}{6} \frac{ql^2}{EJ}$$

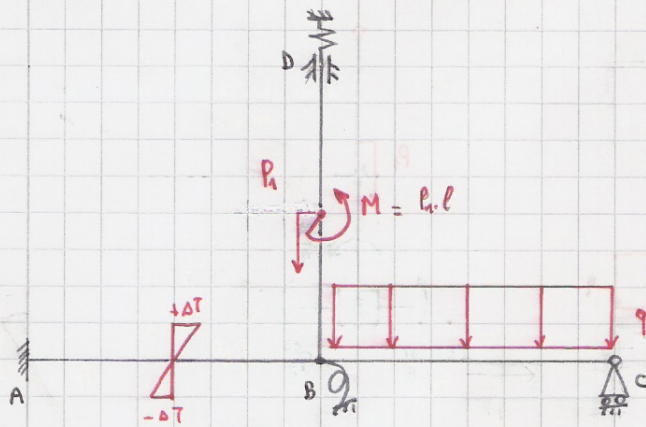
$$k_g = \frac{9}{4} \frac{EJ}{l}$$

$$k_\eta = \frac{11}{2} \frac{EJ}{l^3}$$

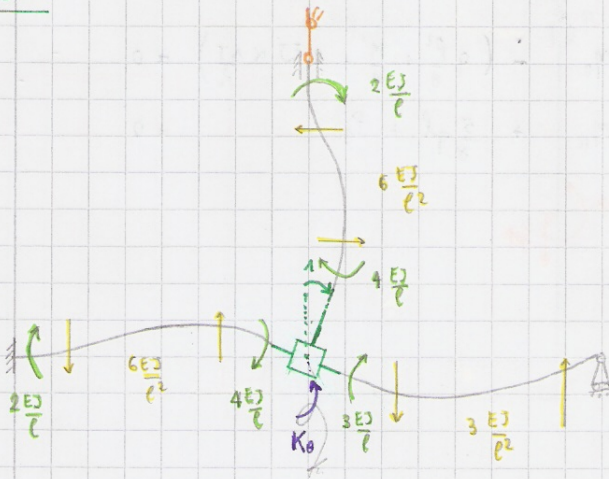
Dato il telaio in figura

Si richiedono i grafici di:

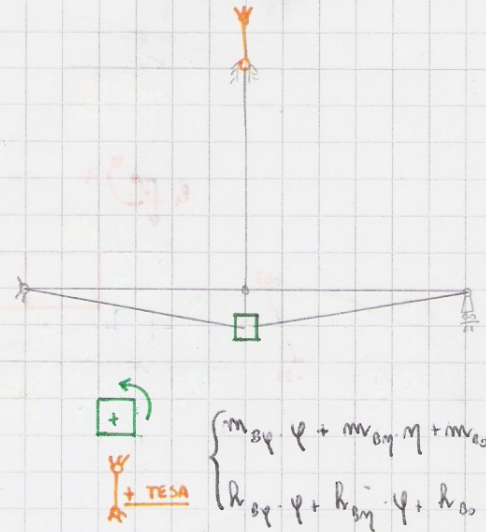
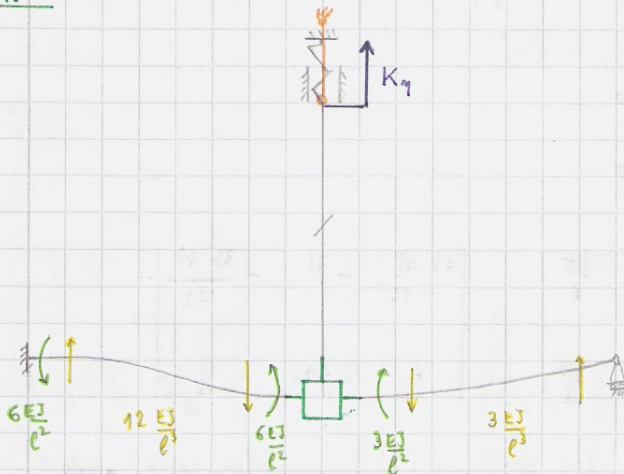
1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.



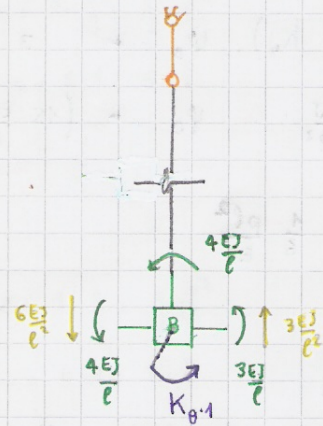
$$\varphi_B = 1$$



$$m_{0\varphi} = 1$$



$$\begin{cases} m_{B\varphi} \cdot \varphi + m_{B\theta} \cdot \theta + m_{B0} = 0 \\ h_{B\varphi} \cdot \varphi + h_{B\theta} \cdot \theta + h_{B0} = 0 \end{cases}$$

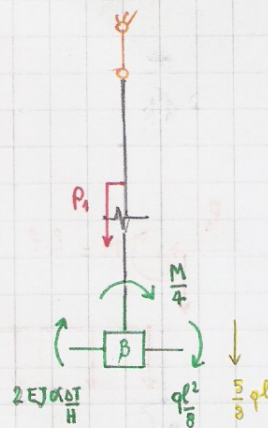
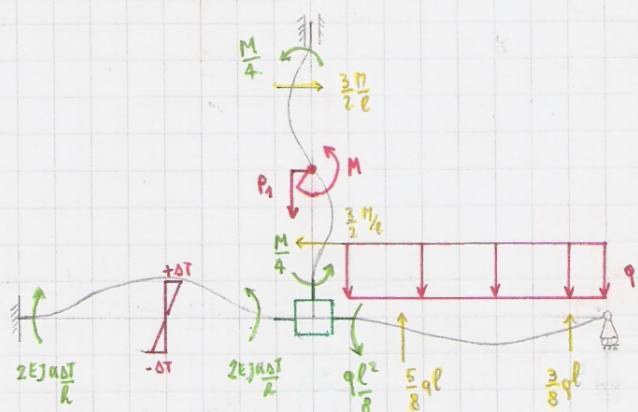


$$\begin{cases} m_{0\varphi} = (4+3+4) \frac{EJ}{l^2} + K_0 = 11 \frac{EJ}{l^2} + K_0 \\ h_{0\varphi} = (-3+6) \frac{EJ}{l^2} = +3 \frac{EJ}{l^2} \end{cases}$$



$$\begin{cases} m_{0\theta} = (3-6) \frac{EJ}{l^2} = -3 \frac{EJ}{l^2} \\ h_{0\theta} = -(12+3) - K_{\theta} = -15 \frac{EJ}{l^3} - K_{\theta} \end{cases}$$

CARICHI $\neq 0$



$$\begin{cases} m_{00} = -ql^2/8 - \frac{M}{4} - \frac{2EJ\alpha\Delta T}{h} \\ h_{00} = \frac{5}{8}ql + P_1 \end{cases}$$

SISTEMA

$$\begin{cases} \left(11 \frac{EJ}{l} + K_0\right) \cdot \psi_0 - 3 \frac{EJ}{l^2} \cdot \eta_0 - \left(ql^2/8 + \frac{M}{4} + 2EJ\alpha\Delta T/h\right) = 0 \\ + 3 \frac{EJ}{l^2} \cdot \psi_0 - \left(15 \frac{EJ}{l^3} + K_0\right) \cdot \eta_0 + \frac{5}{8}ql + P_1 = 0 \end{cases}$$

$$K_0 = \alpha \frac{\Delta T}{h} = \frac{1}{6} \frac{Pl^2}{EJ}$$

$$K_0 = \frac{9}{4} \frac{EJ}{l}$$

$$K_0 = \frac{15}{2} \frac{EJ}{l^3}$$

$$P_1 = 4ql \quad (M = P_1 \cdot l = 4ql^2)$$

$$\begin{cases} \left(11 \frac{EJ}{l} + \frac{9}{4} \frac{EJ}{l}\right) \cdot \psi_0 - 3 \frac{EJ}{l^2} \cdot \eta_0 - \left(ql^2/8 + \frac{4}{3}ql^2 + 2EJ\alpha\Delta T/h\right) = 0 \\ + 3 \frac{EJ}{l^2} \cdot \psi_0 - \left(15 \frac{EJ}{l^3} + \frac{15}{2} \frac{EJ}{l^3}\right) \cdot \eta_0 + \frac{5}{8}ql + 4ql = 0 \end{cases}$$

$$\begin{cases} \frac{53}{4} \frac{EJ}{l} \cdot \psi_0 - 3 \frac{EJ}{l^2} \cdot \eta_0 - \frac{35}{24} ql^2 = 0 \\ + 3 \frac{EJ}{l^2} \cdot \psi_0 - \frac{41}{2} \frac{EJ}{l^3} \cdot \eta_0 + \frac{37}{8} ql = 0 \end{cases}$$

$$\begin{vmatrix} \frac{53}{4} & -3 & -\frac{35}{24} \\ +3 & -\frac{41}{2} & \frac{37}{8} \end{vmatrix} = \begin{vmatrix} 53 & -12 & -\frac{35}{6} \\ 6 & -41 & \frac{37}{4} \end{vmatrix} = \begin{vmatrix} 53 \cdot \frac{41}{12} & -41 & -\frac{35 \cdot 41}{72} \\ 6 & -41 & \frac{37}{4} \end{vmatrix}$$

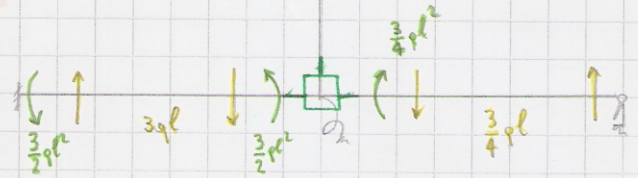
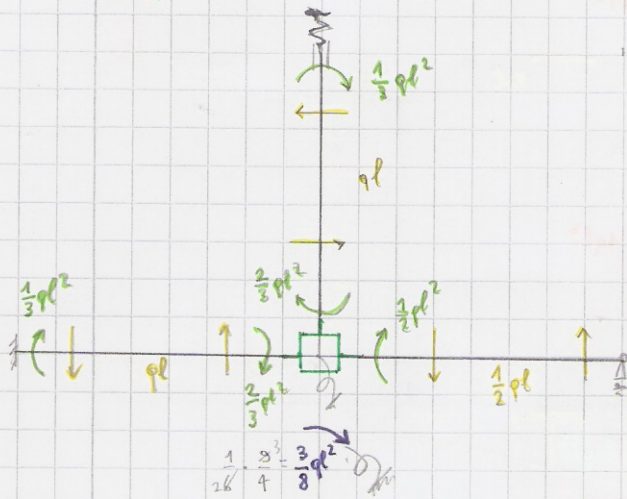
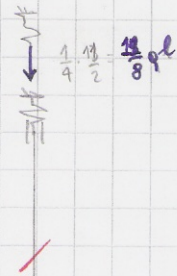
$$\begin{vmatrix} \frac{2173}{12} & -41 & -\frac{1435}{72} \\ 6 & -41 & \frac{37}{4} \end{vmatrix} \quad \left(\frac{2173}{12} - 6\right) \cdot \psi_0 = \frac{1435}{72} + \frac{37}{4} = \frac{1435 + 666}{72} = \frac{2101}{72}$$

$$\frac{2101}{42} \cdot \psi_0 = \frac{2101}{72} \quad \psi_0 = \frac{11}{36} = \frac{1}{6} \frac{ql^3}{EJ}$$

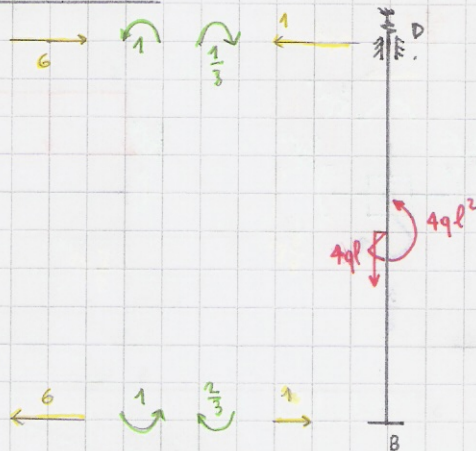
$$\eta_0 = \left(\frac{37}{8} - 3 \cdot \frac{1}{6}\right) \cdot \frac{2}{33} = \frac{37}{8} \cdot \frac{2}{33} \quad \eta_0 \frac{1}{4} \left(\frac{37}{8} - 3 \cdot \frac{1}{6}\right) \cdot \frac{2}{33} = \frac{37}{8} \cdot \frac{2}{33} = \frac{1}{4} \frac{ql^4}{EJ}$$

$$\varphi_0 = \frac{1}{6} \frac{q l^3}{EI}$$

$$M_0 = \frac{1}{4} \frac{q l^4}{EI}$$



SOVRAPPOSIZIONE DEGLI EFFETTI

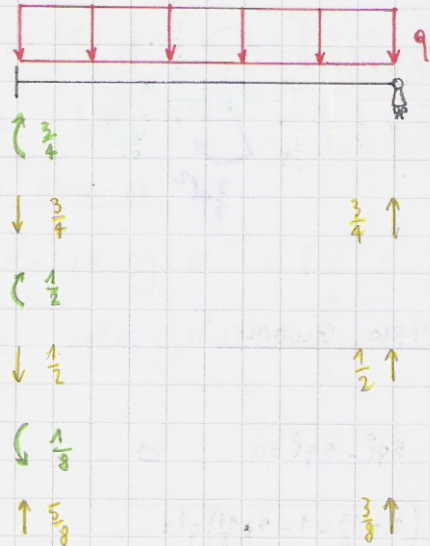
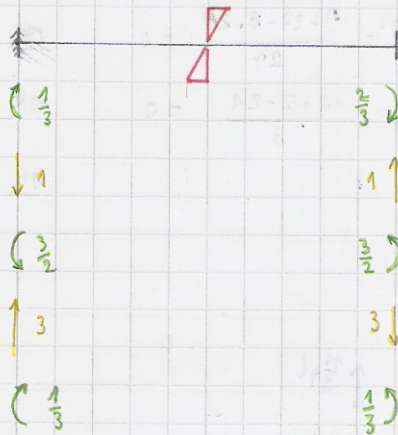


$$M_G = 1 - \frac{1}{3} = \frac{2}{3} q l^2$$

$$T_{\rightarrow} = 6 - 1 = 5 q l$$

$$M_G = 1 - \frac{2}{3} = \frac{1}{3} q l^2$$

$$T_{\leftarrow} = 6 - 1 = 5 q l$$



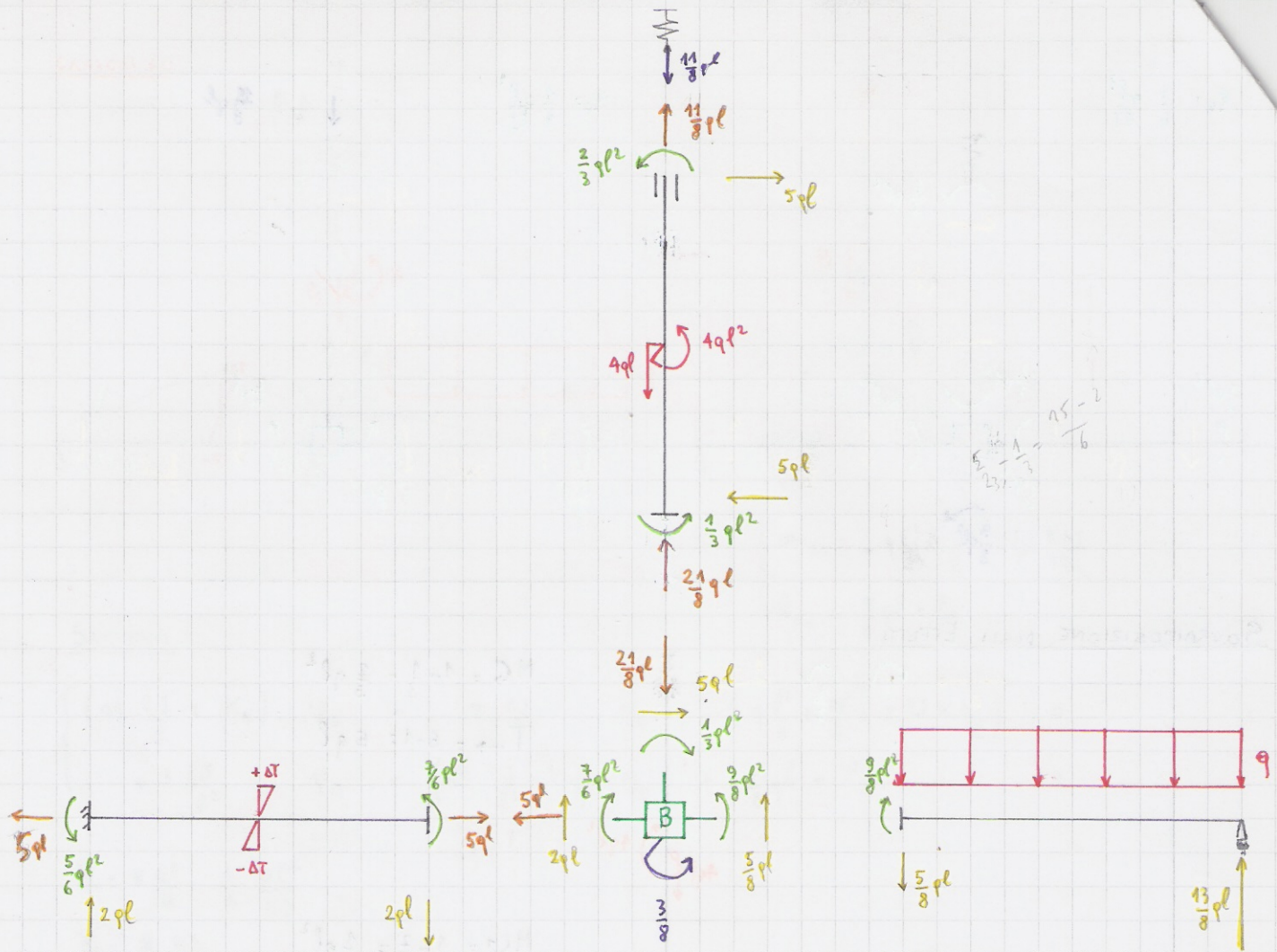
$$M_G: -\frac{1}{3} - \frac{1}{3} + \frac{3}{2} = \frac{-2-2+9}{6} = \frac{5}{6} q l^2 \quad M_R: \frac{2}{3} - \frac{3}{2} - \frac{1}{3} = -\frac{7}{6} q l^2$$

$$T_{\uparrow}: 3 - 1 = 2 q l$$

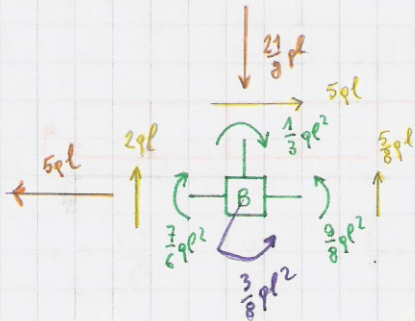
$$T_{\downarrow}: 3 - 1 = 2 q l$$

$$M_R: \frac{3}{4} + \frac{1}{2} - \frac{1}{8} = \frac{9}{8} q l^2 \quad M = 0$$

$$T_{\uparrow}: \frac{5}{8} - \frac{1}{2} - \frac{3}{4} = -\frac{5}{8} q l \quad T_{\downarrow}: \frac{3}{8} + \frac{1}{2} + \frac{3}{4} = \frac{13}{8}$$



EQUILIBRIO NODO:



$$\sum M = \left(\frac{3}{8} + \frac{9}{8} - \frac{1}{3} - \frac{7}{6} \right) ql^2 = \frac{9+27-8-28}{24} = 0 \quad \underline{OK}$$

$$\sum \uparrow = \left(2 + \frac{5}{8} - \frac{21}{8} \right) ql = \frac{16+5-21}{8} = 0 \quad \underline{OK}$$

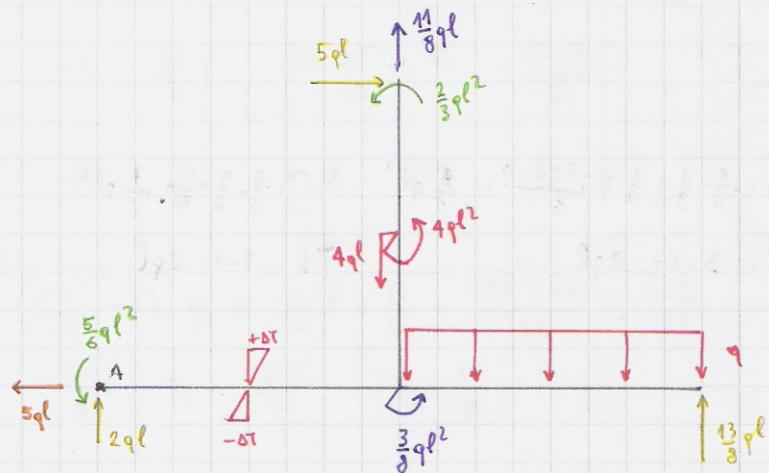
$$\sum \rightarrow = (5 - 5) ql = 0 \quad \underline{OK}$$

EQUILIBRIO GLOBALE:

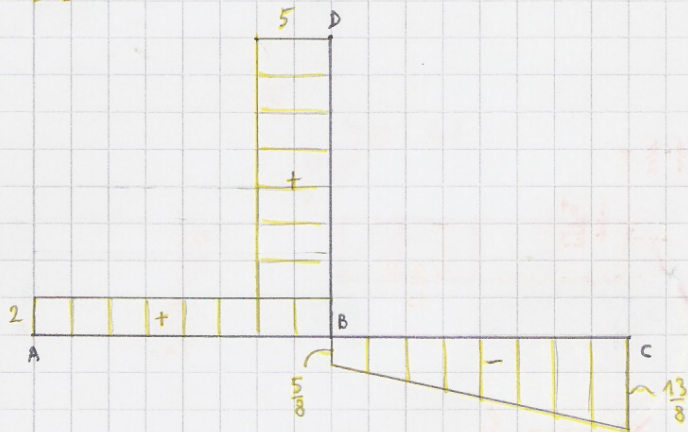
$$\sum \rightarrow: 5ql - 5ql = 0 \quad \underline{OK}$$

$$\begin{aligned} \sum \uparrow: & \left(2 + \frac{13}{8} - 1 - 4 + \frac{11}{8} \right) ql = \\ & = \frac{-16+13-8-32+11}{8} = 0 \quad \underline{OK} \end{aligned}$$

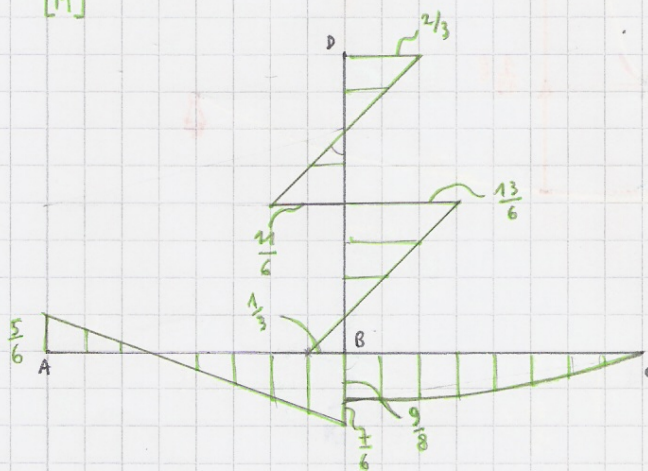
$$\begin{aligned} \sum M^A: & \left(-\frac{3}{8} - \frac{13}{8} \cdot 2 + \frac{3}{2} - 4 - \frac{5}{6} + \right. \\ & \left. + 4 - \frac{2}{3} + 5 - \frac{11}{8} \right) ql^2 = \\ & = \left(-\frac{3}{8} - \frac{13}{4} + \frac{3}{2} - 4 - \frac{5}{6} - \frac{2}{3} + 5 - \frac{11}{8} - \frac{5}{6} \right) ql^2 = \\ & = \frac{-9-78+36-16+120-33-20}{24} = 0 \quad \underline{OK} \end{aligned}$$



[T]



[M]



$$\begin{aligned} M(l/2) &= -\frac{1}{3} + \frac{5}{2} = \frac{-2+15}{6} = \frac{13}{6} ql^2 \\ M^{ss}(l/2) &= -\frac{2}{3} ql^2 + \frac{5}{2} ql^2 = \frac{-4+15}{6} = \frac{11}{6} ql^2 \\ \rightarrow \left(\frac{11}{6} + \frac{13}{6}\right) ql^2 &= \frac{24}{6} ql^2 = 4ql^2 \text{ (OK!)} \end{aligned}$$

FLESSI ASTA BD:

$$\begin{aligned} a) \frac{1}{3} : x &= \left(\frac{1}{3} + \frac{13}{6}\right) : \frac{l}{2} \rightarrow x = \frac{l}{2} \cdot \frac{1}{3} \cdot \frac{2}{5} = \frac{1}{15} l \approx 0,067l \\ \text{oppure } -\frac{1}{3} ql^2 + 5qx &= 0 \quad x = \frac{1}{3} \cdot \frac{1}{5} l = \frac{1}{15} l \end{aligned}$$

$$b) x_2 = l/2 \quad \frac{11+13}{6} = \frac{24}{6} = 4$$

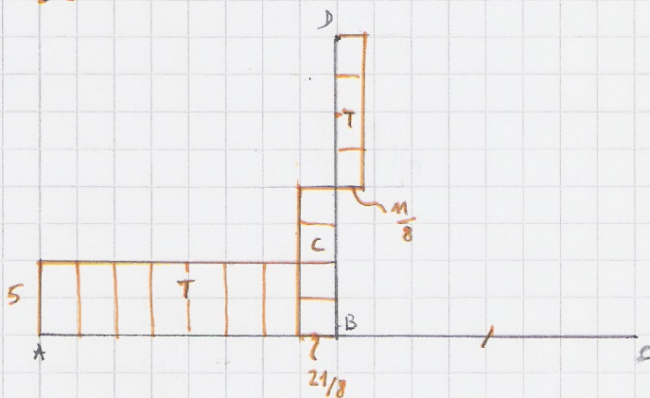
$$\begin{aligned} c) \frac{2}{3} : x_3 &= \left(\frac{11}{6} + \frac{2}{3}\right) : \frac{l}{2} \rightarrow x_3 = \frac{l}{2} \cdot \frac{2}{3} \cdot \frac{2}{5} = \frac{2}{15} l \approx 0,13l \\ \text{oppure } -\frac{2}{3} ql^2 + 5qx &= 0 \quad x = \frac{2}{3} \cdot \frac{1}{5} l = \frac{2}{15} l \text{ (OK)} \end{aligned}$$

FLESSO ASTA AB:

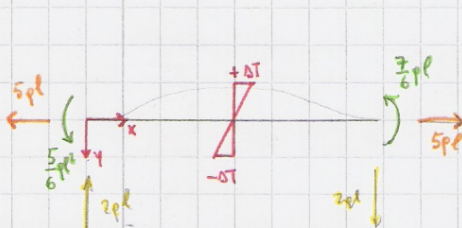
$$\frac{5}{6} : x = \left(\frac{5}{6} + \frac{7}{6}\right) : l \quad x = l \cdot \frac{5}{6} \cdot \frac{1}{2} = \frac{5}{12} l \approx 0,42l$$

$$\text{oppure } 2qlx - \frac{5}{6} ql^2 = 0 \quad x = \frac{5}{6} \cdot \frac{1}{2} l = \frac{5}{12} l \text{ (OK)}$$

[N]



DEFORMATA TERMICA:



$$M(x) = -\frac{5}{6} ql^2 + 2qlx$$

$$y'' = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{R}$$

$$\frac{\alpha\Delta T}{R} = \frac{1}{6} \frac{ql^2}{EJ}$$

$$y'' = \frac{1}{EJ} \left(\frac{5}{6} ql^2 - 2qlx + \frac{1}{3} ql^2 \right) = \frac{1}{EJ} \left(\frac{7}{6} ql^2 - 2qlx \right)$$

$$y'' > 0 \quad \frac{7}{6} ql^2 - 2qlx > 0 \quad 2x < \frac{7}{6} l \quad x < \frac{7}{12} l \approx 0,58l$$

DEFORMATA QUALITATIVA

