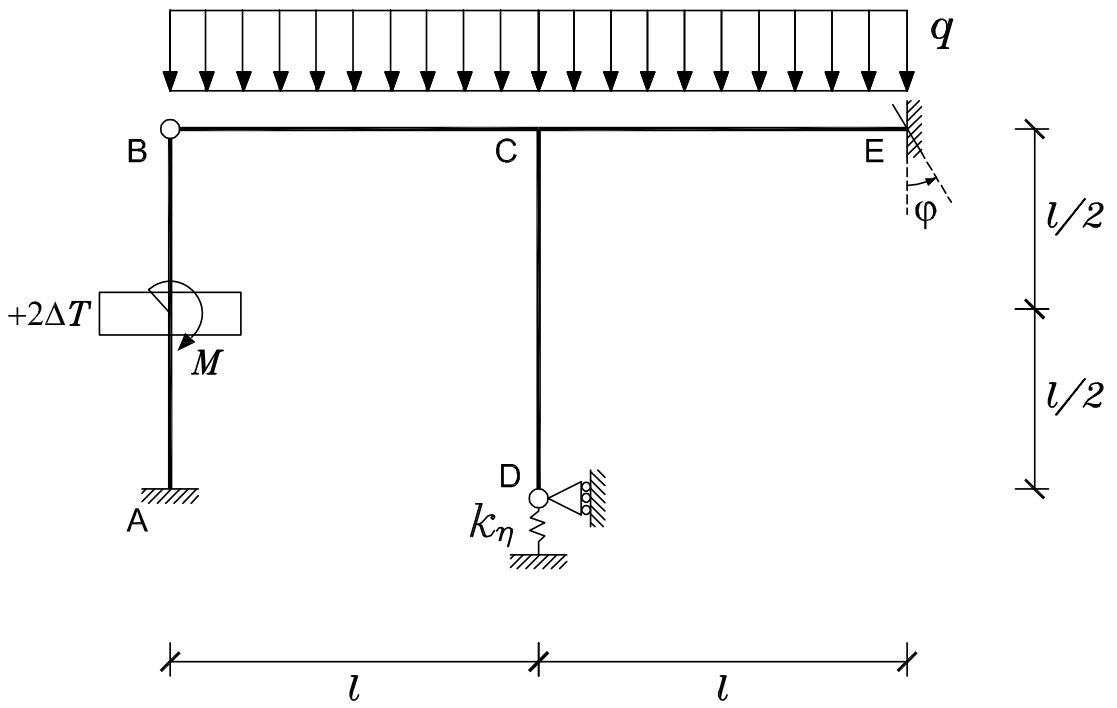
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 8 GIUGNO 2015

DOCENTE: **ING. FAUSTO MINELLI** ESERCITATORE: **ING. ANTONIO CONFORTI**

Esercizio



$$M = 2ql^2$$

$$\varphi = \frac{5}{6}\frac{ql^3}{EJ}$$

$$\frac{\alpha \Delta T}{l} = \frac{1}{16}\frac{ql^2}{EJ}$$

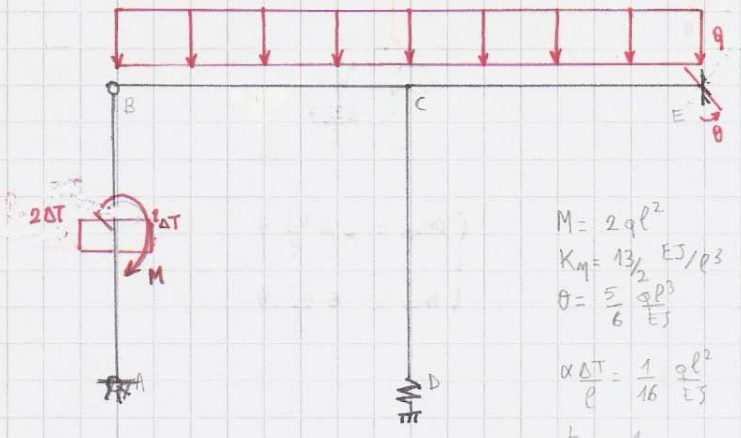
$$k_\eta = \frac{13}{2}\frac{EJ}{l^3}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME TECNICA COSTRUZIONI ALLIEVI EDILI / CIVILI 8-06-2015



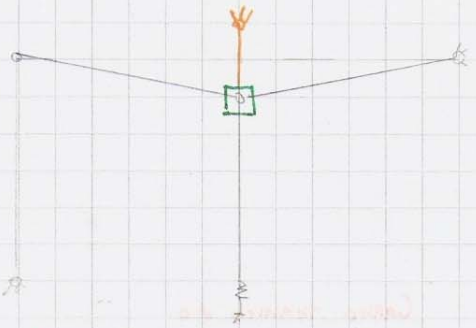
$M = 2ql^2$

$K_\eta = 13/2 \; EJ/l^3$

$\theta = \frac{5}{6}\frac{ql^3}{EJ}$

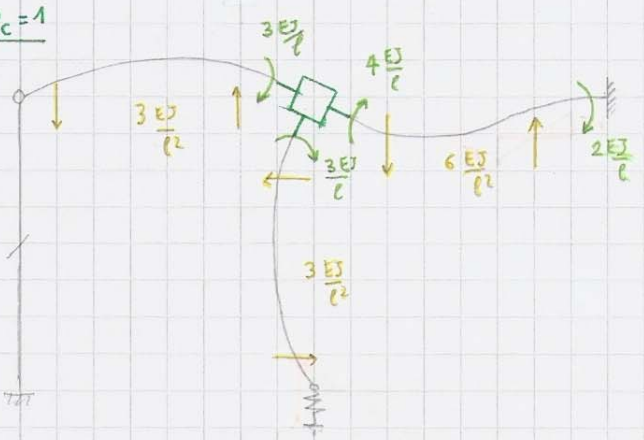
$\alpha\frac{\Delta T}{l} = \frac{1}{16}\frac{ql^2}{EJ}$

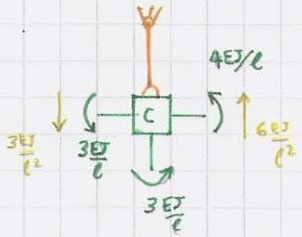
$\frac{t}{l} = \frac{1}{16}$



$$\begin{cases} m_{c\varphi}\cdot\varphi + m_{c\eta}\,\eta + m_{co} = 0 \\ R_{c\varphi}\,\varphi + R_{c\eta}\,\eta + R_{co} = 0 \end{cases}$$

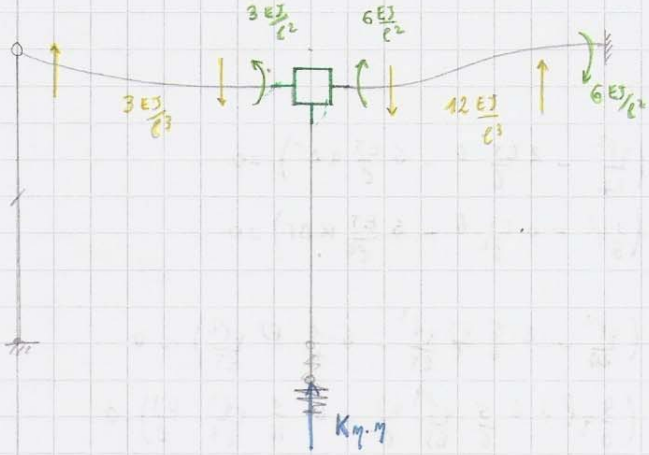
$\varphi_c = 1$

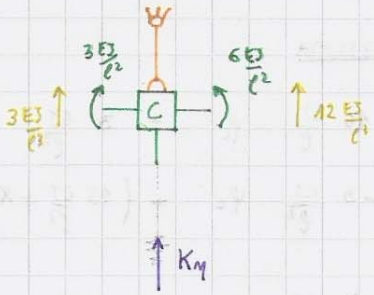




$$\begin{cases} m_{c\varphi} = (3+3+4)\frac{EJ}{l} = 10\,EJ/l \\ R_{c\varphi} = (-6+3)\,EJ/l^2 = -3\,EJ/l^2 \end{cases}$$

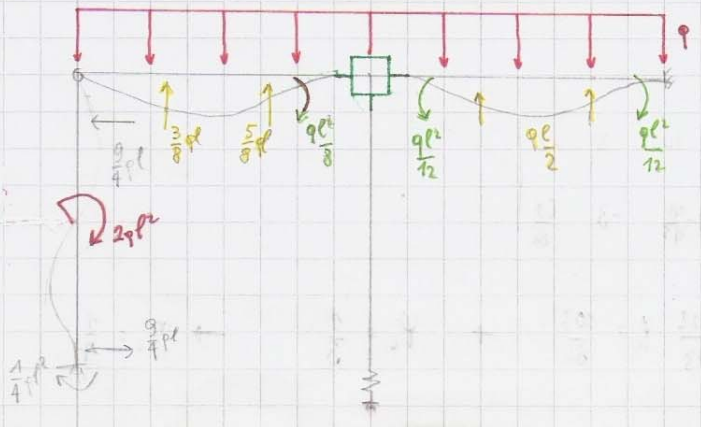
$M_c = 1$

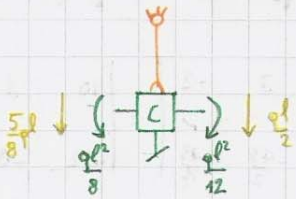




$$\begin{cases} m_{c\eta} = (6-3)\,EJ/l^2 = 3\,EJ/l^2 \\ R_{c\eta} = -(3+12)\frac{EJ}{l^3} - K_\eta = -K_\eta - 15\,EJ/l^3 \end{cases}$$

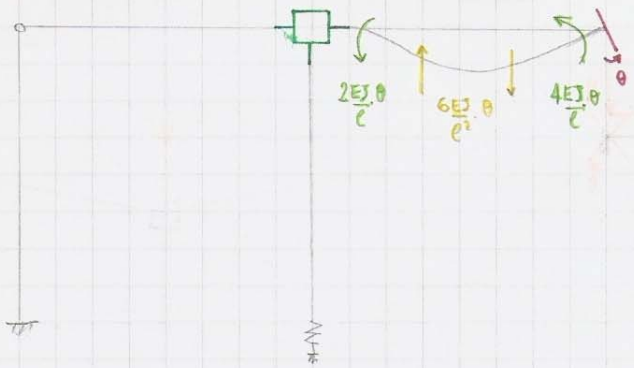
CARICHI ≠ 0

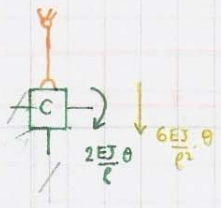




$$\begin{cases} m_{co} = \left(\frac{1}{8} - \frac{1}{12}\right) ql^2 = \frac{ql^2}{24} \\ R_{co} = \left(\frac{5}{8} + \frac{1}{2}\right) ql = \frac{9}{8}\,ql \end{cases}$$

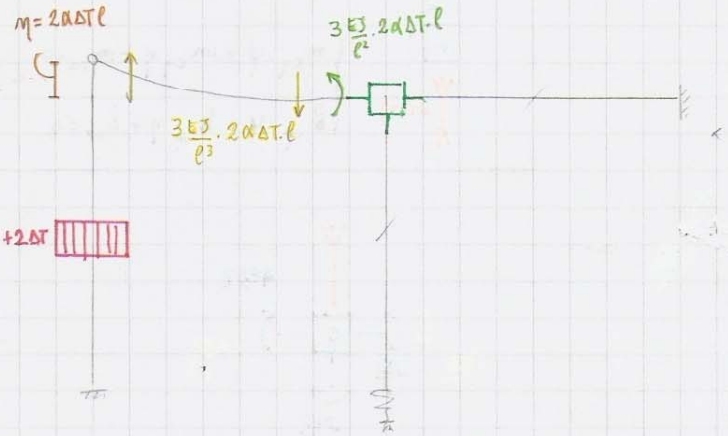
CEDIMENTO $\neq 0$



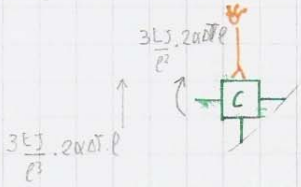


$$\begin{cases} m_{co} = -2\frac{EJ}{\ell}\cdot\theta \\ R_{co} = 6\frac{EJ}{\ell^2}\cdot\theta \end{cases}$$

CARICO TERMICO $\neq 0$



$$\begin{cases} m_{co} = -6\frac{EJ}{\ell}\cdot\alpha\Delta T \\ R_{co} = -6\frac{EJ}{\ell^2}\cdot\alpha\Delta T \end{cases}$$



SISTEMA

$$\begin{cases} 10\frac{EJ}{\ell}\cdot\varphi_c + 3\frac{EJ}{\ell^2}\cdot\eta_c + \left(\frac{q\ell^2}{24} - 2\frac{EJ}{\ell}\cdot\theta - 6\frac{EJ}{\ell}\alpha\Delta T\right) = 0 \\ -3\frac{EJ}{\ell^2}\varphi_c - \left(15\frac{EJ}{\ell^3} + K_m\right)\cdot\eta_c + \left(\frac{9}{8}q\ell + 6\frac{EJ}{\ell^2}\cdot\theta - 6\frac{EJ}{\ell^2}\alpha\Delta T\right) = 0 \end{cases}$$

$$\begin{cases} 10\frac{EJ}{\ell}\varphi_c + 3\frac{EJ}{\ell^2}\eta_c + \left(\frac{q\ell^2}{24} - 2\cdot\frac{5}{6}\frac{EJ}{\ell}\frac{q\ell^3}{EJ} - 6\cdot\frac{1}{16}EJ\frac{q\ell^2}{EJ}\right) = 0 \\ -3\frac{EJ}{\ell^2}\cdot\varphi_c - \left(15+\frac{13}{2}\right)\frac{EJ}{\ell^3}\eta_c + \left(\frac{9}{8}q\ell + 6\cdot\frac{5}{6}\frac{q\ell^3}{EJ}\frac{EJ}{\ell^2} - 6\cdot\frac{1}{16}\frac{q\ell^2}{EJ}\cdot\frac{EJ}{\ell}\right) = 0 \end{cases}$$

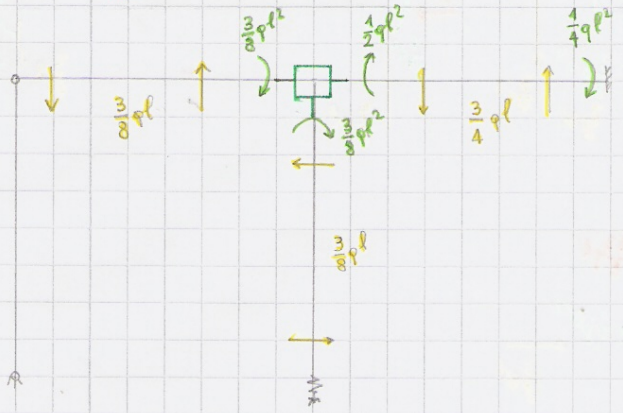
$$\begin{cases} 10\frac{EJ}{\ell}\cdot\varphi_c + 3\frac{EJ}{\ell^2}\eta_c - 2 = 0 \\ -3\frac{EJ}{\ell^2}\varphi_c - \frac{43}{2}\frac{EJ}{\ell^3}\eta_c + \frac{23}{4} = 0 \end{cases}$$

$10\cdot\frac{1}{3} + 3\eta_c = 2 \qquad \eta_c = \left(2 - \frac{5}{4}\right)\frac{1}{3} = \frac{3}{4}\cdot\frac{1}{3} = \frac{1}{4}$

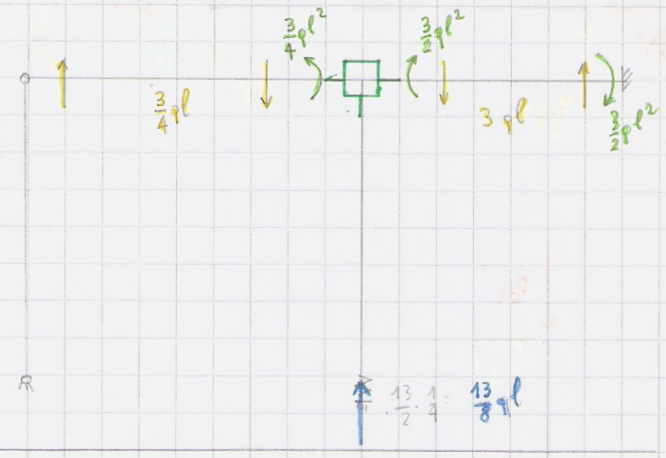
$$\begin{vmatrix} 10 & 3 & -2 \\ -3 & -\frac{43}{2} & \frac{23}{4} \end{vmatrix} = \begin{vmatrix} 10 & 3 & -2 \\ -12 & -86 & 23 \end{vmatrix} \qquad \begin{vmatrix} 10 & 3 & -2 \\ -\frac{18}{43} & -3 & \frac{69}{86} \end{vmatrix}$$

$$\left(10 - \frac{18}{43}\right)\varphi_c = 2 - \frac{69}{86} = \frac{103}{86} \qquad \frac{412}{43}\varphi_c = \frac{103}{86} \rightarrow \varphi_c = \frac{1}{8} \rightarrow \eta_c = \frac{1}{4}$$

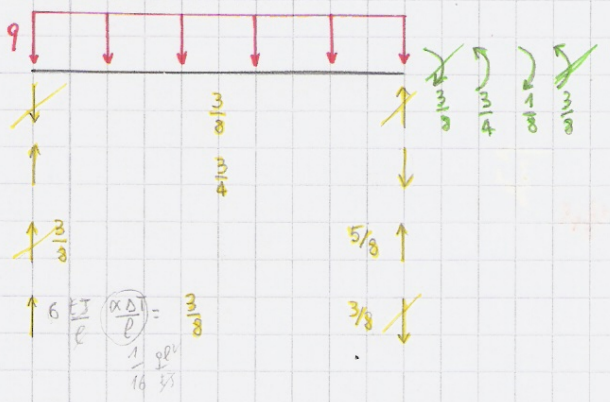
$\varphi_c = \frac{1}{8}\frac{ql^3}{EJ}$

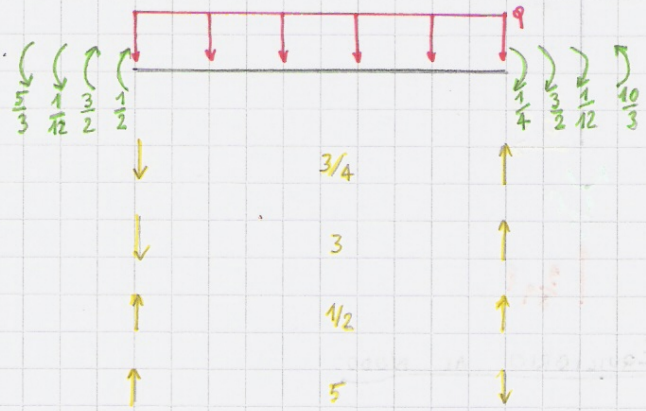


$\eta_c = \frac{1}{4}\frac{ql^4}{EJ}$



SOVRAPPOSIZIONE DEGLI EFFETTI





$M \circlearrowright = 0$

$T \uparrow = \frac{3}{4} + \frac{3}{8} = \frac{9}{8} ql$

$M \circlearrowleft = \frac{3}{4} - \frac{1}{8} = \frac{5}{8} ql^2$

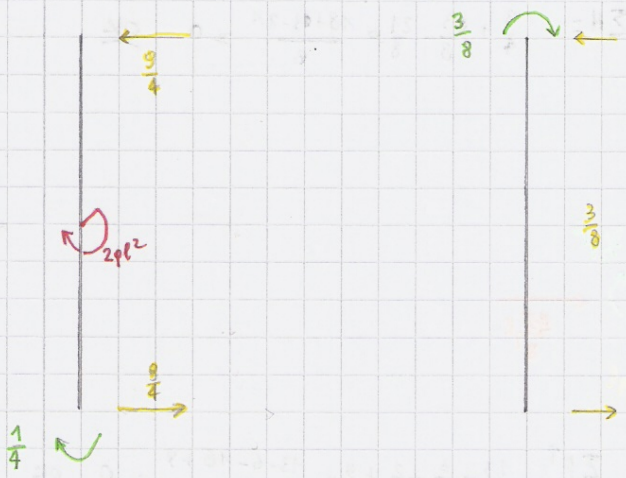
$T \downarrow = \frac{3}{4} - \frac{5}{8} = \frac{1}{8} ql$

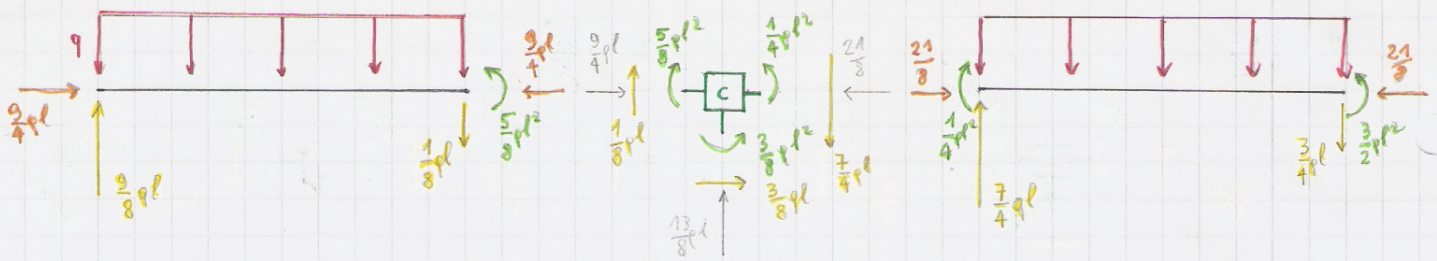
$M \circlearrowright = \frac{3}{2} + \frac{1}{2} - \frac{5}{3} - \frac{1}{12} = \frac{3}{12} = \frac{1}{4} ql^2$

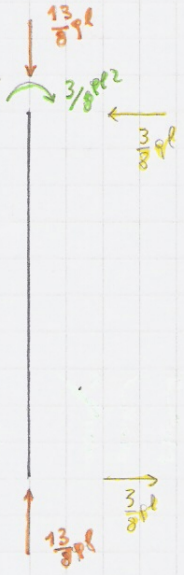
$T \uparrow = 2 + \frac{1}{2} - \frac{3}{4} = \frac{7}{4} ql$

$M \circlearrowright = \frac{1}{4} + \frac{3}{2} + \frac{1}{12} - \frac{10}{3} = -\frac{18}{12} = -\frac{3}{2} ql^2$

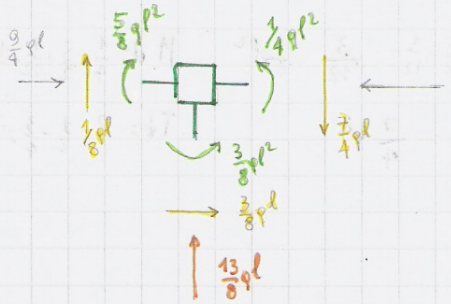
$T \downarrow = 2 - \frac{1}{2} - \frac{3}{4} = \frac{3}{4} ql$







EQUILIBRIO AL NODO:

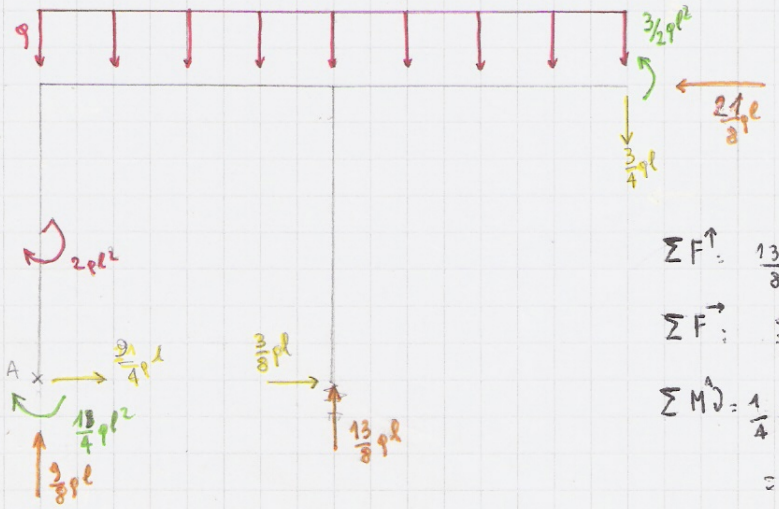


$\Sigma M: \quad \frac{5}{8} - \frac{3}{8} - \frac{1}{4} = \frac{5-3-2}{8} = 0$ OK

$\Sigma V = 0 \quad \left(\frac{1}{8} - \frac{7}{4}\right) ql = \frac{1-14}{8} = -\frac{13}{8} ql$

$\Sigma H: \quad \frac{9}{4} + \frac{3}{8} - \frac{21}{8} = \frac{18+3-21}{8} = 0$ OK

EQUILIBRIO GLOBALE

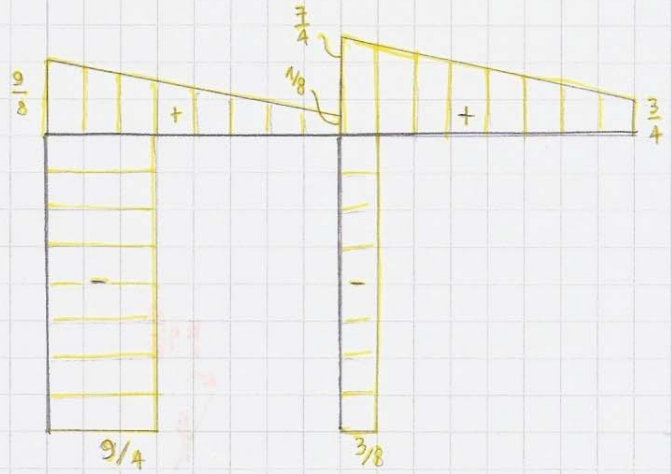


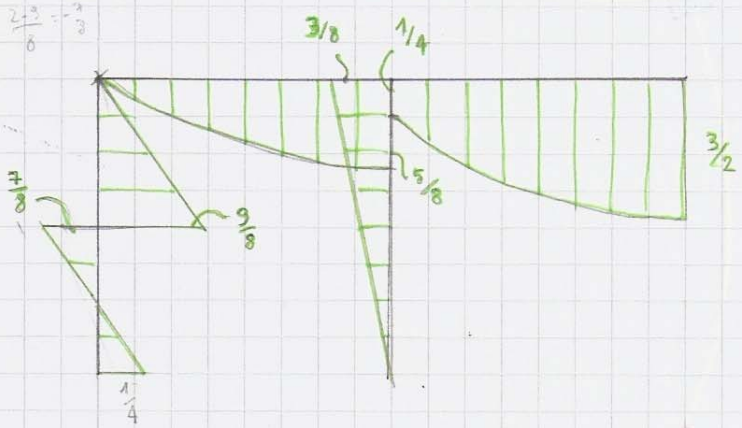
$\Sigma F^{\uparrow}: \quad \frac{13}{8} - \frac{3}{4} - 2 + \frac{9}{8} = \frac{13-6-16+9}{8} = 0$ OK

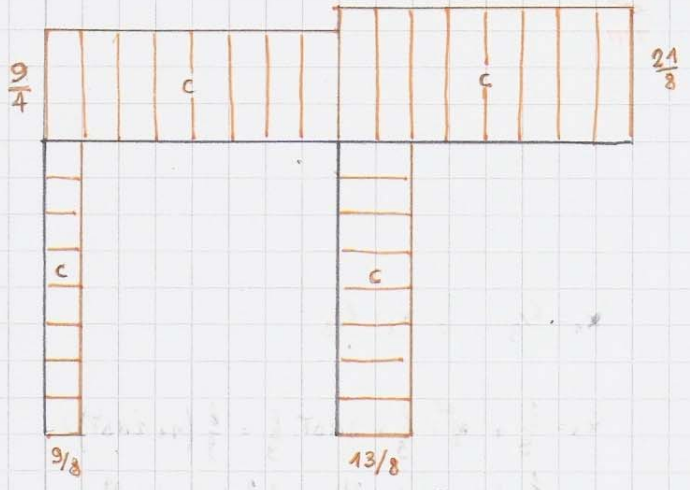
$\Sigma F^{\rightarrow}: \quad \frac{9}{4} + \frac{3}{8} - \frac{21}{8} = \frac{18+3-21}{8} = 0$ OK

$\Sigma M^{\curvearrowleft}_A = \frac{1}{4} + 2 - \frac{13}{8} + \cancel{\frac{3}{4}2} - \frac{21}{8} - \cancel{\frac{3}{2}} + 2 = \frac{1}{4} + 4 - \frac{13}{8} - \frac{21}{8} =$

$= \frac{2+32-13-21}{8} = 0$ OK





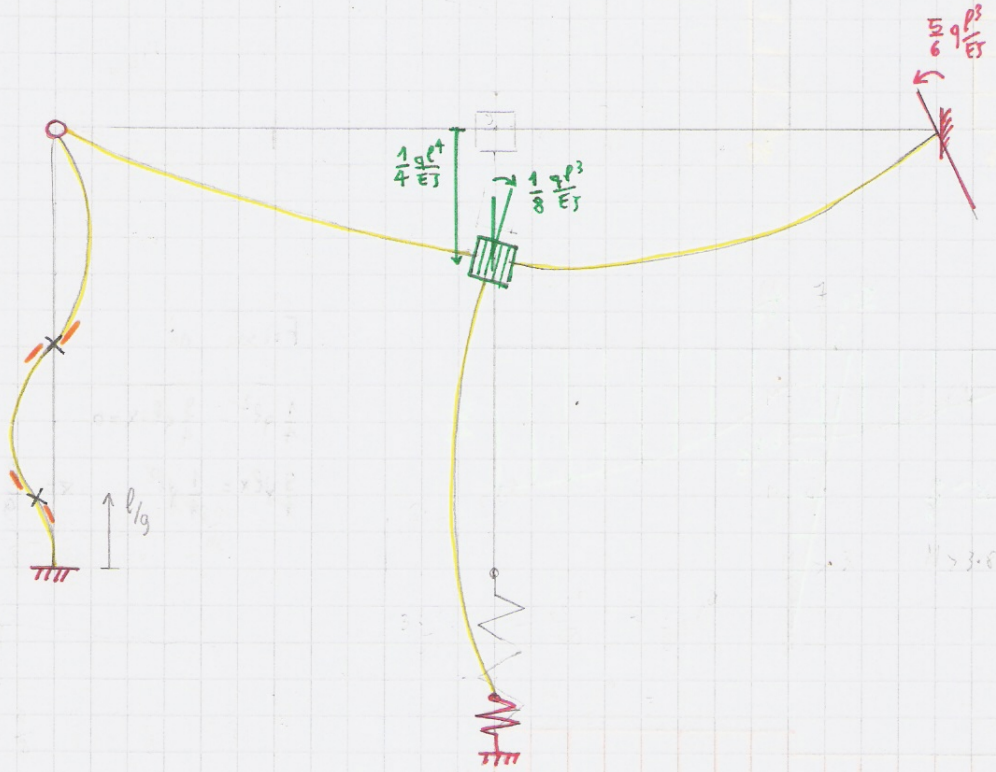


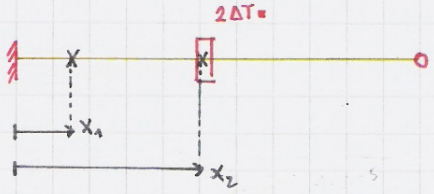
Flesso AB

$$\frac{1}{4} q l^2 - \frac{9}{4} q l \cdot x = 0$$

$$\frac{9}{\cancel{4}} \cancel{q} \cancel{l} x = \frac{1}{\cancel{4}} \cancel{q} l^{\cancel{2}} \qquad x = \frac{l}{9}$$

DEFORMATA QUALITATIVA





$x_1 = \ell/9 \quad ; \quad x_2 = \ell/2$

$$x_1 = \frac{\ell}{9} + \eta^{\Delta T} = \frac{\ell}{9} + 2\alpha\Delta T \cdot \frac{\ell}{9} = \frac{\ell}{9}\left(1 + 2\alpha\Delta T\right) =$$

$$= \frac{\ell}{9}\cdot\left(1 + 2\cdot\frac{1}{16}\frac{q\ell^3}{EJ}\right) = \frac{1}{9}\ell + \frac{1}{72}\frac{q\ell^4}{EJ}$$

$$x_2 = \frac{\ell}{2} + 2\alpha\Delta T\cdot\frac{\ell}{2} = \frac{\ell}{2} + \cancel{2}\cdot\frac{1}{16}\frac{q\ell^3}{EJ}\cdot\frac{\ell}{\cancel{2}} =$$

$$= \frac{\ell}{2} + \frac{1}{16}\frac{q\ell^4}{EJ}$$