TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 14 APRILE 2016

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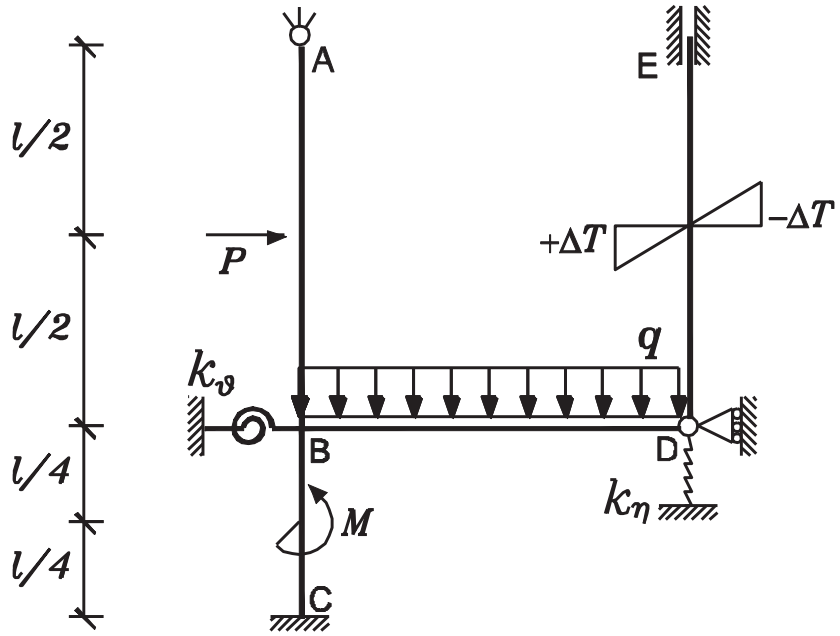
PROF. FAUSTO MINELLI

Esercizio



$M = ql^2$;

$P = 2ql$;

$\frac{\alpha\Delta T}{h} = \frac{1}{2}\frac{ql^2}{EJ}$;

$K_\vartheta = \frac{EJ}{l}$

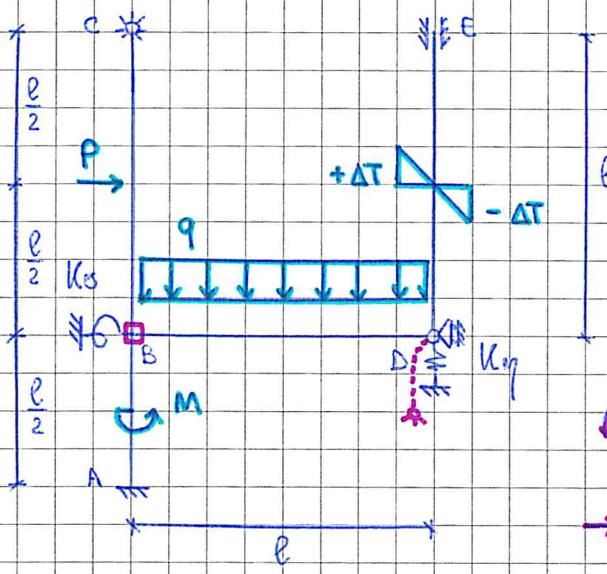
$K_\eta = \frac{3}{4}\frac{EJ}{l^3}$

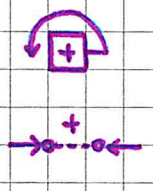
Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

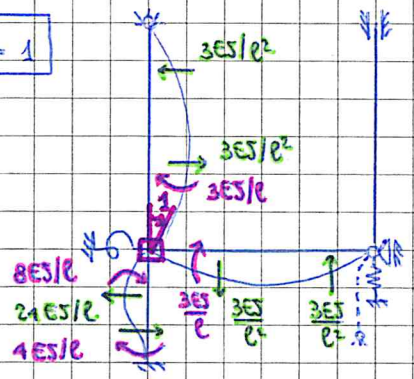
TELAIO 14/04/2016





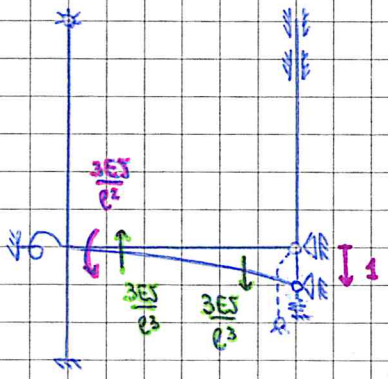
$$\begin{cases} m_{e\varphi}\cdot\varphi_e + m_{B\eta}\,\eta_D + m_{e\phi} = 0 \\ R_{D\varphi}\,\varphi_e + R_{D\eta}\cdot\eta_D + R_{D\phi} = 0 \end{cases}$$

$\varphi_e = 1$



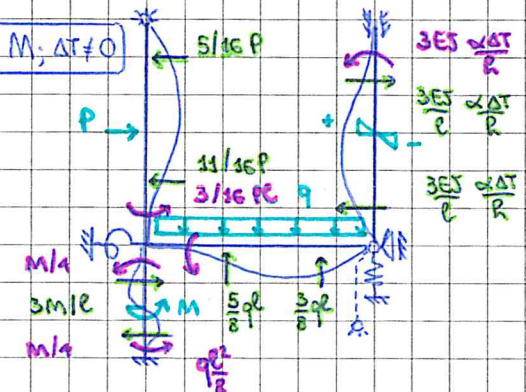
$$\begin{cases} m_{B\varphi} = \frac{3EJ}{\ell} + \frac{3EJ}{\ell} + \frac{4EJ}{\ell/2} + K_\theta = \frac{14EJ}{\ell} + K_\theta \\ R_{D\varphi} = \frac{3EJ}{\ell^2} \end{cases}$$

$\eta_D = 1$



$$\begin{cases} m_{B\eta} = -\frac{3EJ}{\ell} \\ R_{D\eta} = -\frac{3EJ}{\ell^3} - K_\eta \end{cases}$$

$P;q;M;\Delta T \neq 0$



$$\begin{cases} m_{e\phi} = -\frac{3}{16}P\ell - \frac{q\ell^2}{8} - \frac{M}{4} \\ R_{D\phi} = \frac{3}{8}q\ell \end{cases}$$

SISTEMA:

$$\begin{cases} \left(14\frac{EJ}{\ell} + K_\Theta\right)\varphi_B - \frac{3EJ}{\ell^2}M_D - \frac{3}{16}P\ell - \frac{q\ell^2}{8} - \frac{M}{4} = 0 \\ \frac{3EJ}{\ell^2}\varphi_B + \left(-\frac{3EJ}{\ell^3} - K_M\right)M_D + \frac{3}{8}q\ell = 0 \end{cases}$$

- $P = 2q\ell$
- $M = q\ell^2$
- $\frac{\alpha\Delta T}{h} = \frac{1}{2}\frac{q\ell^2}{EJ}$
- $K_\Theta = \frac{EJ}{\ell}$
- $K_M = \frac{3}{4}\frac{EJ}{\ell^3}$

$$\begin{cases} \left(14\frac{EJ}{\ell} + \frac{EJ}{\ell}\right)\varphi_B - \frac{3EJ}{\ell^2}M_D - \frac{3}{16}\cdot 2q\ell\cdot\ell - \frac{q\ell^2}{8} - \frac{q\ell^2}{4} = 0 \\ \frac{3EJ}{\ell^2}\varphi_B - \left(\frac{3EJ}{\ell^3} + \frac{3}{4}\frac{EJ}{\ell^3}\right)M_D + \frac{3}{8}q\ell = 0 \end{cases}$$

$$\begin{cases} 15\frac{EJ}{\ell}\varphi_B - 3\frac{EJ}{\ell^2}M_D - \frac{3}{4}q\ell^2 = 0 \\ \frac{3EJ}{\ell^2}\varphi_B - \frac{15}{4}\frac{EJ}{\ell^3}M_D + \frac{3}{8}q\ell = 0 \end{cases}$$

$$\begin{vmatrix} 15 & -3 & -\frac{3}{4} \\ 3 & -\frac{15}{4} & +\frac{3}{8} \end{vmatrix} \quad \begin{matrix} \times 1 \\ \times 5 \end{matrix} \qquad \begin{vmatrix} 15 & -3 & -\frac{3}{4} \\ 15 & -\frac{75}{4} & +\frac{15}{8} \end{vmatrix} \quad (-)$$

$$\left(-3 + \frac{75}{4}\right)M_D + \left(-\frac{3}{4} - \frac{15}{8}\right) = 0$$

$$M_D = \frac{21}{8}\cdot\frac{4}{63} = \frac{1}{6}\frac{q\ell^4}{EJ}$$

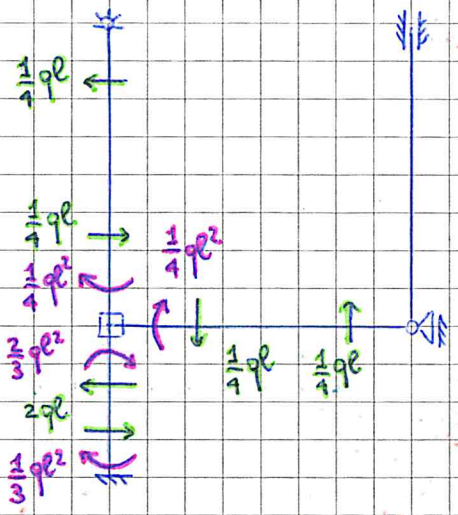
$$\varphi_B = \left(\frac{3EJ}{\ell^2}\cdot\frac{1}{6}\frac{q\ell^4}{EJ} + \frac{3}{4}q\ell^2\right)\cdot\frac{\ell}{15EJ} = \frac{1}{12}\frac{q\ell^3}{EJ}$$

⇓

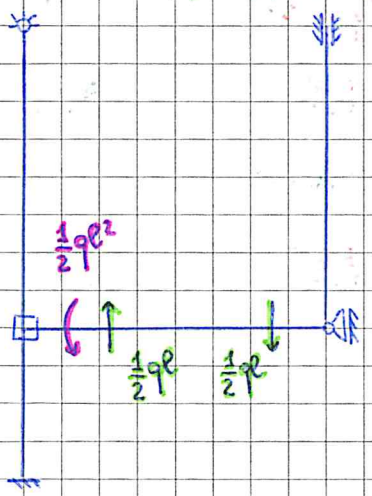
$$\boxed{\varphi_B = \frac{1}{12}\frac{q\ell^3}{EJ}}$$ ciné ORARIA

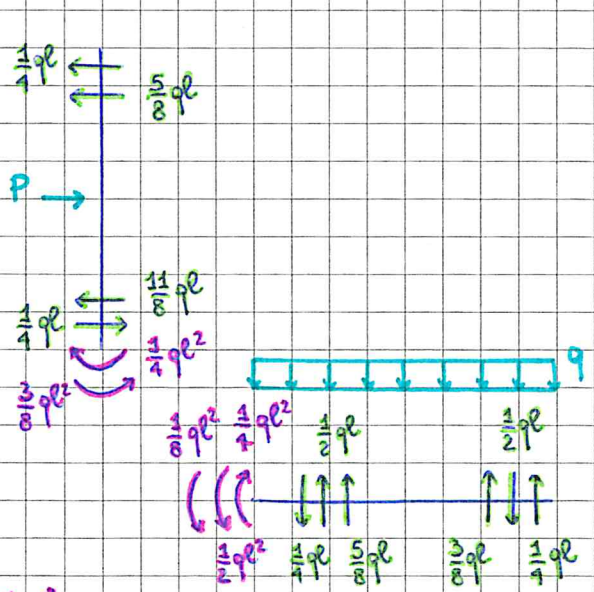
$$\boxed{M_D = \frac{1}{6}\frac{q\ell^4}{EJ}}$$ ciné verso il BASSO

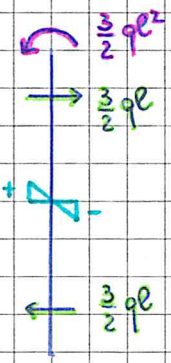
$\varphi_B = \frac{1}{12} \frac{ql^3}{EJ}$:

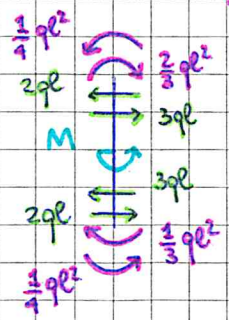


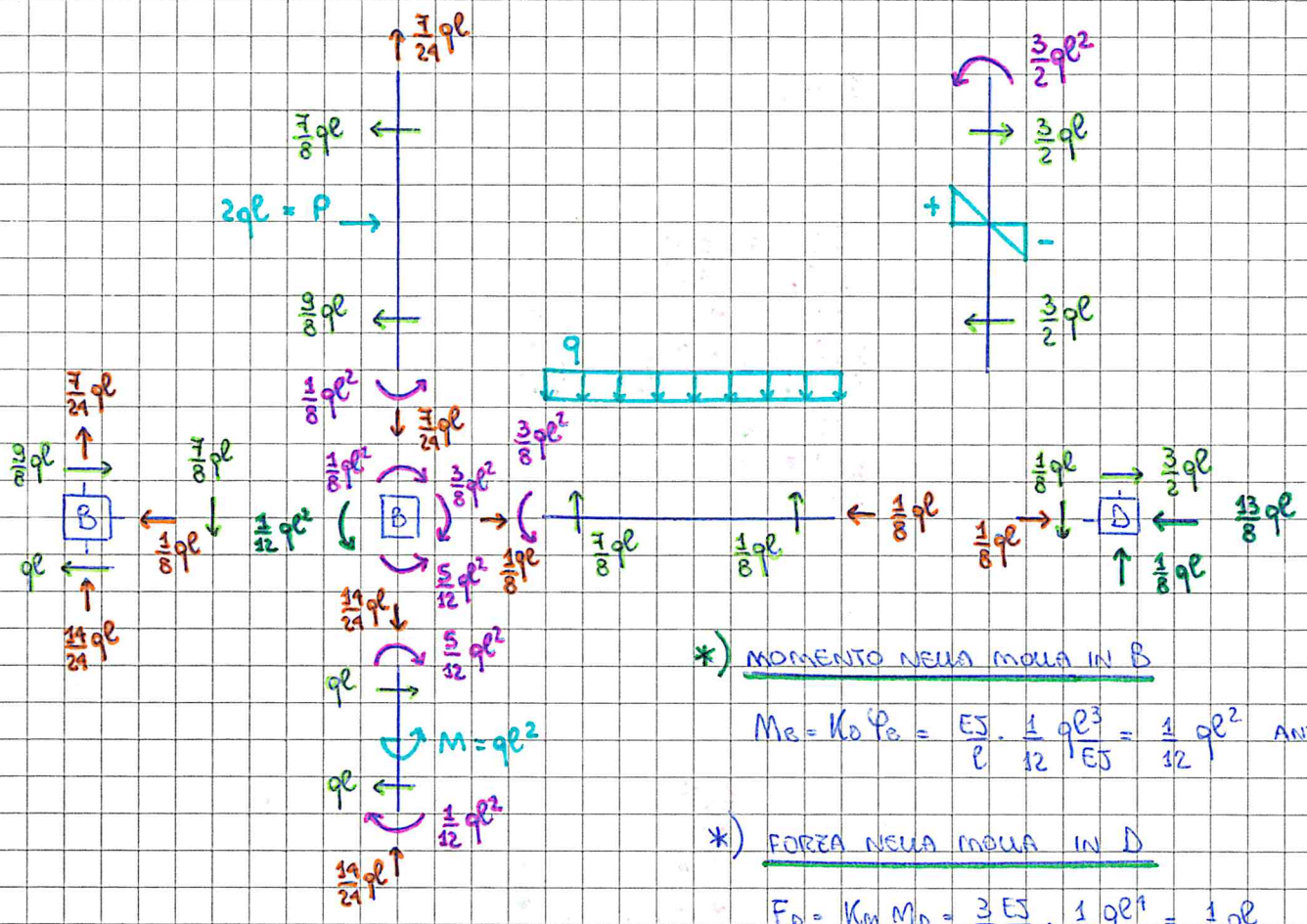
$M_D = \frac{1}{6} \frac{ql^4}{EJ}$:









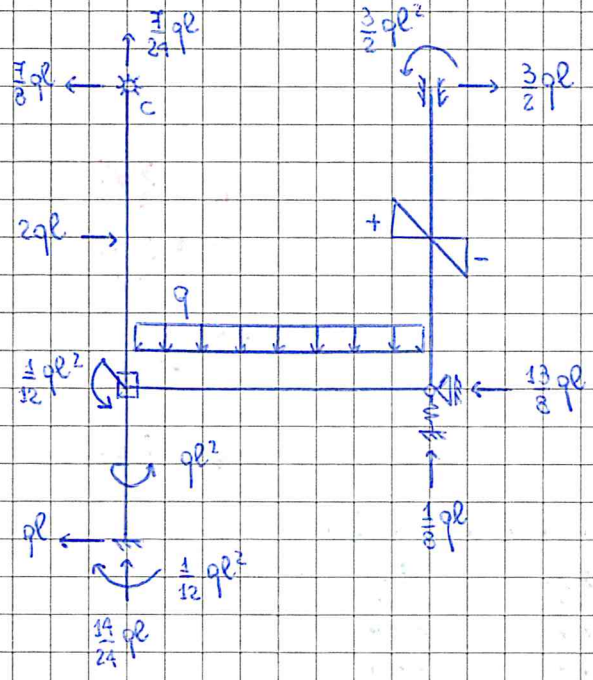


*) MOMENTO NELLA MOLLA IN B

$$M_B = K_\theta \varphi_B = \frac{EJ}{\ell} \cdot \frac{1}{12} \frac{q\ell^3}{EJ} = \frac{1}{12} q\ell^2$$ ANTIORARIO ↺

*) FORZA NELLA MOLLA IN D

$$F_D = K_m v_D = \frac{3}{4} \frac{EJ}{\ell^3} \cdot \frac{1}{6} \frac{q\ell^4}{EJ} = \frac{1}{8} q\ell$$ VERSO L'ALTO ↑

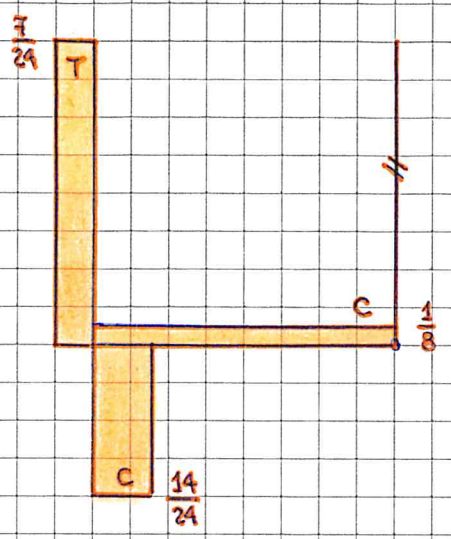


$$\sum F^{\rightarrow} = 0 : \frac{3}{2}q\ell - \frac{7}{8}q\ell + 2q\ell - \frac{13}{8}q\ell - q\ell = 0$$

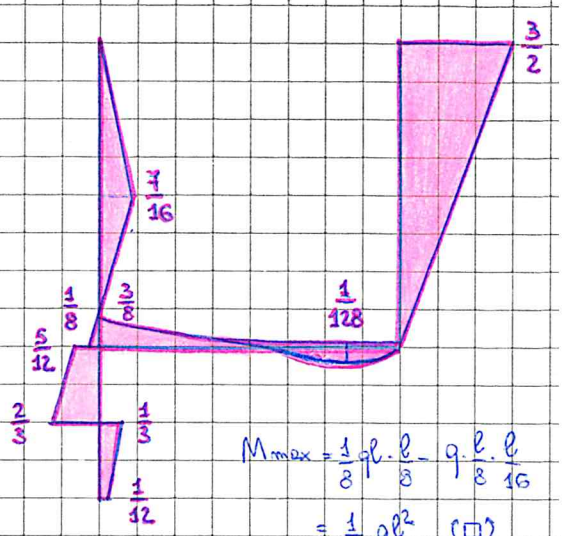
$$\sum F^{\uparrow} = 0 : \frac{7}{24}q\ell - q\ell + \frac{1}{8}q\ell + \frac{14}{24}q\ell = 0$$

$$\sum M^{\circlearrowright}_C = 0 : -\frac{3}{2}q\ell^2 - 2q\ell \cdot \frac{\ell}{2} - \frac{1}{12}q\ell^2 + q\ell \cdot \frac{\ell}{2} + \frac{13}{8}q\ell \cdot \ell - \frac{1}{8}q\ell\ell - q\ell^2 + q\ell \cdot \frac{3\ell}{2} + \frac{1}{12}q\ell^2 = 0$$

(N)

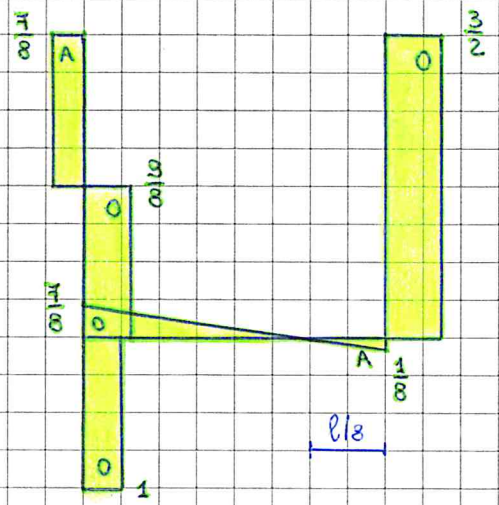


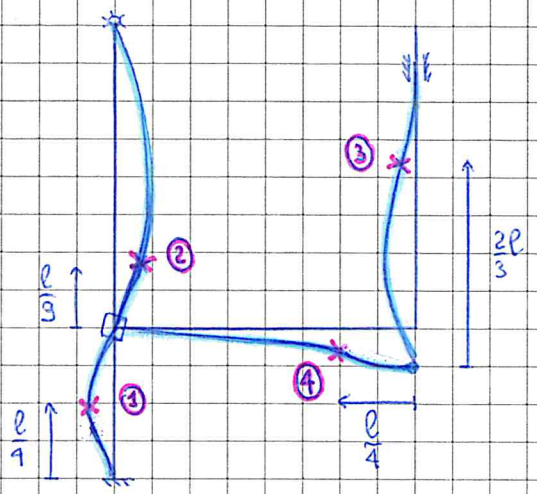
(M)



$$M_{max} = \frac{1}{8}ql \cdot \frac{l}{8} - q\frac{l}{8}\cdot\frac{l}{16} = \frac{1}{128}ql^2 \quad (\square)$$

(V)





FLESSI :

(1) asta $\overline{AB}$: $x = \frac{l}{4}$

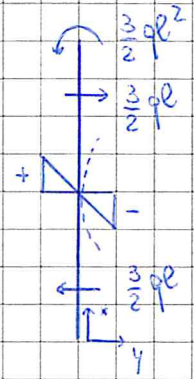
(2) asta $\overline{BC}$: $\dfrac{\frac{1}{8}ql}{x} = \dfrac{\frac{7}{16}ql}{\frac{l}{2}-x} \longrightarrow \frac{1}{8}\left(\frac{l}{2}-x\right) = \frac{7}{16}x \longrightarrow \frac{9}{16}x = \frac{l}{16} \longrightarrow x = \frac{l}{9}$

(3) asta $\overline{DE}$: $M(x) = \frac{3}{2}qlx$

$$y'' = -\frac{3}{2}\frac{qlx}{EJ} + 2\cdot\frac{1}{2}\frac{ql^2}{EJ} \geq 0 \longrightarrow \frac{3}{2}x \leq l$$

$x \leq \frac{2}{3}l$ da D (+)

$x > \frac{2}{3}l$ (−)



(4) asta $\overline{BD}$: $M(x) = q\frac{x^2}{2} - \frac{1}{8}qlx = 0 \longrightarrow \frac{q}{2}x\left(x - \frac{1}{4}l\right) = 0 \longrightarrow x = \frac{l}{4}$ da D