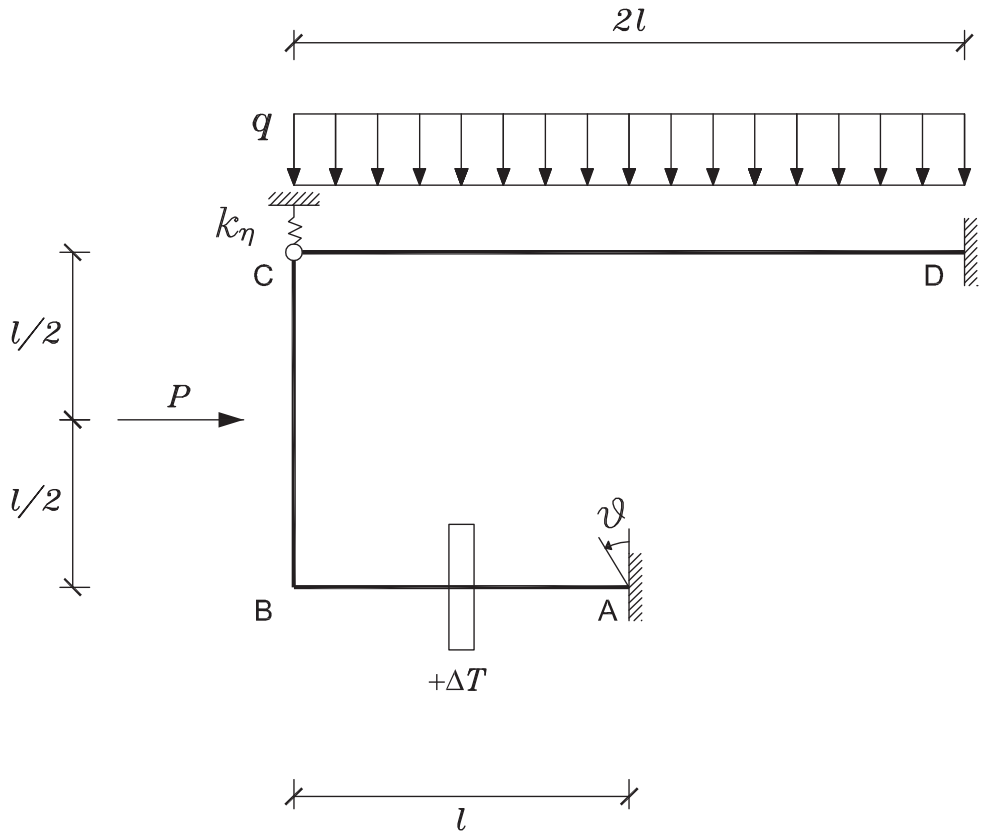
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 29 NOVEMBRE 2016

DOCENTE: **PROF. FAUSTO MINELLI** ESERCITATORE: **ING. LUCA FACCONI**

Esercizio



$$\vartheta = \frac{2}{3}\frac{ql^3}{EJ};$$

$$P = 4ql;$$

$$\alpha\Delta T = \frac{7}{36}\frac{ql^3}{EJ};$$

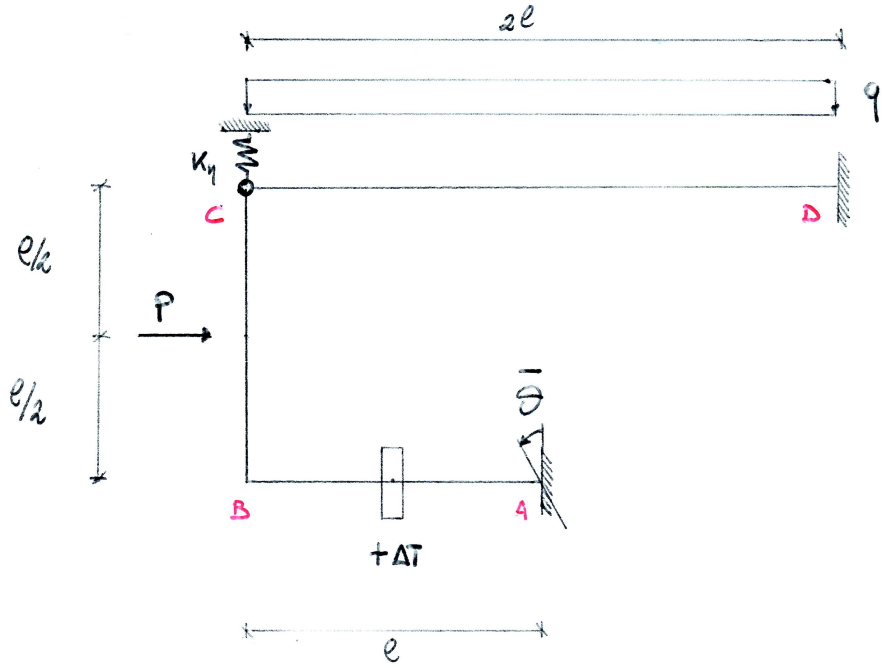
$$K_\eta = \frac{21}{8}\frac{EJ}{l^3}$$

Dato il telaio in figura

Si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

* TEMA ESAME 29/11/2016



$\bar{\theta} = \frac{2}{3}\frac{ql^3}{EJ}$

$P = 4ql$

$\alpha\Delta T = \frac{7}{36}\frac{ql^3}{EJ}$

$K_\eta = \frac{21}{8}\frac{EJ}{l^3}$

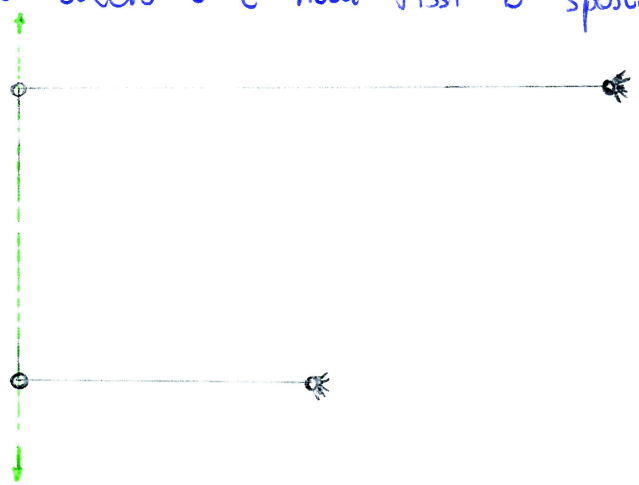
- Si valuta il grado di iperstaticità della struttura:

G.d.V = 3 + 3 + 2 + 1 = 9

G.d.L = 3 * 2 = 6

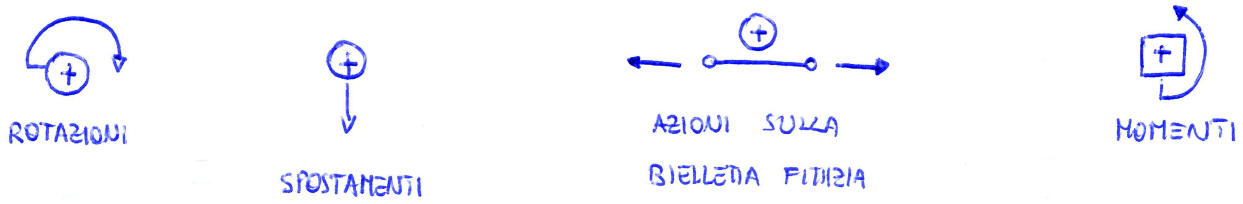
➡ struttura 3 volte iperstatica.

- Si valuta se il telaio è a nodi fissi o spostabili:

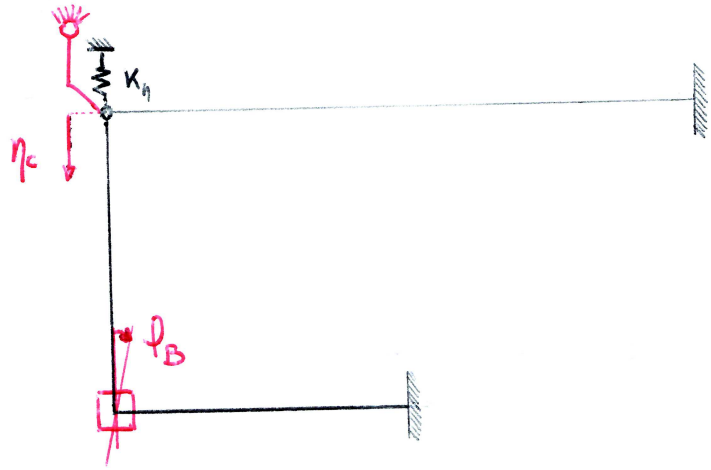


➡ si individua chiaramente un cinematismo possibile per l'asta BC. Il telaio è quindi a nodi spostabili.

- Convenzioni adottate:



- Si valutano le incognite necessarie alla risoluzione:

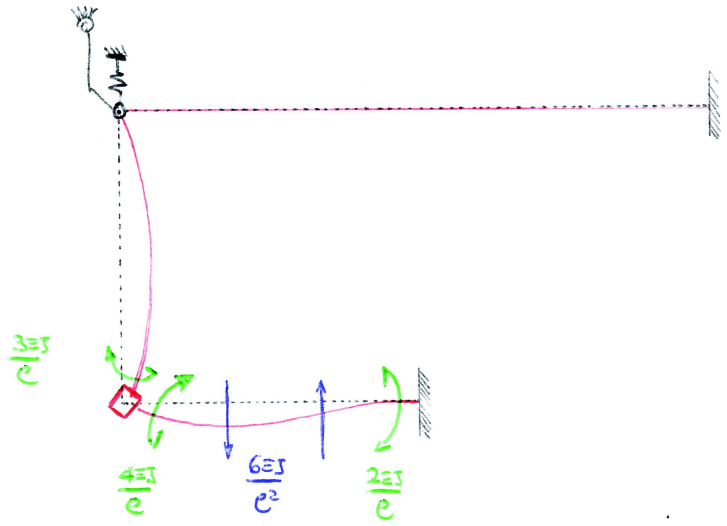


➡ sono necessarie 2 incognite per risolvere il sistema applicando il metodo degli spostamenti: $\begin{cases} \varphi_B \\ \eta_c \end{cases}$

- Sistema risolvente:

$$\begin{cases} m_{BB}\,\varphi_B + m_{Bc}\,\eta_c + m_{B0} = 0 \\ h_{cB}\,\varphi_B + h_{cc}\,\eta_c + h_{c0} = 0 \end{cases}$$

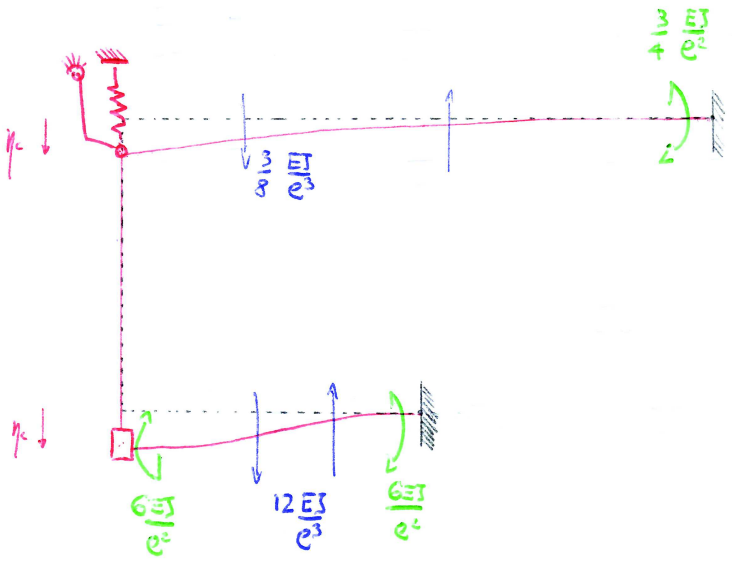
- CASO 1: $\varphi_B = 1$



$$m_{BB} = +7\,\frac{EJ}{\ell}$$

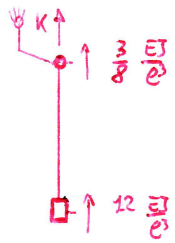
$$h_{CB} = -6\,\frac{EJ}{\ell^2}$$

- CASO 2: $\eta_C = 1$



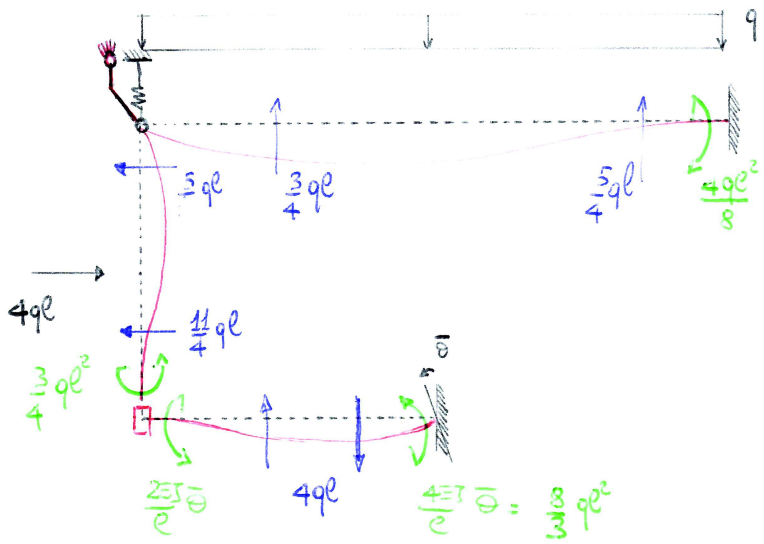
$$m_{BC} = +6\,\frac{EJ}{\ell^2}$$

$$h_{CC} = ?$$



$$h_{CC} = -12\,\frac{EJ}{\ell^3} - \frac{3}{8}\,\frac{EJ}{\ell^3} - \frac{21}{8}\,\frac{EJ}{\ell^3} = -15\,\frac{EJ}{\ell^3}$$

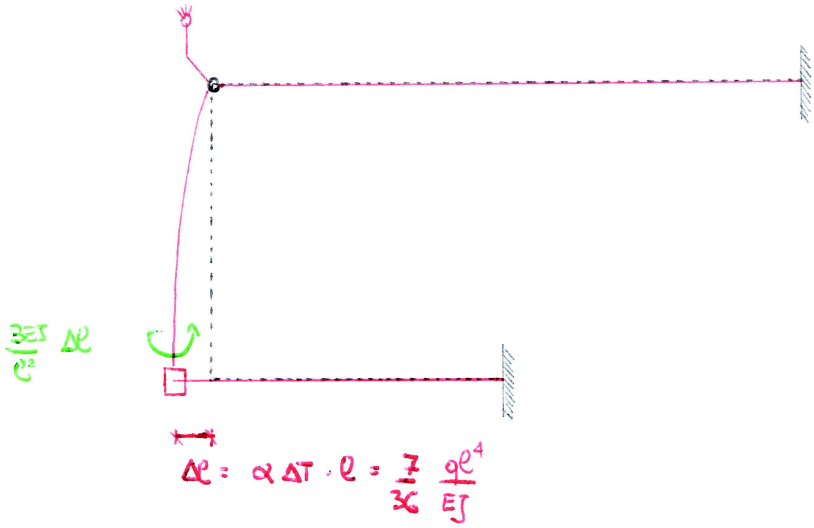
- CASO 3: carichi esterni 1



$$m^1_{BO} = -\frac{4}{3}q\ell^2 - \frac{3}{4}q\ell^2 = -\frac{25}{12}q\ell^2$$

$$h^1_{CO} = \frac{3}{4}q\ell + 4q\ell = \frac{19}{4}q\ell$$

• CASO 4: carichi esterni 2



$$m^2_{BO} = -\frac{3EJ}{\ell^2}\cdot\frac{7}{36}\frac{q\ell^4}{EJ} = -\frac{7}{12}q\ell^2$$

$$h^2_{CO} = 0$$

➡ Sistema Risolvente:

$$\begin{cases} 7\frac{EJ}{\ell}\varphi_B + 6\frac{EJ}{\ell^2}\eta_C - \frac{25}{12}q\ell^2 - \frac{7}{12}q\ell^2 = 0 & 1) \\ -6\frac{EJ}{\ell^2}\varphi_B - 15\frac{EJ}{\ell^3}\eta_C + \frac{19}{4}q\ell + 0 = 0 & 2) \end{cases}$$

moltiplico l'equazione 1) per $\frac{15}{6\ell}$ e la sommo alla 2)

$$\frac{35}{2}\frac{EJ}{\ell^2}\varphi_B - 6\frac{EJ}{\ell^2}\varphi_B - \frac{125}{24}q\ell - \frac{35}{24}q\ell + \frac{19}{4}q\ell = 0$$

$$\frac{23}{2}\frac{EJ}{\ell^2}\varphi_B - \frac{23}{12}q\ell = 0$$

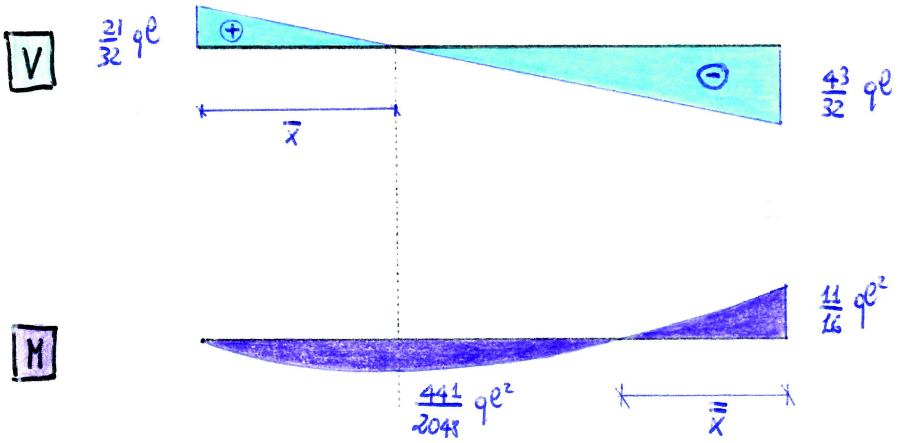
da cui ricavo:

$$\varphi_B = +\frac{1}{6}\frac{q\ell^3}{EJ}$$

$$\eta_C = +\frac{1}{4}\frac{q\ell^4}{EJ}$$

- Si passa quindi al calcolo delle azioni interne:

ASTA CD



$$\bar{x} = \frac{21}{32}\ell$$

$$M_{max} = q\left(\frac{21}{32}\ell\right)^2 - \frac{q}{2}\left(\frac{21}{32}\ell\right)^2 = \frac{441}{2048} q\ell^2$$

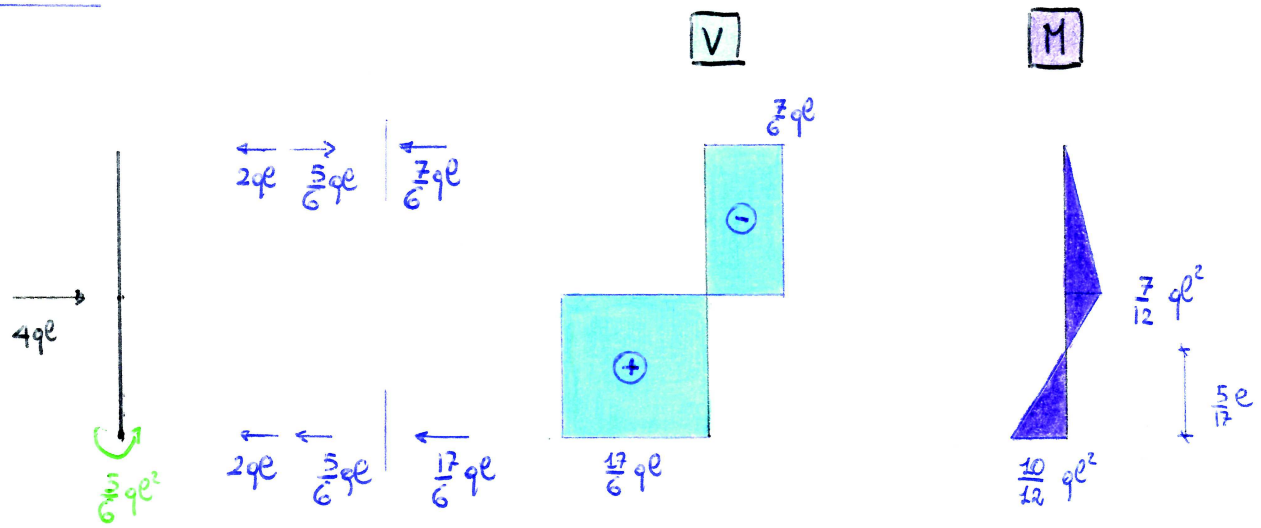
$$M(x) = \frac{43}{32} q\ell\, x - q\frac{x^2}{2} - \frac{11}{16} q\ell^2$$

pongo $M(\bar{\bar{x}}) = 0$

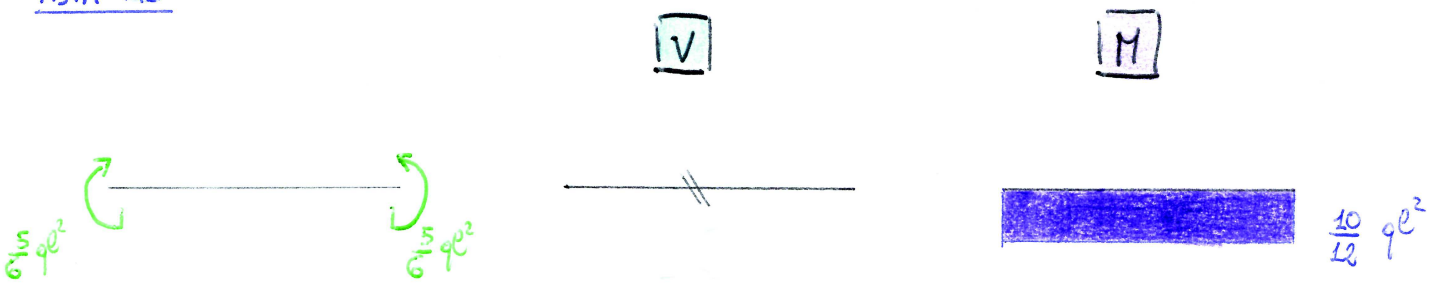
$$\bar{\bar{x}}^2 - \frac{43}{16}\ell\,\bar{\bar{x}} + \frac{11}{8}\ell^2 = 0$$

$$\bar{\bar{x}} = \frac{11}{16}\ell$$

ASTA BC

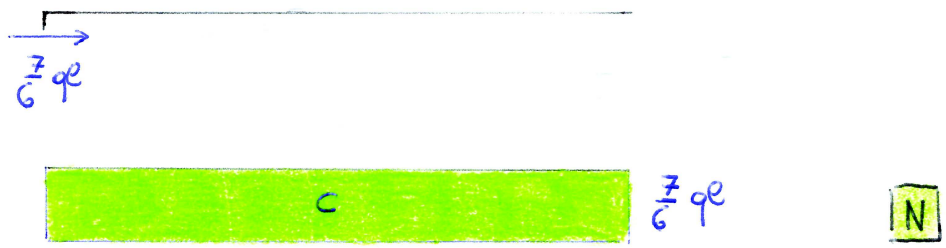


ASTA AB

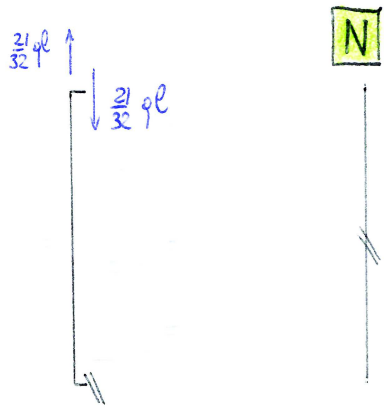


Si valutano quindi le azioni assiali

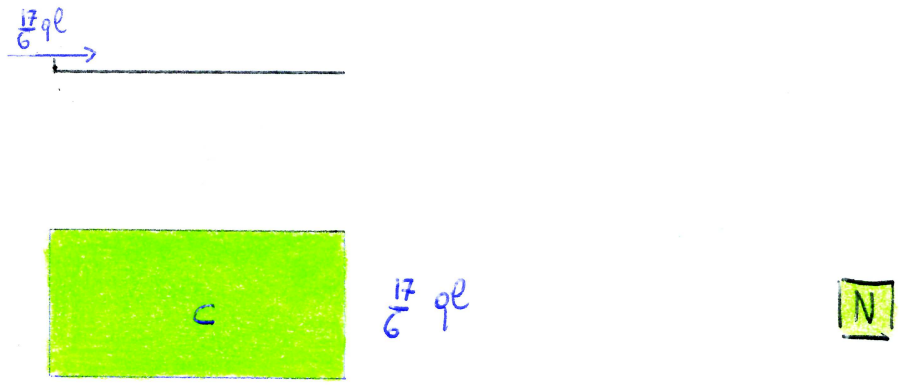
ASTA CD



ASTA BC

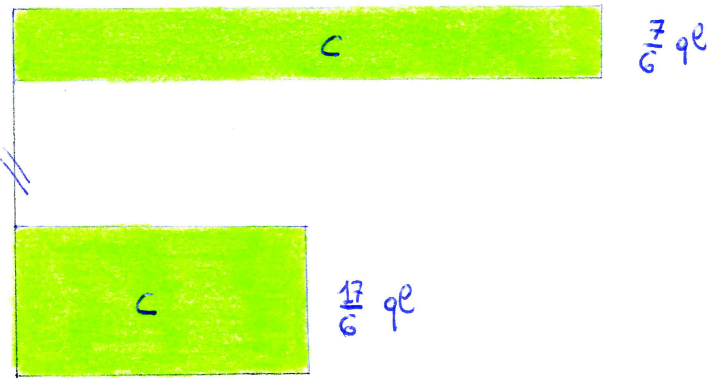


ASTA AB

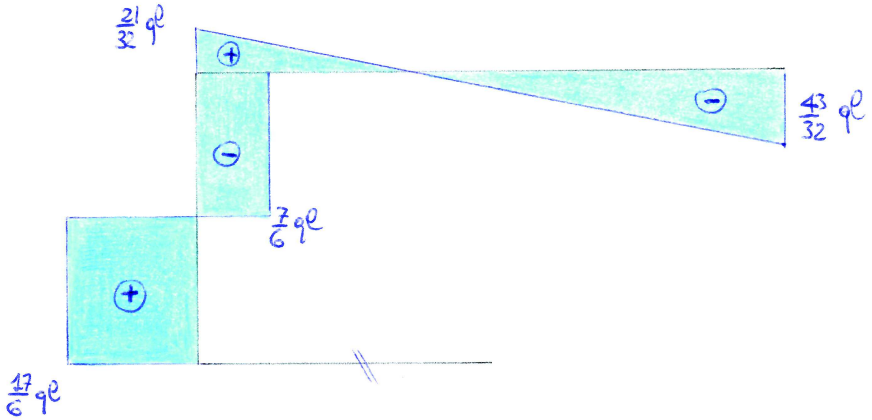


- Grafici finali:

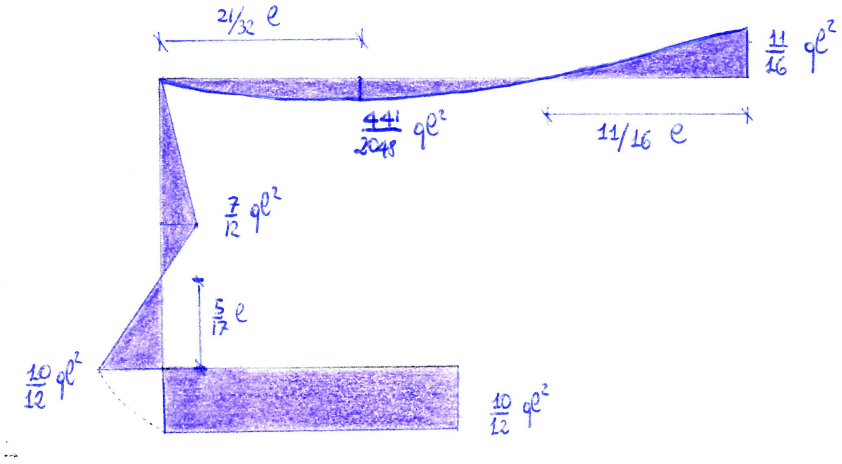
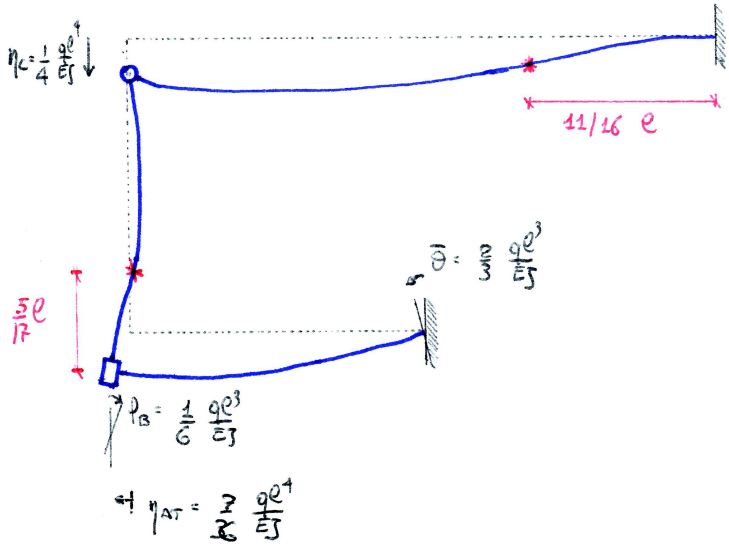
AZIONE ASSIALE



TAGLIO



MOMENTO



DEFORMATA QUALITATIVA