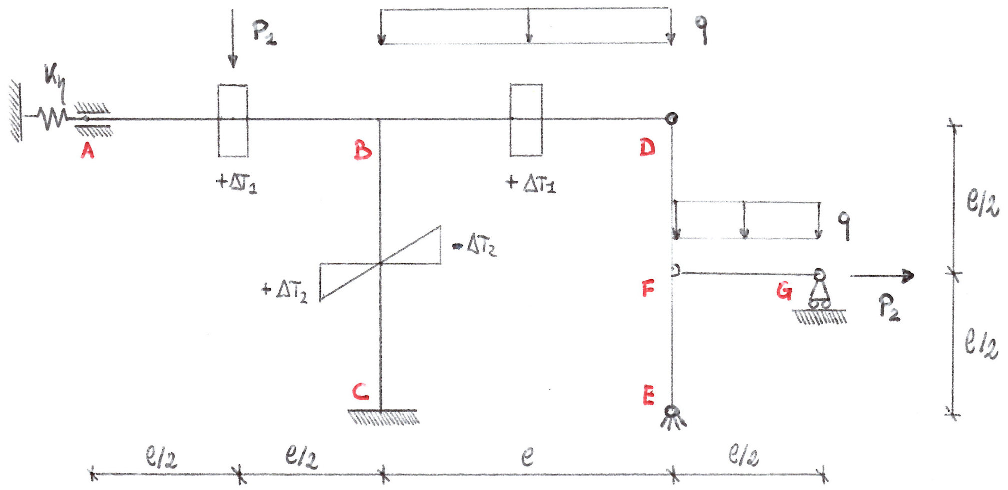
* TEMA ESAME 09/01/2017




Dati:

$P_1 = 21\, q\ell$

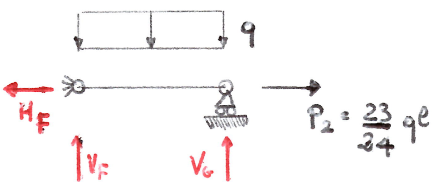
$P_2 = \frac{23}{24}\, q\ell$

$\alpha \Delta T_1 = \frac{1}{6} \frac{q\ell^3}{EJ}$

$\frac{\alpha \Delta T_2}{h} = \frac{1}{8} \frac{q\ell^2}{EJ}$

$K_\eta = \frac{1}{12} \frac{EJ}{\ell^3}$

- Si osserva come l'asta FG sia un'appendice isostatica e può quindi essere risolta con gli equilibri e "staccata" dalla struttura principale:

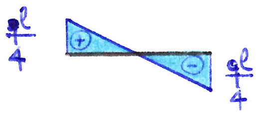



$V_F = V_G = \frac{q\ell}{4}$

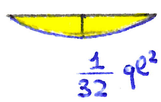
$H_F = \frac{23}{24}\, q\ell$

grafici delle azioni interne:

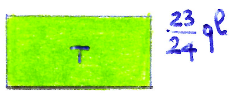
Taglio.




Momento.




Azione Assiale



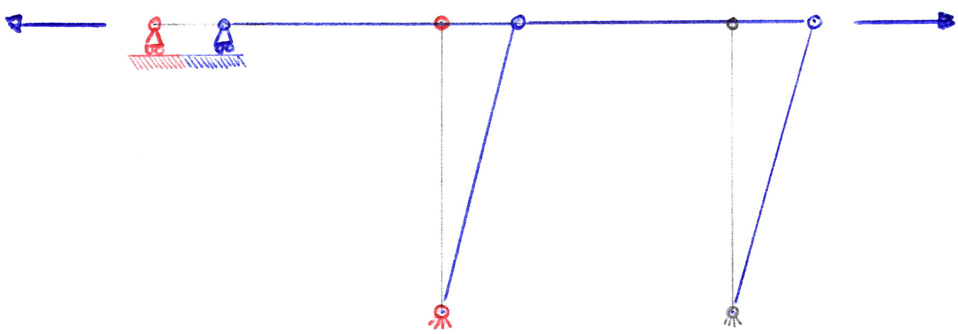
• Si valuta quindi il grado di iperstaticità della struttura:

G.d.V: $2+1+3+2+2 = 10$

G.d.L: $2 \times 3 = 6$

⇒ STRUTTURA 4 VOLTE IPERSTATICA

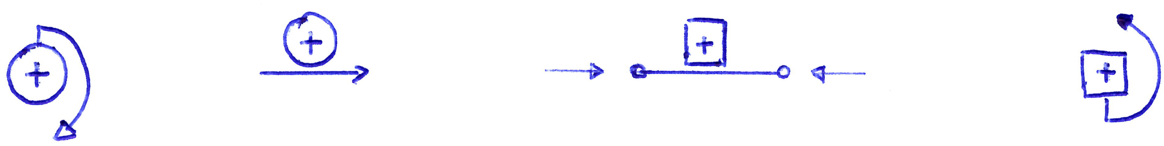
• Si valuta se il telaio sia a nodi fissi o spostabili:



<u>OSSERVAZIONE</u>: IN ROSSO SI SONO RIPORTATI I "VINCOLI MODIFICATI" PER ENFATIZZARE LA DEFORMABILITÀ FLESSIONALE DELLE ASTE CHE COMPONGONO LA STRUTTURA.

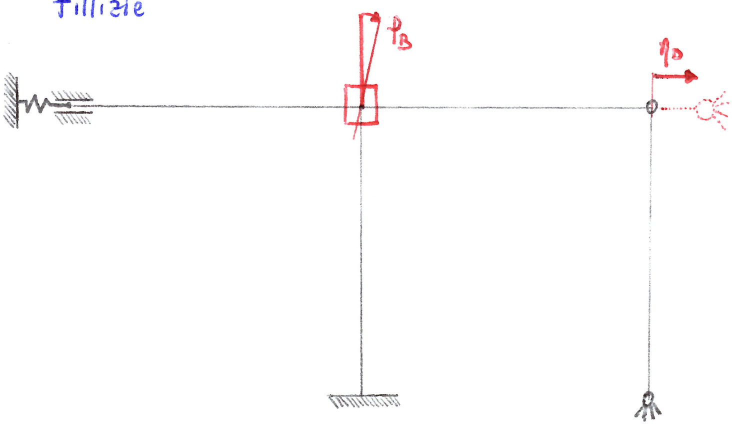
⇒ Si individua chiaramente un possibile cinematismo per le aste AB e BD.
<u>Il telaio è quindi a nodi spostabili.</u>

• Convenzioni adottate per la risoluzione del telaio:



rotazioni — spostamenti — azioni sulle bielle fittizie — momenti

• Si decide di risolvere il telaio adottando il <u>metodo degli spostamenti</u>, le incognite sono:



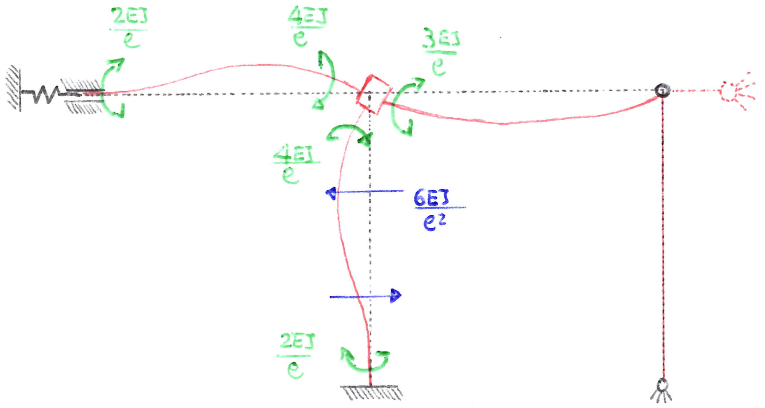

* $\varphi_B$

* $\eta_D$

Da cui il sistema risolvente (SISTEMA DELLE EQUAZIONI DI EQUILIBRIO)

$$\begin{cases} \Sigma M_B = 0 \\ \Sigma R_D = 0 \end{cases} \qquad \begin{cases} m_{BB}\,\varphi_B + m_{BD}\,\eta_D + m_0 = 0 \\ h_{DB}\,\varphi_B + h_{DD}\,\eta_D + h_{D0} = 0 \end{cases}$$

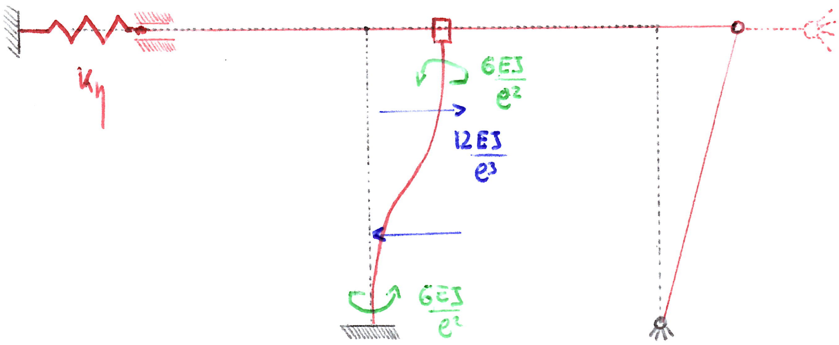
• CASO 1: $\varphi_B = 1$




$$m_{BB} = +11\,\frac{EJ}{\ell}$$

$$h_{DB} = +6\,\frac{EJ}{\ell^2}$$

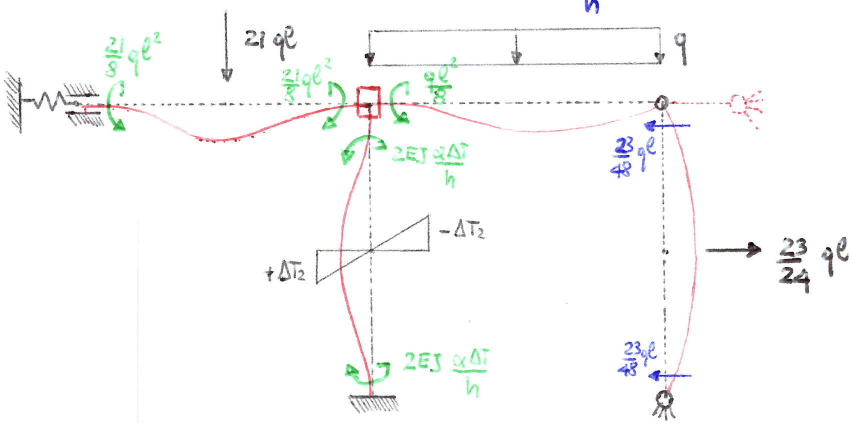
• CASO 2: $\eta_D = 1$




$$m_{BD} = -6\,\frac{EJ}{\ell^2}$$

$$h_{DD} = -12\,\frac{EJ}{\ell^3} - k_\eta \cdot 1 = -12\,\frac{EJ}{\ell^3} - \frac{1}{12}\,\frac{EJ}{\ell^3} = -\frac{145}{12}\,\frac{EJ}{\ell^3}$$

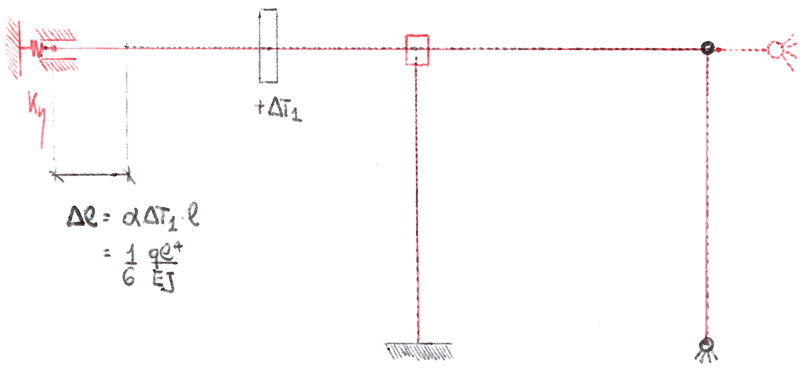
• CASO 3: $P_1 \neq 0$; $P_2 \neq 0$; $q \neq 0$; $\frac{\alpha \Delta T_2}{h} \neq 0$




$$m^3_{B0} = +\frac{21}{8} q\ell^2 - 2EJ \cdot \frac{1}{8} \cdot \frac{q\ell^2}{EJ} - \frac{q\ell^2}{8} = +\frac{9}{4} q\ell^2$$

$$h^3_{D0} = +\frac{23}{48} q\ell$$

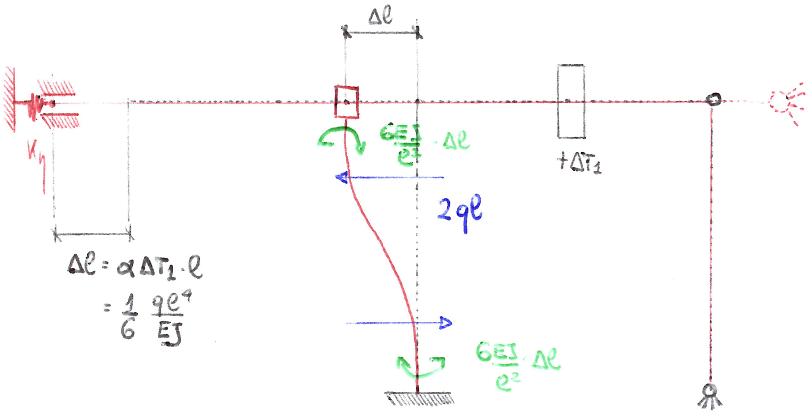
• CASO 4: $\Delta T_1 \neq 0$




$$m_{BO}^{4} = 0$$

$$h_{DO}^{4} = + K_\eta \cdot \Delta \ell = + \frac{1}{6} \frac{q\ell^4}{EJ} \cdot \frac{1}{12} \frac{EJ}{\ell^3} = + \frac{1}{72} q\ell$$

• CASO 5: $\Delta T_1 \neq 0$




$$m_{BO}^{5} = \frac{6EJ}{\ell^2} \cdot \frac{1}{6} \frac{q\ell^4}{EJ} = q\ell^2$$

$$h_{DO}^{5} = +2q\ell + K_\eta \cdot \Delta \ell = +2q\ell + \frac{1}{72} q\ell = \frac{145}{72} q\ell$$

Sistema risolvente:

$$\begin{cases} 11 \frac{EJ}{\ell} \varphi_B - 6 \frac{EJ}{\ell^2} \eta_D + \frac{9}{4} q\ell^2 + 0 + q\ell^2 = 0 & 1) \\ 6 \frac{EJ}{\ell^2} \varphi_B - \frac{145}{12} \frac{EJ}{\ell^3} \eta_D + \frac{23}{48} q\ell + \frac{1}{72} q\ell + \frac{145}{72} q\ell = 0 & 2) \end{cases}$$

moltiplico l'equazione 2) per $\frac{11}{6} \ell$ e la sottraggo alla 1)

$$- 6 \frac{EJ}{\ell^2} \eta_D + \frac{145}{12} \cdot \frac{11}{6} \frac{EJ}{\ell^2} \eta_D + \frac{13}{4} q\ell^2 - \frac{361}{144} \cdot \frac{11}{6} q\ell^2 = 0$$

$$\frac{1163}{72} \frac{EJ}{\ell^2} \eta_D - \frac{1163}{864} q\ell^2 = 0 \longrightarrow \eta_D = \frac{1}{12} \frac{q\ell^4}{EJ}$$

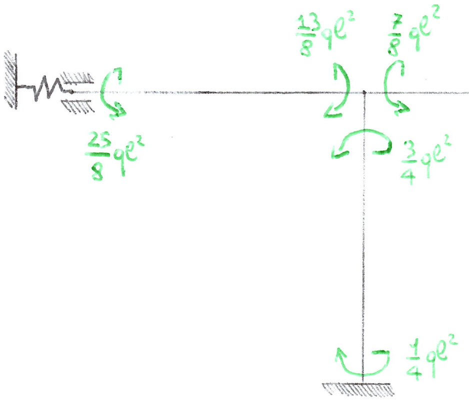
sostituendo $\eta_D$ in 1) ricavo:

$$\varphi_B = - \frac{1}{4} \frac{q\ell^3}{EJ}$$

Da cui in conclusione:

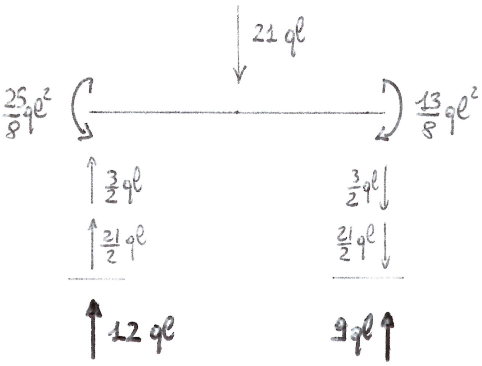
$$\begin{cases} \varphi_B = -\dfrac{1}{4}\dfrac{q\ell^3}{EJ} \\[2ex] \eta_D = +\dfrac{1}{12}\dfrac{q\ell^4}{EJ} \end{cases}$$

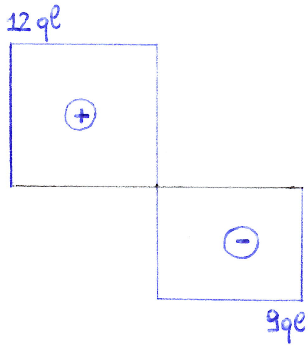
- Si passa quindi al calcolo delle azioni interne

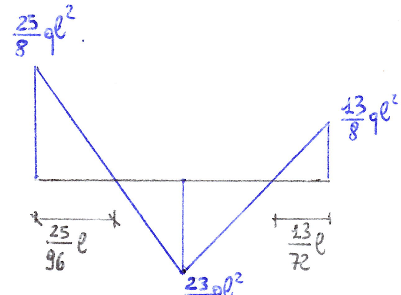



SOMMANDO I CONTRIBUTI DAI 5 CASI PRECEDENTI SI OTTENGONO I SEGUENTI MOMENTI ALLE ESTREMITÀ DELLE ASTE

ASTA AB

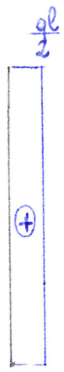


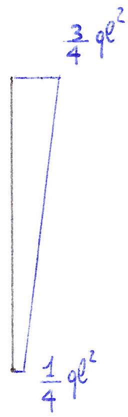




$\boxed{V}$ $\boxed{M}$

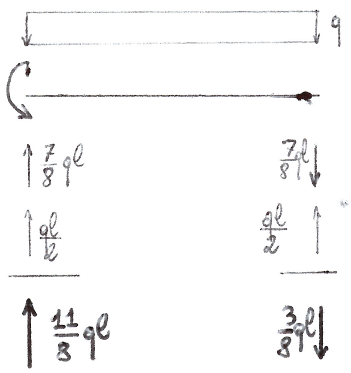
ASTA BC

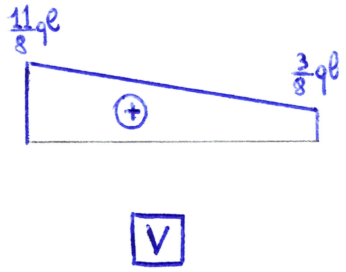



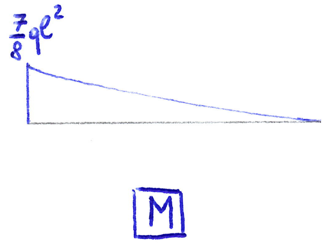




$\boxed{V}$ $\boxed{M}$

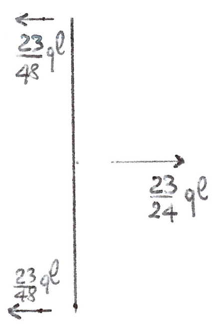
## ASTA BD

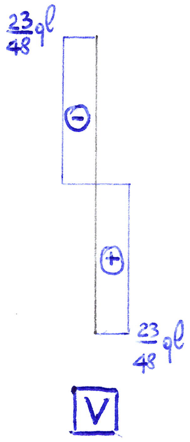


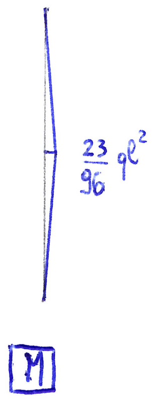




## ASTA DE

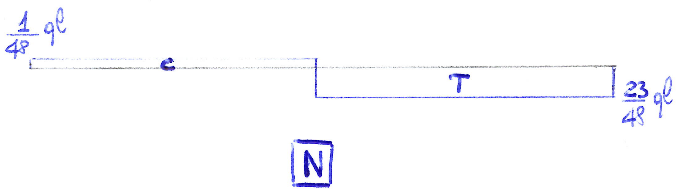


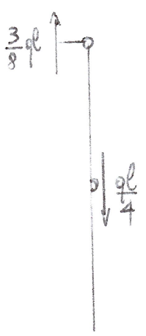



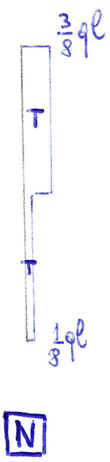

## Azioni Assiali:

## ASTE AB + BD

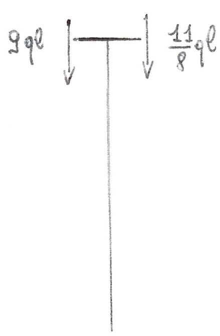



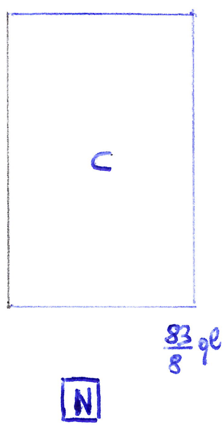

## ASTA DE



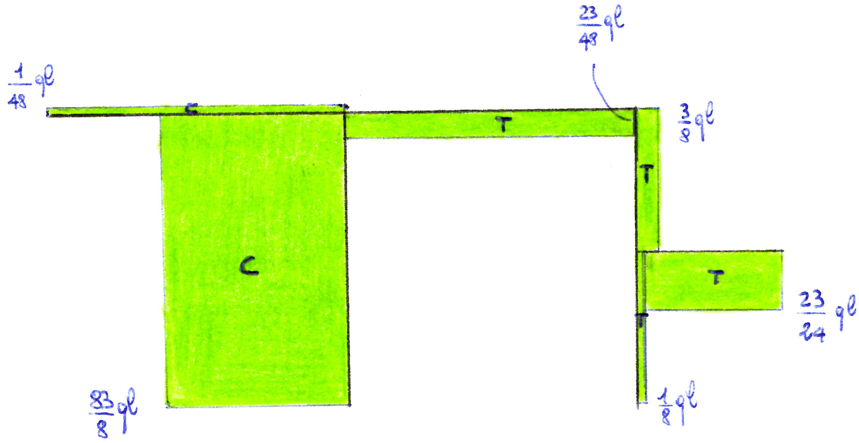

## ASTA BC



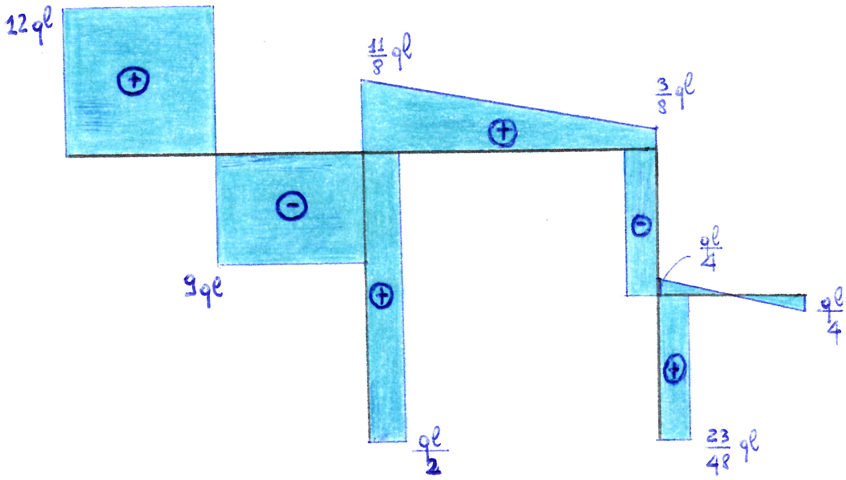



• Grafici finali:

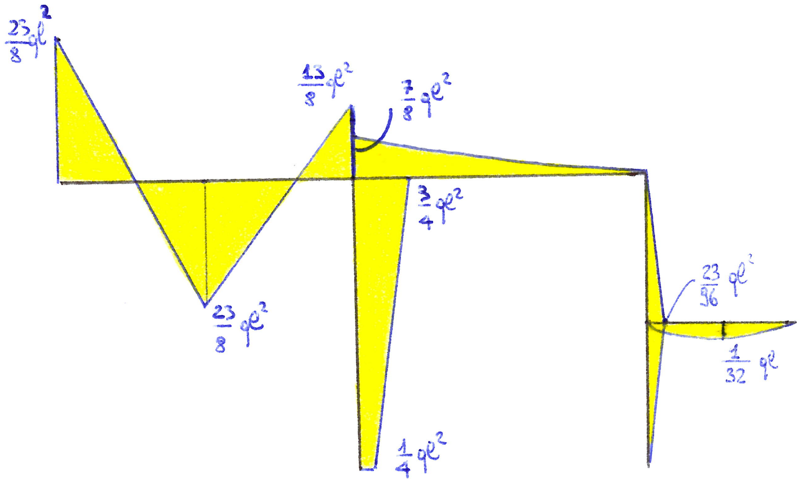
Azione assiale




Taglio




Momento

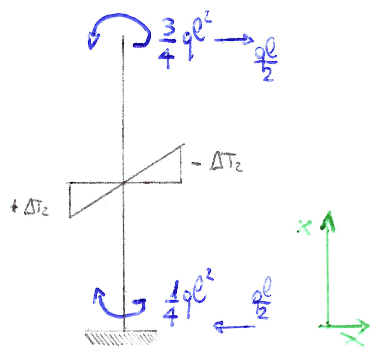


• Deformata qualitativa

Per il tracciamento della deformata qualitativa è necessario valutare:

1. La deformata dell'asta BC a causa della variazione termica a farfalla
2. Spostamenti nodali in A e B

1. DEFORMATA PER ASTA BC




Si valuta il segno della curvatura della trave sfruttando l'equazione della linea elastica:

$$y'' = -\frac{M(x)}{EJ} \pm 2\frac{\alpha \Delta T}{h}$$

$$M(x) = \frac{1}{4}ql^2 + \frac{ql}{2}x$$

$$y'' = -\frac{1}{EJ}\left[\frac{1}{4}ql^2 + \frac{ql}{2}x\right] + 2\cdot\frac{1}{8}\frac{ql^2}{EJ}$$

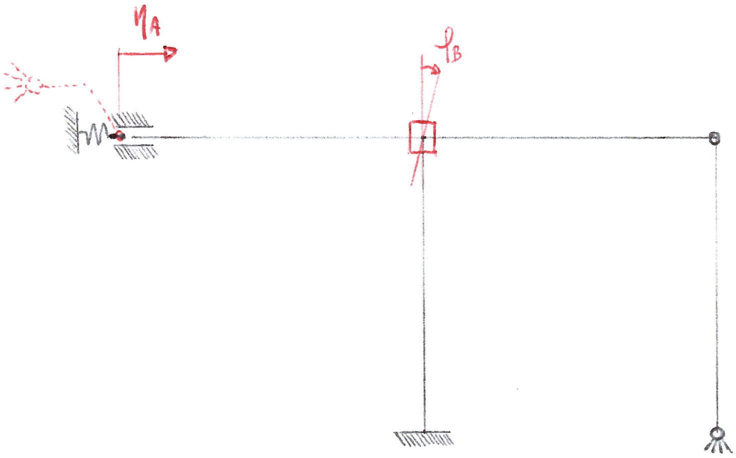
$$y''EJ = -\frac{1}{4}ql^2 - \frac{ql}{2}x + \frac{1}{4}ql^2$$

$$y''EJ = -\frac{ql}{2}x \Rightarrow \underline{\underline{\forall x>0 \quad y''<0}}$$

2. SPOSTAMENTI NODALI IN A E B

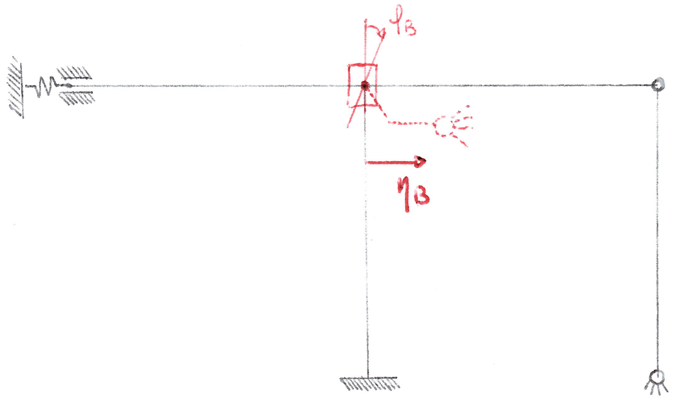
A causa delle variazioni termiche costanti: $\eta_0 \neq \eta_B \neq \eta_A$.

Per valutare gli spostamenti $\eta_A$ e $\eta_B$ si può riapplicare il metodo degli spostamenti andando a mettere una volta la biella fittizia in A e poi (ripetendo i calcoli) mettendo la biella in B.




Incognite:

$$\begin{cases} \eta_A \\ \varphi_B \end{cases}$$




Incognite

$$\begin{cases} \eta_B \\ \varphi_B \end{cases}$$

Sviluppando i calcoli si ottiene:

$$\begin{cases} \eta_A = -\dfrac{1}{4}\dfrac{q\ell^4}{EJ} \\ \eta_B = -\dfrac{1}{12}\dfrac{q\ell^4}{EJ} \end{cases}$$

Tuttavia gli spostamenti in A e B si possono valutare anche in modo più veloce. Infatti nell'ipotesi che $EA \to \infty$ si può scrivere:

$$\eta_A = \eta_D - \alpha \Delta T_1 \cdot \ell_{AB} - \alpha \Delta T_1 \cdot \ell_{BD} = \frac{1}{12}\frac{q\ell^4}{EJ} - \frac{1}{6}\frac{q\ell^3}{EJ}\cdot\ell - \frac{1}{6}\frac{q\ell^3}{EJ}\ell = -\frac{1}{4}\frac{q\ell^4}{EJ}$$

$$\eta_B = \eta_D - \alpha \Delta T_1 \ell_{BD} = \frac{1}{12}\frac{q\ell^4}{EJ} - \frac{1}{6}\frac{q\ell^3}{EJ}\ell = -\frac{1}{12}\frac{q\ell^4}{EJ}$$

Deformata qualitativa: