

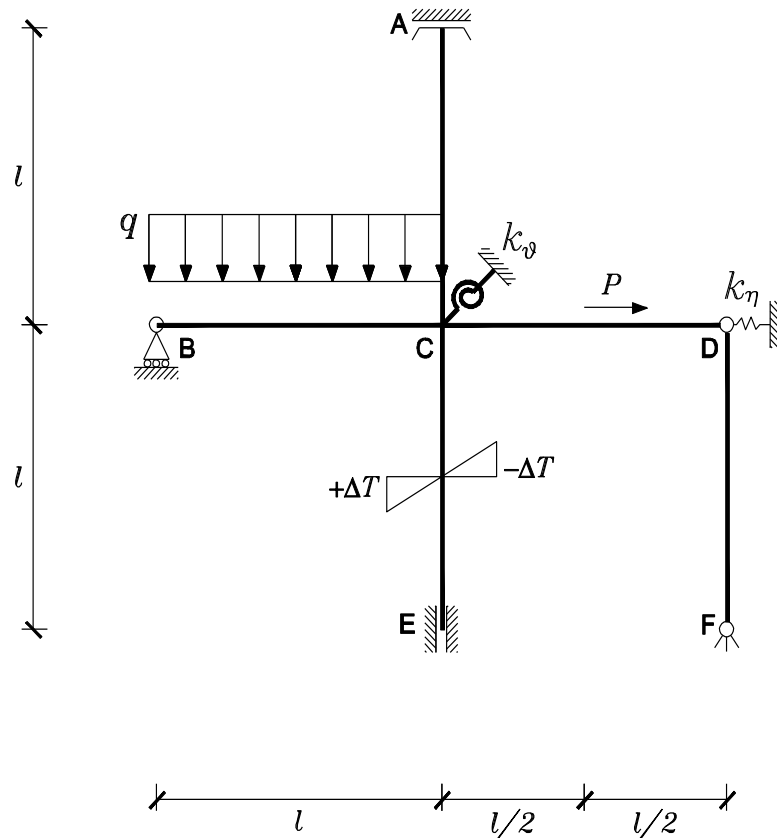
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 12 SETTEMBRE 2016

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PROF. FAUSTO MINELLI

ESERCITATORE: ING. LUCA FACCONI

Esercizio



$$K_{\theta} = 15 \frac{EJ}{l};$$

$$K_{\eta} = \frac{11}{2} \frac{EJ}{l^3};$$

$$P = 4ql;$$

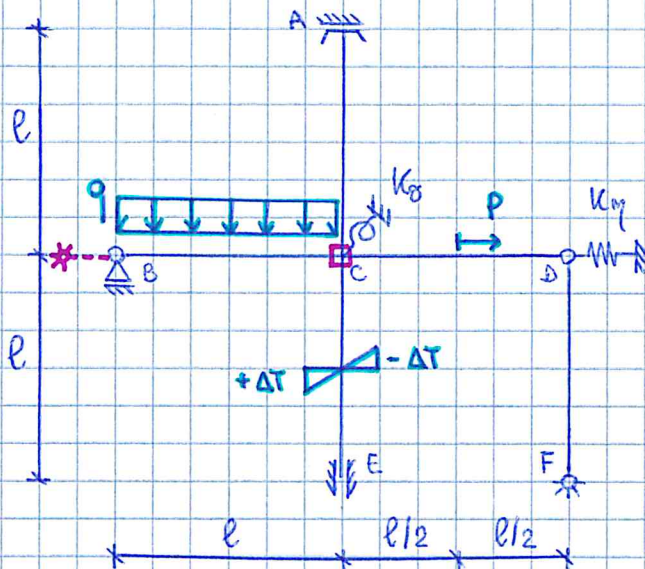
$$\frac{\alpha \Delta T}{h} = \frac{1}{8} \frac{ql^2}{EJ}.$$

Dato il telaio in figura

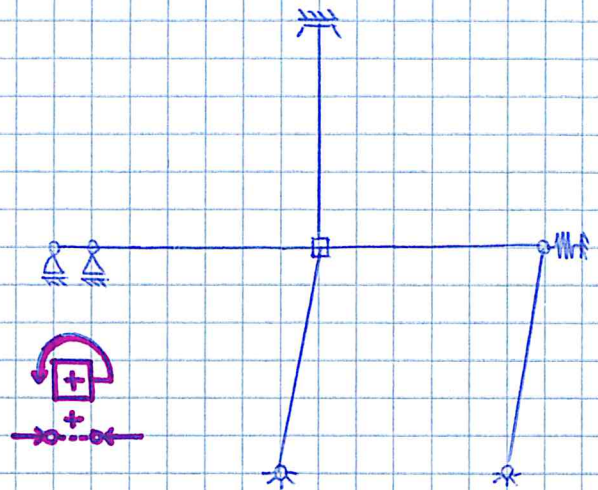
Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TELAIO 12/09/2016

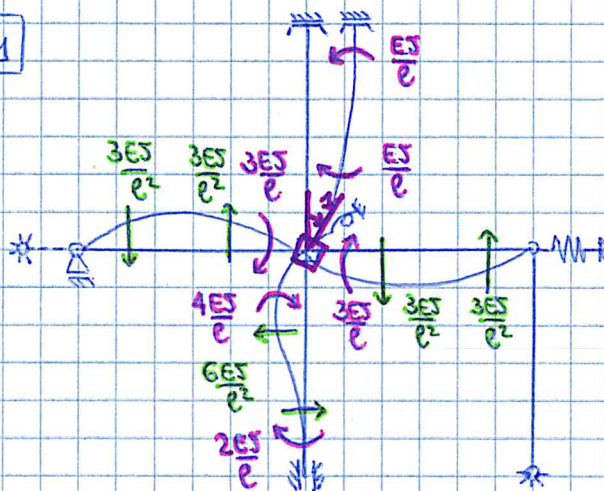


Nodi spostabili:



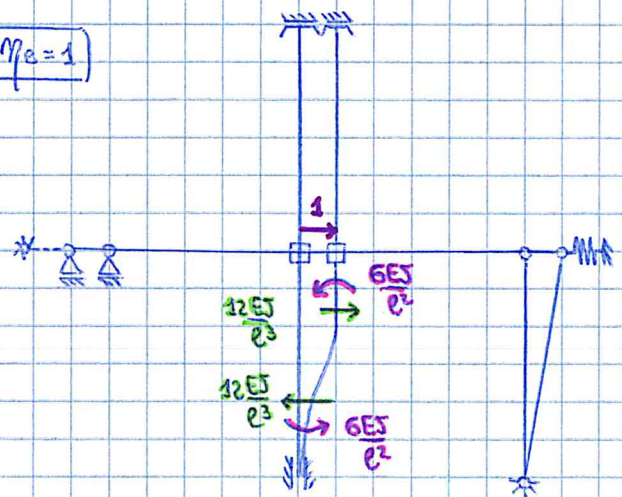
$$\begin{cases} m_{cc} \varphi_c + m_{ce} \eta_e + m_{cd} = 0 \\ h_{cc} \varphi_c + h_{ce} \eta_e + h_{cd} = 0 \end{cases}$$

$\varphi_c = 1$



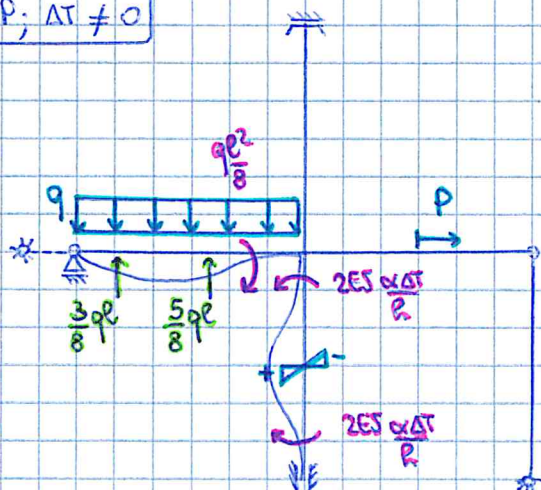
$$\begin{cases} m_{cc} = \frac{3ES}{l} + \frac{3ES}{l} + \frac{4ES}{l} + K_D + \frac{ES}{l} \\ h_{cc} = -\frac{6ES}{l^2} \end{cases}$$

$\eta_e = 1$



$$\begin{cases} m_{ce} = -\frac{6ES}{l^2} \\ h_{ce} = \frac{12ES}{l^3} + K_D \end{cases}$$

$q, P, \Delta T \neq 0$



$$\begin{cases} m_{cd} = -\frac{2ES \alpha \Delta T}{l} + \frac{ql^2}{8} \\ h_{cd} = -P \end{cases}$$

SISTEMA:

$$\begin{cases} \left(\frac{11 EJ}{l^2} + K_{\alpha} \right) \varphi_c - \frac{6 EJ}{l^2} \eta_B + \left(-2 EJ \frac{\alpha \Delta T}{R} + \frac{q l^2}{8} \right) = 0 \\ -\frac{6 EJ}{l^2} \varphi_c + \left(\frac{12 EJ}{l^3} + K_{\eta} \right) \eta_B - P = 0 \end{cases}$$

- $P = 4 q l$
- $K_{\alpha} = 15 \frac{EJ}{l}$
- $K_{\eta} = \frac{11 EJ}{2 l^3}$
- $\frac{\alpha \Delta T}{R} = \frac{1}{8} \frac{q l^2}{EJ}$

$$\begin{cases} \left(\frac{11 EJ}{l} + 15 \frac{EJ}{l} \right) \varphi_c - \frac{6 EJ}{l^2} \eta_B + \left(-2 EJ \cdot \frac{1}{8} \frac{q l^2}{EJ} + \frac{1}{8} q l^2 \right) = 0 \\ -\frac{6 EJ}{l^2} \varphi_c + \left(\frac{12 EJ}{l^3} + \frac{11 EJ}{2 l^3} \right) \eta_B - 4 q l = 0 \end{cases}$$

$$\begin{cases} 26 \frac{EJ}{l} \varphi_c - \frac{6 EJ}{l^2} \eta_B - \frac{1}{8} q l^2 = 0 \\ -\frac{6 EJ}{l^2} \varphi_c + \frac{35 EJ}{2 l^3} \eta_B - 4 q l = 0 \end{cases}$$

$$\begin{vmatrix} +26 & -6 & -\frac{1}{8} \\ -6 & +\frac{35}{2} & -4 \end{vmatrix} \begin{matrix} \times 1 \\ \times \frac{26}{6} \end{matrix} \quad \begin{vmatrix} +26 & -6 & -\frac{1}{8} \\ -26 & \frac{455}{6} & -\frac{52}{3} \end{vmatrix} \begin{matrix} + \\ + \end{matrix} \quad \begin{cases} \left(-6 + \frac{455}{6} \right) \eta_B + \left(-\frac{1}{8} - \frac{52}{3} \right) = 0 \\ \eta_B = \frac{413}{24} \cdot \frac{6}{413} = \frac{1}{4} \frac{q l^4}{EJ} \end{cases}$$

$$\varphi_c = \left(\frac{6 EJ}{l^2} \cdot \frac{1}{4} \frac{q l^4}{EJ} + \frac{1}{8} q l^2 \right) \cdot \frac{l}{26 EJ} = \frac{1}{16} \frac{q l^3}{EJ}$$

↓

$$\boxed{\varphi_c = \frac{1}{16} \frac{q l^3}{EJ}}$$

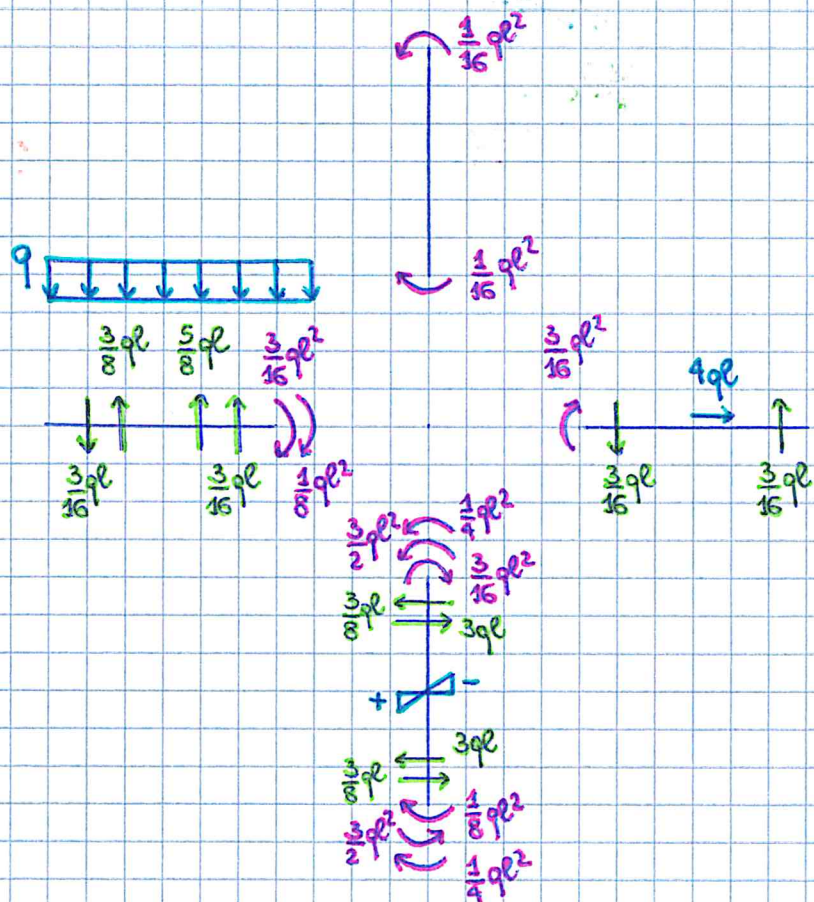
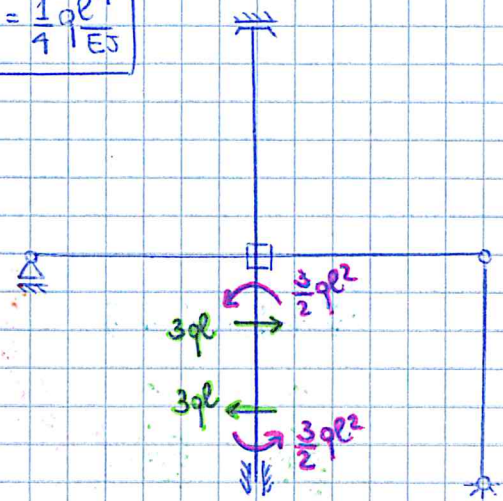
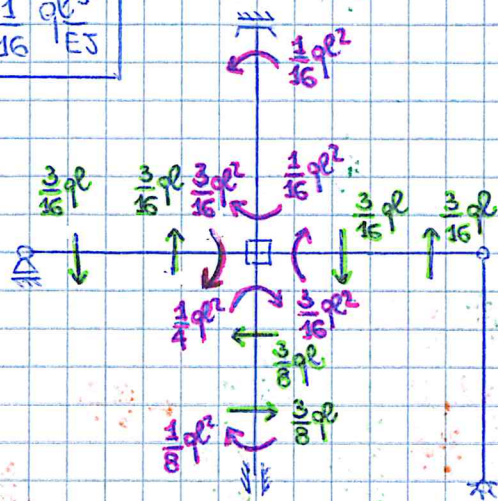
ORARIA

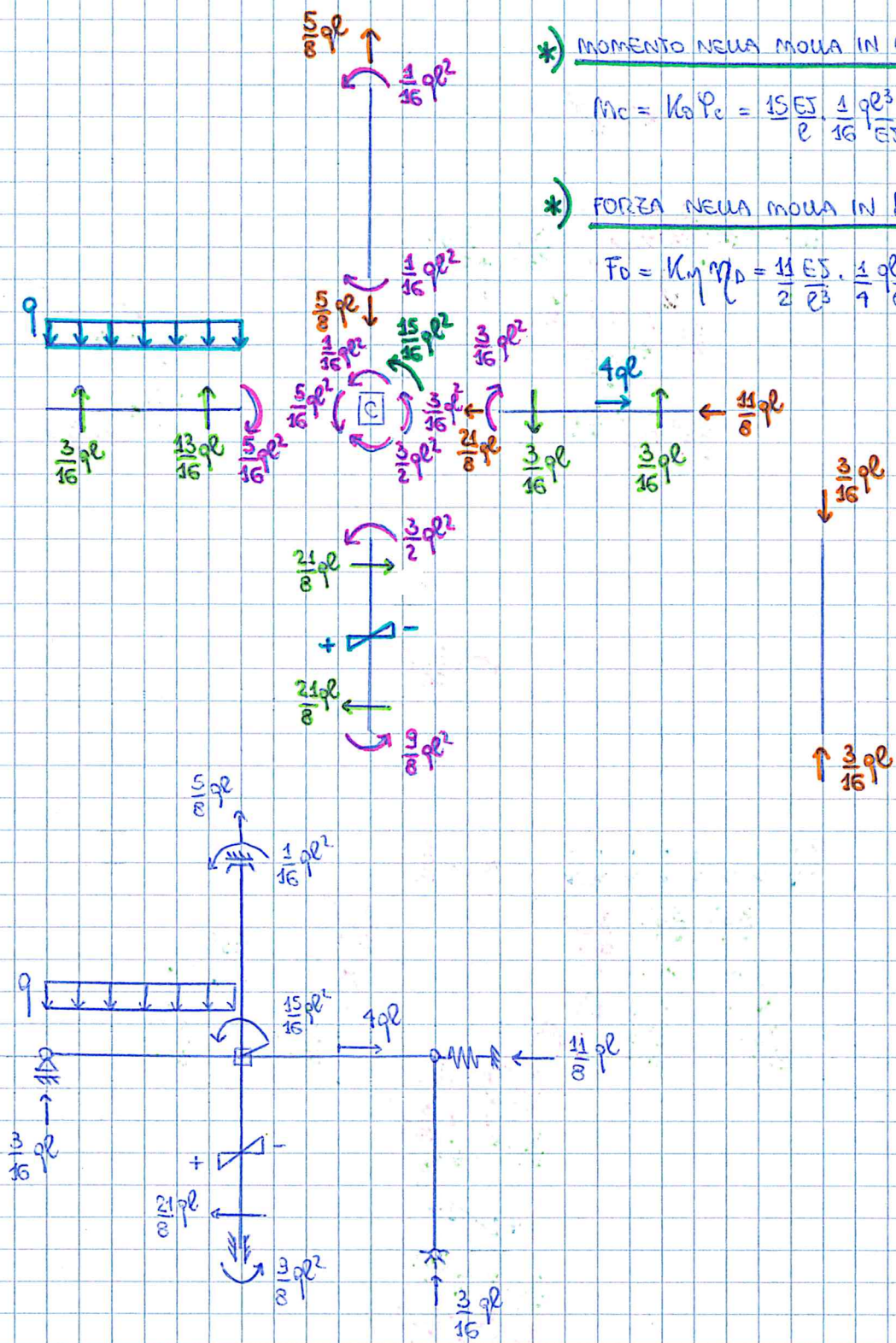
$$\boxed{\eta_B = \frac{1}{4} \frac{q l^4}{EJ}}$$

VERNO DESTRA

$$\varphi_c = \frac{1}{16} \frac{ql^3}{EI}$$

$$\eta_B = \frac{1}{4} \frac{ql^4}{EI}$$





*) MOMENTO NELLA MOLLA IN C:

$$M_C = K_\theta \varphi_C = \frac{15 ES}{l} \cdot \frac{1}{16} \frac{ql^3}{ES} = \frac{15}{16} ql^2 \quad \curvearrowright$$

*) FORZA NELLA MOLLA IN D:

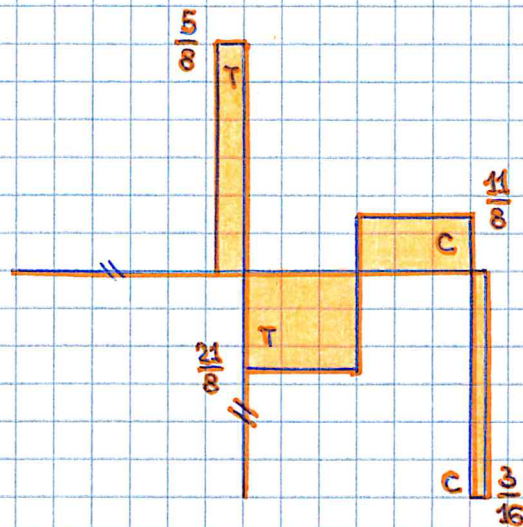
$$F_D = K_\eta \eta_D = \frac{11 ES}{2 l^3} \cdot \frac{1}{4} \frac{ql^4}{ES} = \frac{11}{8} ql \quad \leftarrow$$

$$\sum F^x = 0 : 4ql - \frac{21}{8} ql - \frac{11}{8} ql = 0 \quad \checkmark$$

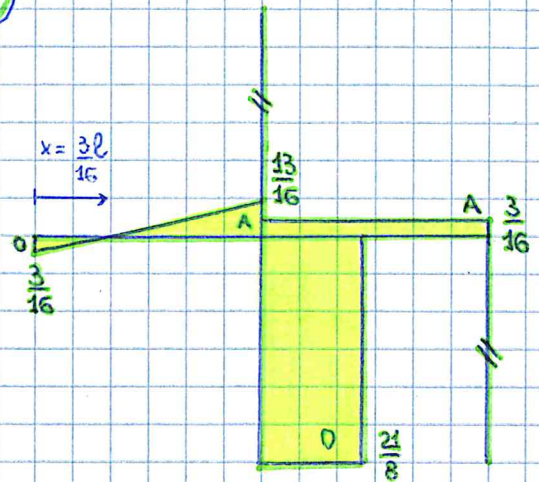
$$\sum F^y = 0 : \frac{3}{16} ql + \frac{5}{8} ql + \frac{3}{16} ql - ql = 0 \quad \checkmark$$

$$\sum M^{B \curvearrowright} = 0 : ql \cdot \frac{l}{2} - \frac{15}{16} ql^2 - \frac{1}{16} ql^2 - \frac{5}{8} ql \cdot l + \frac{21}{8} ql \cdot l - \frac{3}{8} ql^2 - \frac{3}{16} ql \cdot 2l = 0 \quad \checkmark$$

(N)



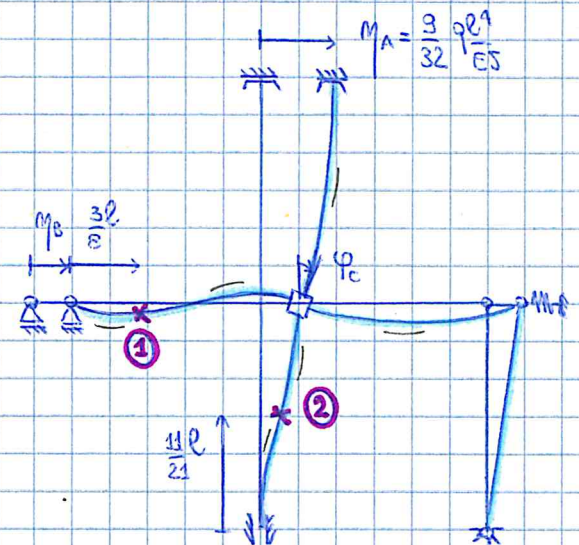
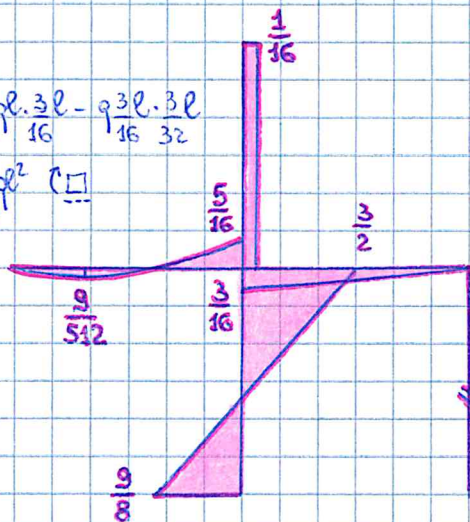
(V)



(M)

$$M_{max ec} = \frac{3ql}{16} \cdot \frac{3l}{16} - \frac{q \cdot 3l \cdot 3l}{16 \cdot 32}$$

$$= \frac{9}{512} ql^2 \quad \square$$



FLESSI:

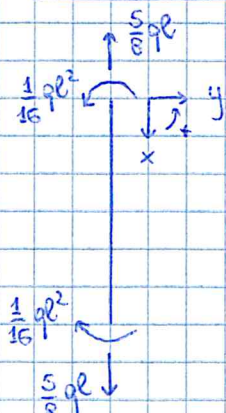
*) asta $\overline{BC}$: $M(x) = \frac{3ql}{16}x - \frac{qx^2}{2} = 0 \rightarrow \frac{1}{2}x\left(\frac{3l}{8} - x\right) = 0 \rightarrow x_1 = \frac{3l}{8}$

*) asta $\overline{CE}$: $M(x) = -\frac{9}{8}ql^2 + \frac{21}{8}qlx$

$$y'' = \frac{9}{8} \frac{ql^2}{ES} - \frac{21}{8} \frac{qlx}{ES} + 2 \cdot \frac{1}{8} \frac{ql^2}{ES} = \frac{11}{8} \frac{ql^2}{ES} - \frac{21}{8} \frac{qlx}{ES} \geq 0$$

$$\frac{1}{8} \frac{ql}{ES} (11l - 21x) \geq 0 \rightarrow x_2 = \frac{11l}{21}$$

SPOSTAMENTO IN A:



$$M(x) = \frac{1}{16} ql^2$$

$$y''(x) = -\frac{ql^2}{16ES}$$

$$y'(x) = -\frac{ql^2}{16ES}x + C_1$$

$$y(x) = -\frac{ql^2}{16ES} \frac{x^2}{2} + C_1x + C_2$$

• $y'(0) = 0 \rightarrow C_1 = 0$

• $y(l) = \frac{1}{4} \frac{ql^4}{ES} \rightarrow -\frac{1}{32} \frac{ql^4}{ES} + C_2 = \frac{1}{4} \frac{ql^4}{ES} \rightarrow C_2 = \frac{9}{32} \frac{ql^4}{ES}$

$M_A = y(0) = C_2 = \frac{9}{32} \frac{ql^4}{ES} > M_B$ + A DESTRA!