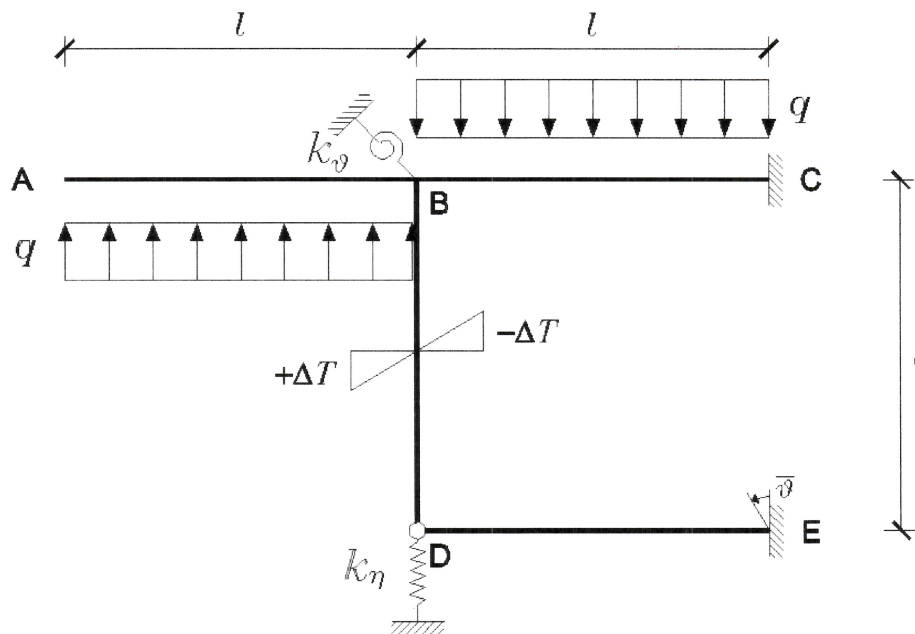


TEMA ESAME DEL 10 APRILE 2017

DOCENTE: **PROF. FAUSTO MINELLI**

ESERCITATORE: **ING. LUCA FACCONI**

Esercizio



$$\frac{\alpha \Delta T}{h} = \frac{1}{6} \frac{q l^2}{E J}$$

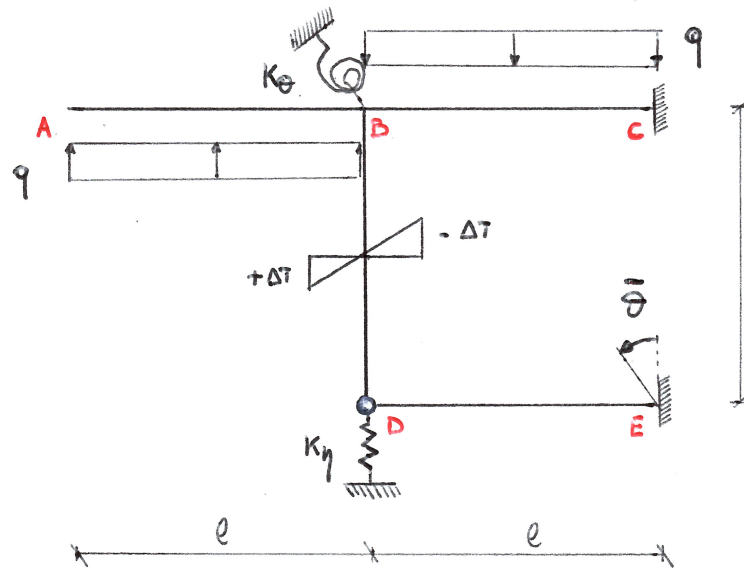
$$\bar{\theta} = \frac{1}{36} \frac{ql^3}{EJ}$$

$$K_{\theta} = \frac{1}{3} \frac{EJ}{l}$$

$$K_{\eta} = \frac{1}{3} \frac{EJ}{l^3}$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.



Dati:

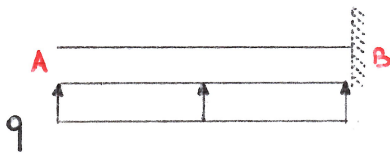
$$\frac{\alpha \Delta T}{h} = \frac{1}{6} \frac{q l^2}{E S}$$

$$K_{\theta} = \frac{1}{3} \frac{E S}{l}$$

$$\bar{\theta} = \frac{1}{36} \frac{q l^3}{E S}$$

$$K_{\eta} = \frac{1}{3} \frac{E S}{l^3}$$

- Si osserva come l'asta AB sia un'appendice isostatica, può quindi essere risolta semplicemente con gli equilibri e quindi "staccata" dalla struttura principale:

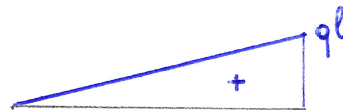


$$M_B^{sx} = + \frac{q l^2}{2}$$

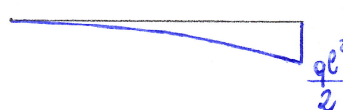
$$V_B^{sx} = + q l$$



AZIONE ASSIALE



TAGLIO



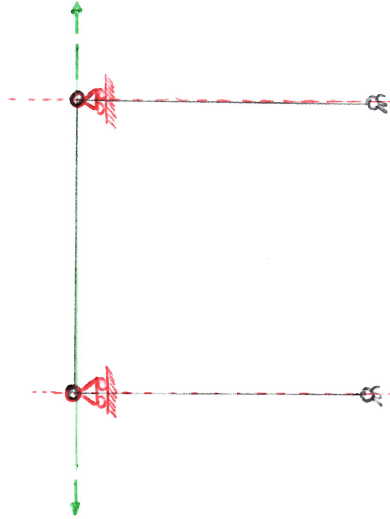
MOMENTO

- Si valuta il grado di iperstaticità della struttura:

$$G.d.V = 3 + 3 + 2 = 8$$

$$G.d.L = 3 \times 2 = 6$$

- Si valuta se il telaio sia a nodi fissi o spostabili.



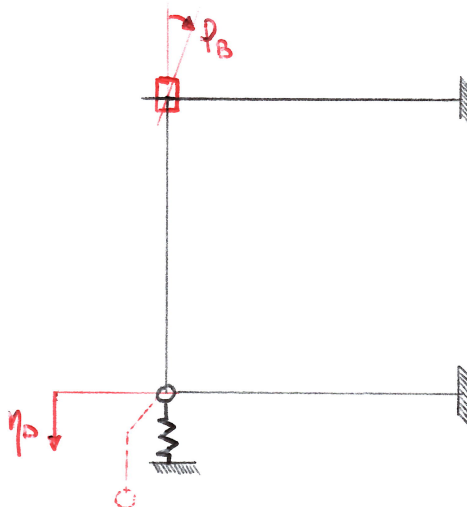
- Si osserva chiaramente un possibile cinetismo per l'asta BD.
Il telaio è quindi a nodi spostabili.

- Convenzioni adottate nella risoluzione del telaio:



- Si decide di risolvere il telaio adottando il metodo degli spostamenti,
 le incognite sono:

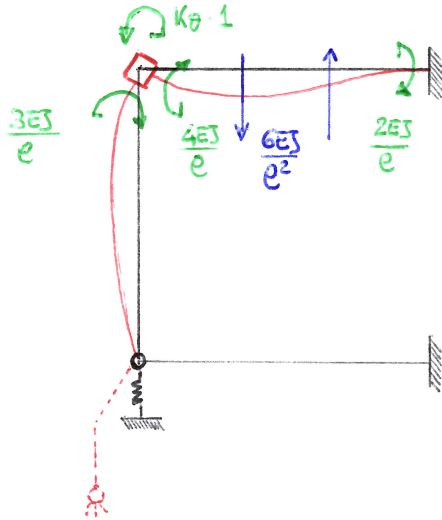
$$\begin{cases} \theta_B \\ \eta_D \end{cases}$$



Da cui il sistema risolvente (SISTEMA DELLE EQUAZIONI DI EQUILIBRIO)

$$\begin{cases} \sum M_D = 0 \\ \sum R_D = 0 \end{cases} \quad \begin{cases} m_{DB} \varphi_B + m_{DD} \eta_D + m_{DO} = 0 \\ h_{DB} \varphi_B + h_{DD} \eta_D + h_{DO} = 0 \end{cases}$$

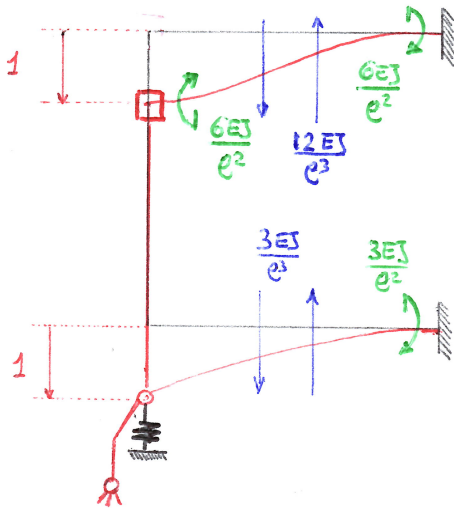
* CASO 1: $\varphi_B \neq 0$; $\eta_D = 0$; $\forall P_{ext} = 0$



$$\begin{aligned} m_{DB} &= \frac{3EJ}{e} + \frac{4EJ}{e} + K\theta \\ &= + \frac{22}{3} \frac{EJ}{e} \end{aligned}$$

$$h_{DB} = - \frac{6EJ}{e^2}$$

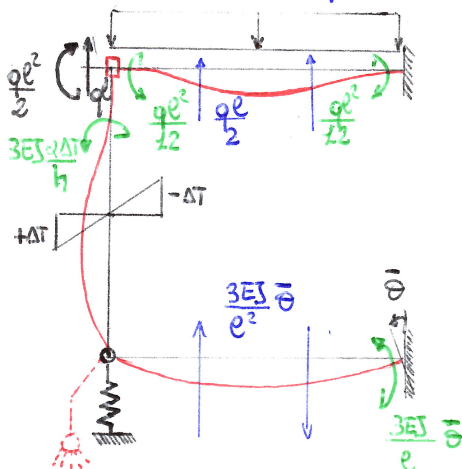
* CASO 2: $\eta_D \neq 0$; $\varphi_B = 0$; $\forall P_{ext} = 0$



$$m_{DB} = + \frac{6EJ}{e^2}$$

$$\begin{aligned} h_{DD} &= - \frac{12EJ}{e^3} - \frac{3EJ}{e^3} - K\eta \\ &= - \frac{46}{3} \frac{EJ}{e^3} \end{aligned}$$

* CASO 3: $P_{ext} \neq 0$; $\varphi_B = 0$; $\eta_D = 0$



$$\begin{aligned} m_{DB} &= - \frac{qe^2}{12} - \frac{qe^2}{2} - \frac{3EJ \alpha \Delta T}{h} \\ &= - \frac{13}{12} qe^2 \end{aligned}$$

$$\begin{aligned} h_{DD} &= - qe + \frac{qe}{2} + \frac{3EJ}{e^2} \cdot \frac{1}{36} \frac{qe^3}{EJ} \\ &= - \frac{5}{12} qe \end{aligned}$$

Sistema risolvibile:

$$\begin{cases} \frac{22}{3} \frac{EI}{l} \varphi_B + 6 \frac{EI}{l^2} \eta_D - \frac{13}{12} ql^2 = 0 & 1) \\ -6 \frac{EI}{l^2} \varphi_B - \frac{46}{3} \frac{EI}{l^3} \eta_D - \frac{5}{12} ql = 0 & 2) \end{cases}$$

moltiplico l'equazione 2) per $\frac{11}{9}$ e la sommo a 1)

$$6 \frac{EI}{l^2} \eta_D - \frac{506}{27} \frac{EI}{l^2} \eta_D - \frac{13}{12} ql^2 - \frac{55}{108} ql^2 = 0$$

$$\rightarrow -\frac{344}{27} \frac{EI}{l^2} \eta_D - \frac{43}{27} ql^2 = 0 \quad \rightarrow \quad \eta_D = -\frac{1}{8} \frac{ql^4}{EI}$$

sostituisco ora η_D in 1):

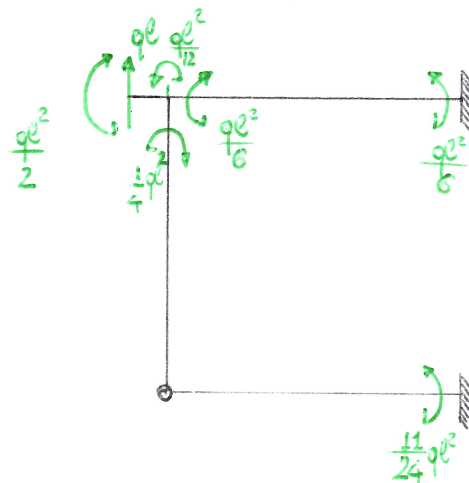
$$\frac{22}{3} \frac{EI}{l} \varphi_B + 6 \frac{EI}{l^2} \left(-\frac{1}{8} \frac{ql^4}{EI} \right) - \frac{13}{12} ql^2 = 0$$

$$\rightarrow \frac{22}{3} \frac{EI}{l} \varphi_B - \frac{11}{6} ql^2 = 0 \quad \rightarrow \quad \varphi_B = +\frac{1}{4} \frac{ql^3}{EI}$$

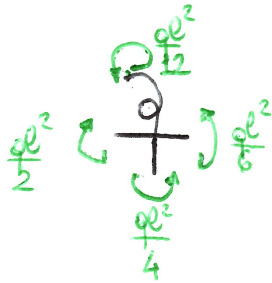
Da cui in conclusione:

$$\begin{cases} \varphi_B = +\frac{1}{4} \frac{ql^3}{EI} \\ \eta_D = -\frac{1}{8} \frac{ql^4}{EI} \end{cases}$$

• Si passa quindi al calcolo delle azioni interne:



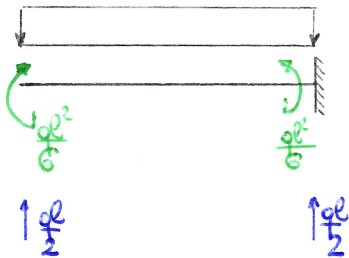
Verifica equilibrio rotazione nel nodo B:



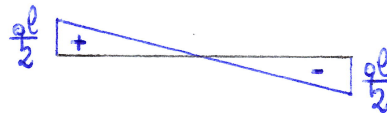
$$\sum M_B \stackrel{?}{=} 0$$

$$\frac{qe^2}{2} - \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{12} \right) qe^2 = 0 \quad \underline{OK}$$

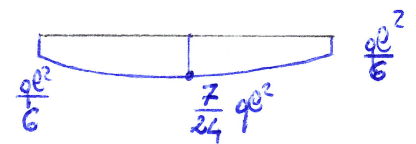
ASTA BC



TAGLIO

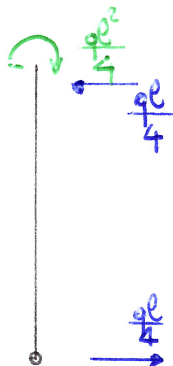


MOMENTO



$$V(l/2) = 0 \Rightarrow M_{max} = M(l/2) = \frac{qe^2}{6} + \frac{ql}{2} \cdot \frac{l}{2} - \frac{ql}{2} \cdot \frac{l}{4} = \frac{7}{24} qe^2$$

ASTA BD



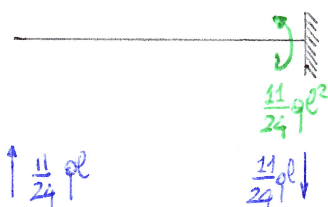
TAGLIO



MOMENTO



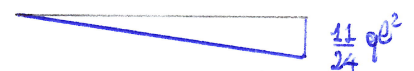
ASTA DE



TAGLIO

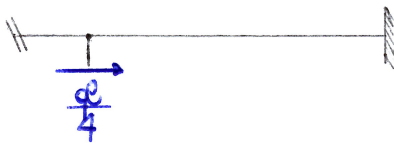


MOMENTO

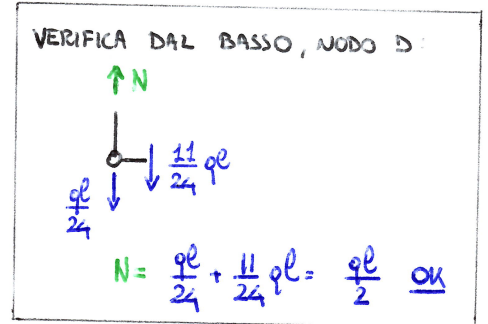
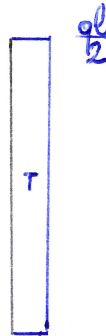
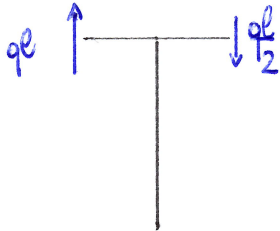


Azioni Assiali

ASTA BC



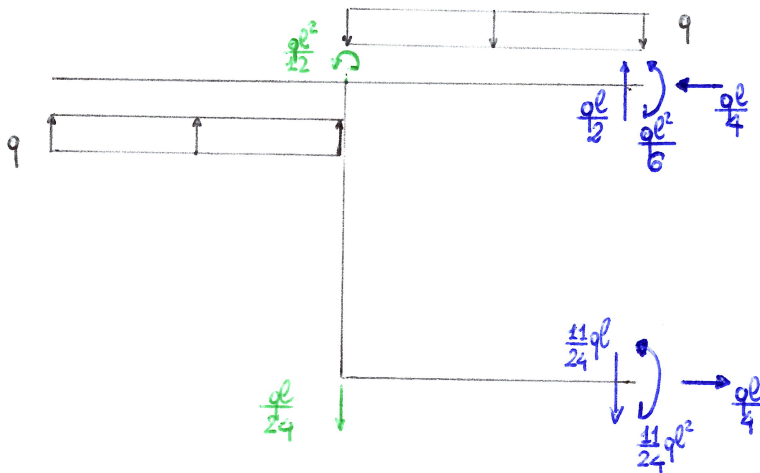
ASTA BD



ASTA DE



• Verifica equilibrio globale:



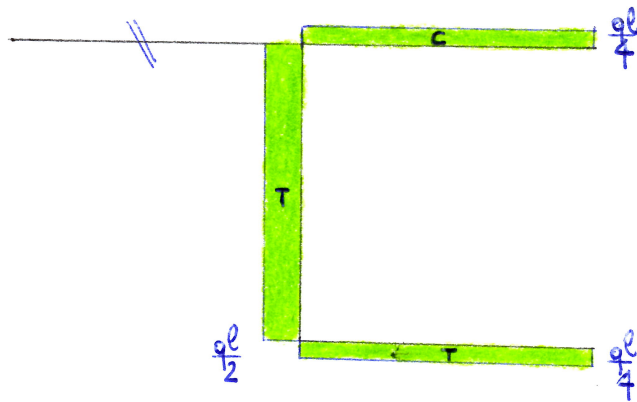
$$\sum F_H \stackrel{?}{=} 0 \rightarrow \frac{ql}{4} - \frac{ql}{4} \stackrel{?}{=} 0 \rightarrow 0 = 0 \quad \underline{OK}$$

$$\sum F_V \stackrel{?}{=} 0 \rightarrow -ql + ql - \frac{ql}{2} + \frac{ql}{24} + \frac{11}{24}ql \stackrel{?}{=} 0 \rightarrow 0 = 0 \quad \underline{OK}$$

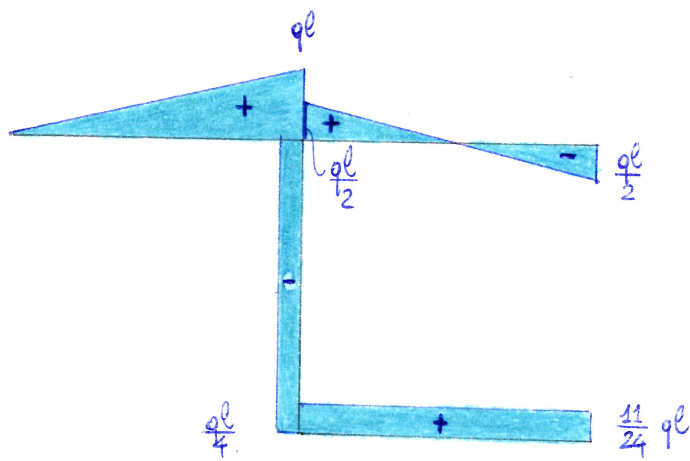
$$\sum M_B \stackrel{?}{=} 0 \rightarrow ql \cdot \frac{l}{2} + ql \cdot \frac{l}{2} - \frac{ql^2}{12} - \frac{ql}{2} \cdot l - \frac{ql^2}{6} + \frac{11}{24}ql \cdot l - \frac{11}{24}ql^2 - \frac{ql}{4} \cdot l \stackrel{?}{=} 0 \rightarrow 0 = 0 \quad \underline{OK}$$

• Grafici Finali

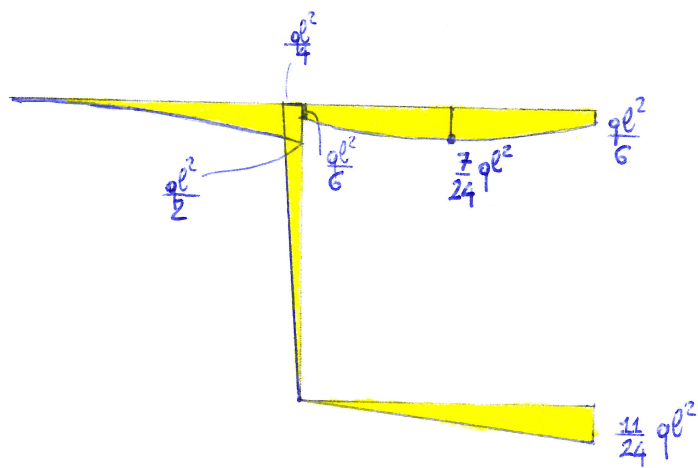
AZIONE
ASSIALE



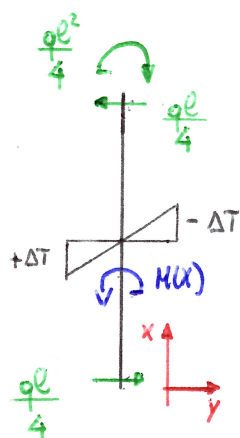
TAGLIO



MOMENTO



- Studio della deformata per l'este BD



$$M(x) = ?$$

$$M(x) = -\frac{1}{4} qe \cdot x$$

$$y''(x) = -\frac{M(x)}{EI} \pm 2 \frac{\alpha \Delta T}{h}$$

$$= \frac{1}{4} \frac{qe}{EI} x + 2 \cdot \frac{1}{6} \frac{qe}{EI} = \frac{1}{4} \frac{qe}{EI} x + \frac{1}{3} \frac{qe}{EI}$$

$$\text{quando } y''(x) > 0 \longrightarrow \frac{1}{4} \frac{qe}{EI} x + \frac{1}{3} \frac{qe}{EI} > 0$$

$$\longrightarrow x > -\frac{4}{3}e \Rightarrow \underline{\underline{\forall x \in [0; l] \quad y'' > 0}}$$

- Deformata qualitativa:

