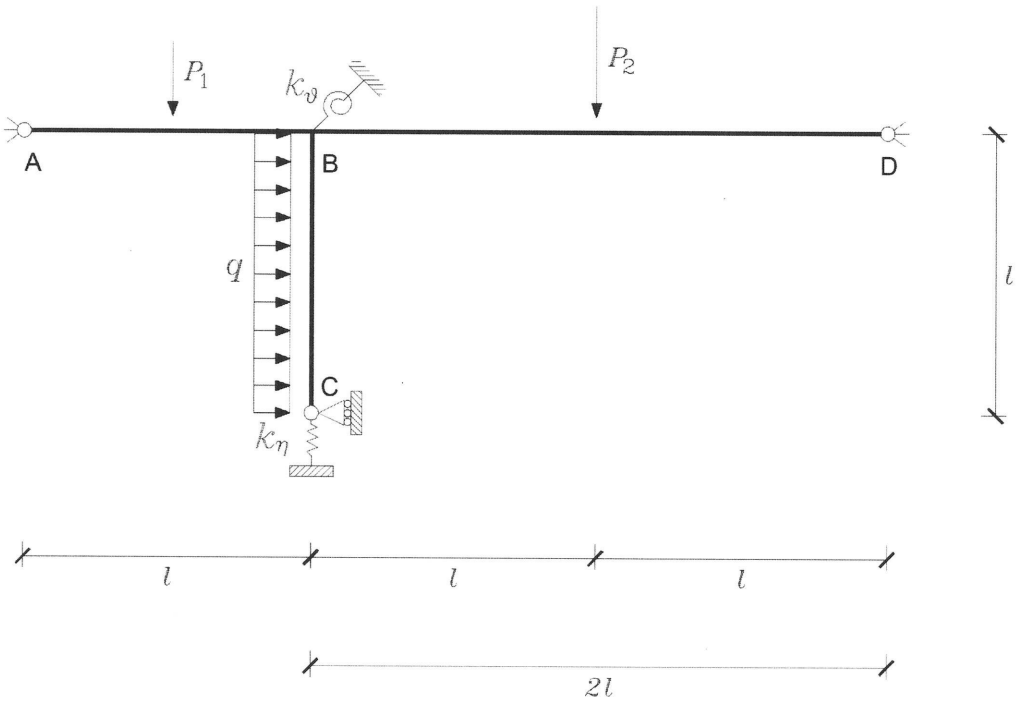
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 19 GIUGNO 2017

DOCENTE: **PROF. FAUSTO MINELLI** ESERCITATORE: **ING. LUCA FACCONI**

DURATA: **2 ORE**

Esercizio



$$K_\theta = \frac{13}{4}\frac{EJ}{l}$$

$$K_\eta = \frac{45}{8}\frac{EJ}{l^3}$$

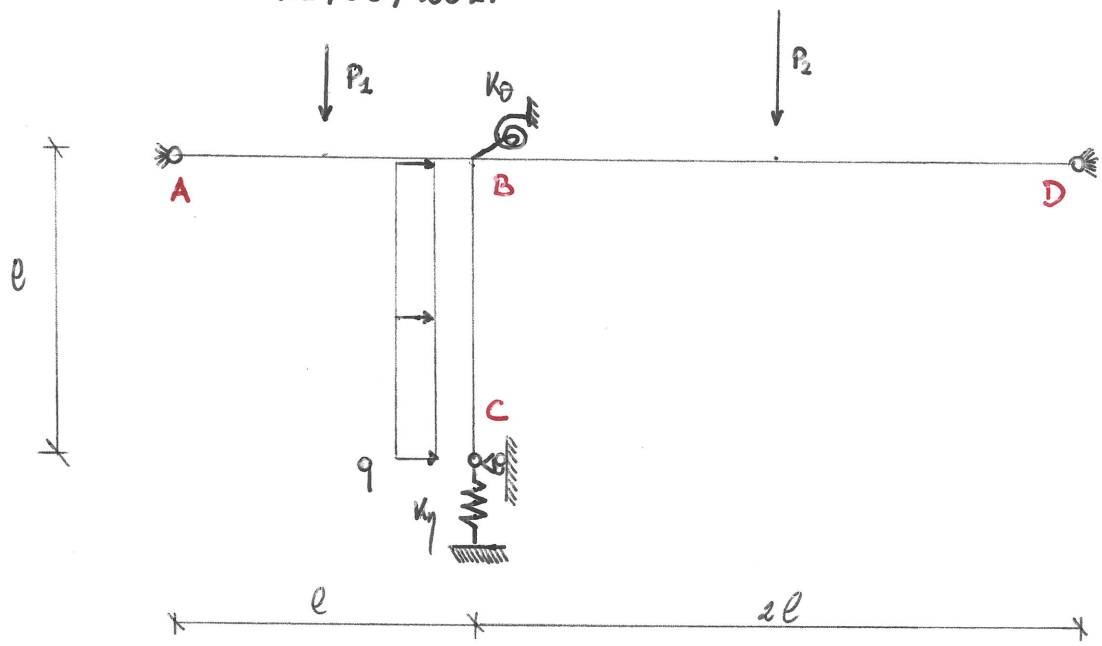
$$P_1 = 4ql$$

$$P_2 = 5ql$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

* TEMA ESAME 19/06/2017



Dati:

$K_\theta = \frac{13}{4}\frac{EJ}{\ell}$ $\quad P_1 = 4q\ell$

$K_\eta = \frac{45}{8}\frac{EJ}{\ell^3}$ $\quad P_2 = 5q\ell$

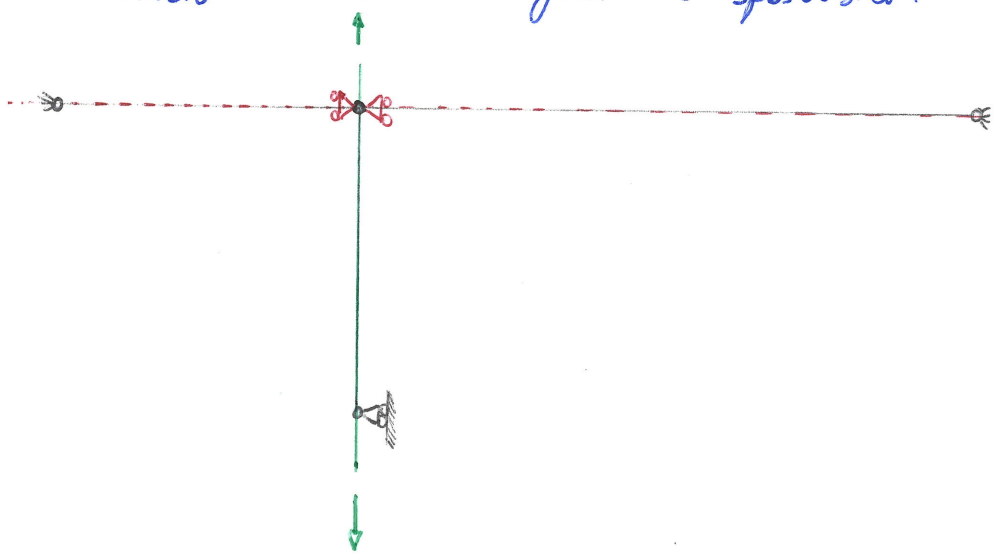
- Si valuta il grado di iperstaticità della struttura:

G.d.V. = 2 + 1 + 2 = 5

G.d.L. = 3 * 1 = 3

➡ struttura 2 volte iperstatica

- Si valuta se il telaio sia a nodi fissi o spostabili:

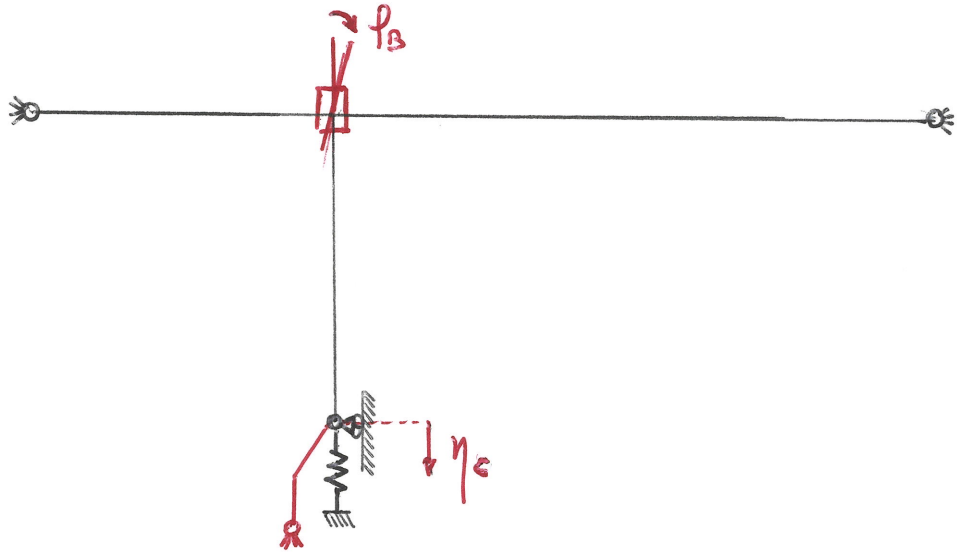


➡ Si osserva un possibile cinematismo per l'asta BC. Il telaio è a nodi spostabili

- Convenzioni adottate nella risoluzione del telaio:

- Si decide di risolvere il telaio adottando il <u>metodo degli spostamenti</u>, le incognite sono:

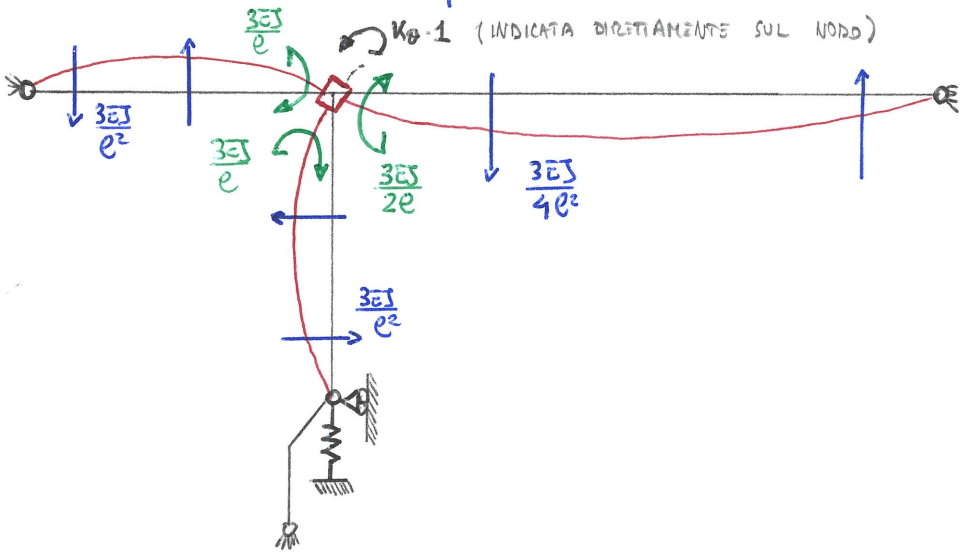
$$\begin{cases} \varphi_B \\ \eta_C \end{cases}$$



da cui il sistema risolvente (SISTEMA DELLE EQUAZIONI DI EQUILIBRIO)

$$\begin{cases} \Sigma M_B = 0 \\ \Sigma R_C = 0 \end{cases} \qquad \begin{cases} m_{BB}\,\varphi_B + m_{BC}\,\eta_C + m_{Bp} = 0 \\ h_{CB}\,\varphi_B + h_{CC}\,\eta_C + h_{Cp} = 0 \end{cases}$$

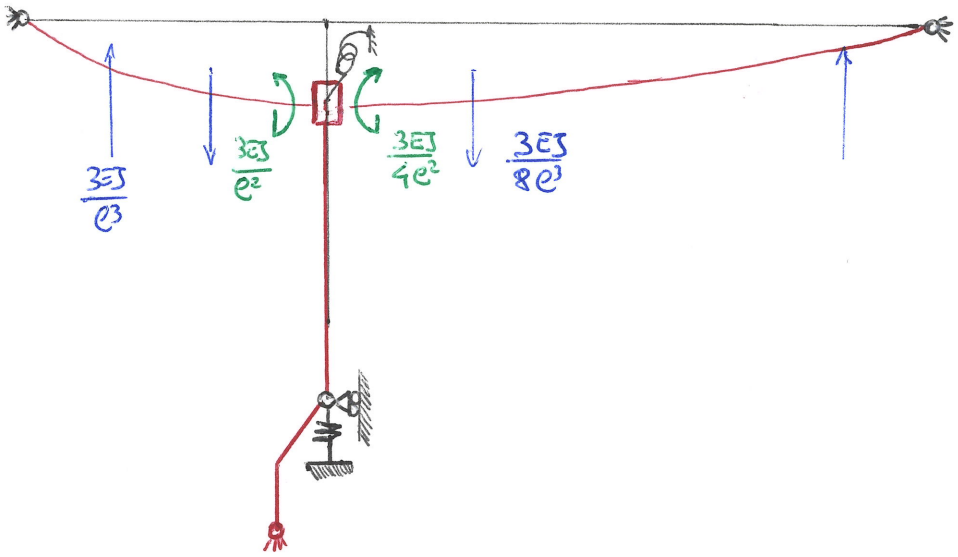
* CASO 1: $\varphi_B = 1 \; ; \; \eta_D = 0 \; ; \; \forall P_{ext} = 0$



$$m_{BB} = K_\theta + \left(\frac{3}{2} + 3 + 3\right)\frac{EJ}{\ell} = \frac{43}{4}\,\frac{EJ}{\ell}$$

$$h_{CB} = \frac{9}{4}\,\frac{EJ}{\ell^2}$$

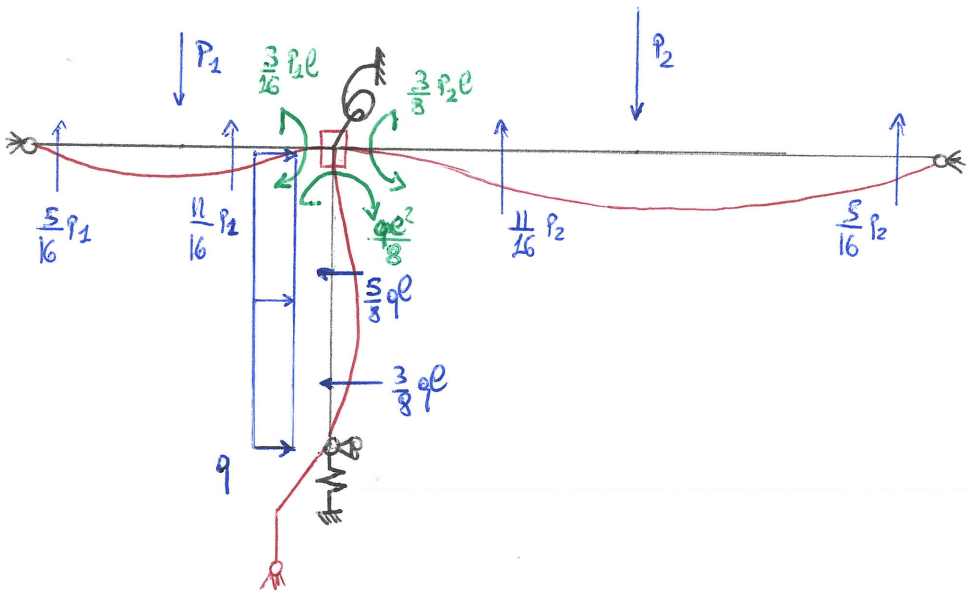
* CASO 2: $\varphi_B = 0 \; ; \; \eta_C = 1 \; ; \; \forall P_{ext} = 0$



$$m_{BC} = -\frac{9}{4}\frac{EJ}{\ell^2}$$

$$h_{CC} = -\frac{3EJ}{\ell^2} - \frac{3EJ}{\ell^2} - K\eta = -9\frac{EJ}{\ell^2}$$

* CASO 3: $\varphi_B = 0 \; ; \; \eta_C = 0 \; ; \; \forall P_{ext} \neq 0$



$$m_{BO} = \frac{3}{16}P_1\ell - \frac{3}{8}P_2\ell + \frac{q\ell^2}{8} = \frac{3}{4}q\ell^2 - \frac{15}{8}q\ell^2 + \frac{q\ell^2}{8} = -q\ell^2$$

$$h_{CO} = \frac{11}{16}P_1 + \frac{11}{16}P_2 = +\frac{99}{16}q\ell$$

Sistema risolvente:

$$\begin{cases} \frac{43}{4}\frac{EJ}{\ell}\varphi_B - \frac{9}{4}\frac{EJ}{\ell^2}\eta_C - q\ell^2 = 0 & 1) \\ \frac{9}{4}\frac{EJ}{\ell^2}\varphi_B - 9\frac{EJ}{\ell^3}\eta_C + \frac{99}{16}q\ell = 0 & 2) \end{cases}$$

moltiplico l'equazione 1) per $9/43\ell$ poi sottraggo la 2) dalla 1):

$$-\frac{81}{172}\frac{EJ}{\ell^3}\eta_C + 9\frac{EJ}{\ell^3}\eta_C - \frac{9}{43}q\ell - \frac{99}{16}q\ell = 0$$

$$\rightarrow \frac{1467}{172}\frac{EJ}{\ell^3}\eta_C - \frac{4401}{688}q\ell = 0 \rightarrow \eta_C = \frac{3}{4}\frac{q\ell^4}{EJ}$$

Sostituisco ora η_D in 2):

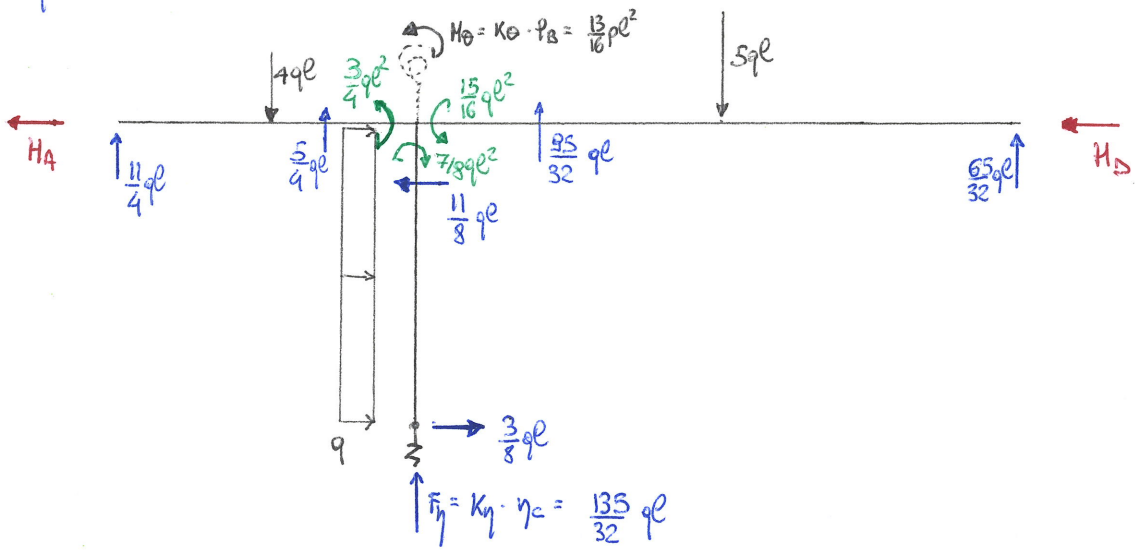
$$\frac{9}{4}\frac{EJ}{\ell^2}\varphi_B - 9\frac{EJ}{\ell^3}\cdot\frac{3}{4}\frac{q\ell^4}{EJ} + \frac{99}{16}q\ell = 0$$

$$\rightarrow \frac{9}{4}\frac{EJ}{\ell^2}\varphi_B - \frac{9}{16}q\ell = 0 \rightarrow \varphi_B = \frac{1}{4}\frac{q\ell^3}{EJ}$$

Da cui in conclusione:

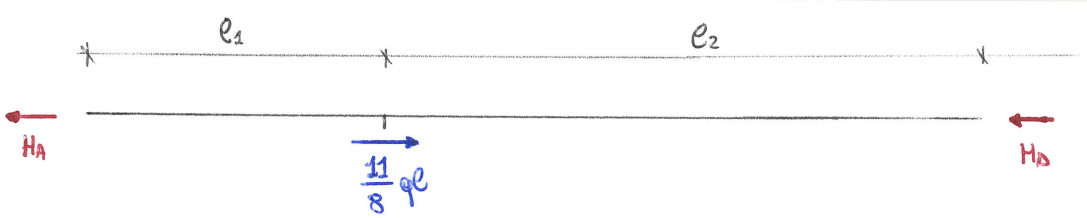
$$\begin{cases} \varphi_B = \frac{1}{4}\frac{q\ell^3}{EJ} \\ \eta_C = \frac{3}{4}\frac{q\ell^4}{EJ} \end{cases}$$

- Si passa quindi al calcolo delle azioni interne:



oss. Come ricavare il valore delle azioni assiali H_A e H_D?

→ L'AZIONE ASSIALE ($11/8\ q\ell$) SI RIPARTISCE IN BASE ALLE RIGIDEZZE:



$$H_A = P \cdot \frac{K_1}{K_1+K_2}$$

$$= \frac{11}{8}q\ell \cdot \frac{EA/\ell_1}{EA\cdot\left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)}$$

$$= \frac{11}{8}q\ell \cdot \frac{\ell_2}{\ell_1+\ell_2}$$

$$= \frac{11}{8}q\ell \cdot \frac{2\ell}{3\ell}$$

$$= \frac{11}{12}q\ell$$

$$H_D = P \cdot \frac{K_2}{K_1+K_2}$$

$$= \frac{11}{8}q\ell \cdot \frac{EA/\ell_2}{EA\cdot\left(\frac{1}{\ell_1}+\frac{1}{\ell_2}\right)}$$

$$= \frac{11}{8}q\ell \cdot \frac{\ell_1}{\ell_1+\ell_2}$$

$$= \frac{11}{8}q\ell \cdot \frac{\ell}{3\ell}$$

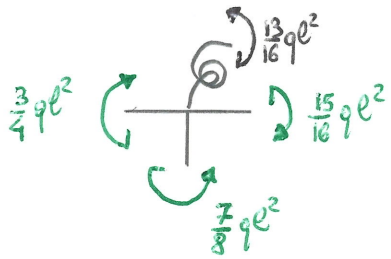
$$= \frac{11}{24}q\ell$$

A questo punto posso verificare l'equilibrio globale: $\Sigma M_C = 0$

$$\frac{11}{4}q\ell^2 - \frac{11}{12}q\ell^2 - 2q\ell^2 - \frac{13}{16}q\ell^2 + \frac{q\ell^2}{2} + 5q\ell^2 - \frac{65}{16}q\ell^2 - \frac{11}{24}q\ell^2 \stackrel{?}{=} 0$$

$0 = 0$ ✓

Verifica dell'equilibrio locale nel nodo in B: $\Sigma M_B^{back} = 0$



$$\frac{3}{4}q\ell^2 + \frac{15}{16}q\ell^2 - \frac{7}{8}q\ell^2 - \frac{13}{16}q\ell^2 \stackrel{?}{=} 0$$

$0 = 0$ ✓

Verifica alla traslazione verticale: $\Sigma F_V = 0$

$$\frac{11}{4}q\ell - 4q\ell - 5q\ell + \frac{65}{32}q\ell + \frac{135}{32}q\ell \stackrel{?}{=} 0$$

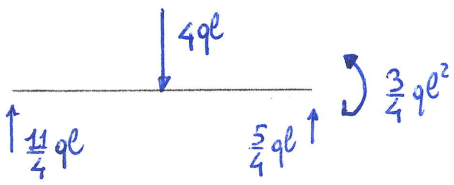
$0 = 0$ ✓

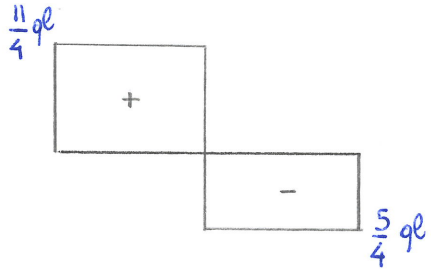
Verifica alla traslazione orizzontale: $\Sigma F_H = 0$

$$q\ell + \frac{3}{8}q\ell - \frac{11}{12}q\ell - \frac{11}{24}q\ell \stackrel{?}{=} 0$$

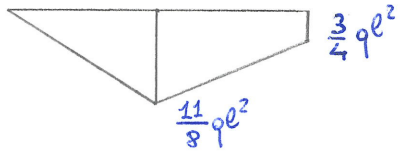
$0 = 0$ ✓

ASTA AB:



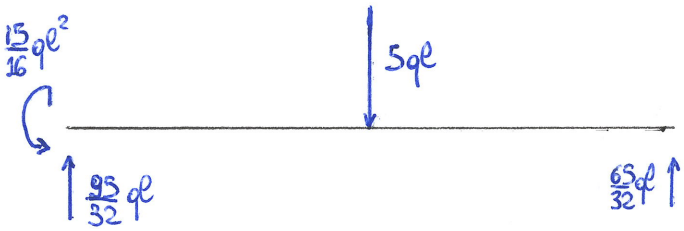


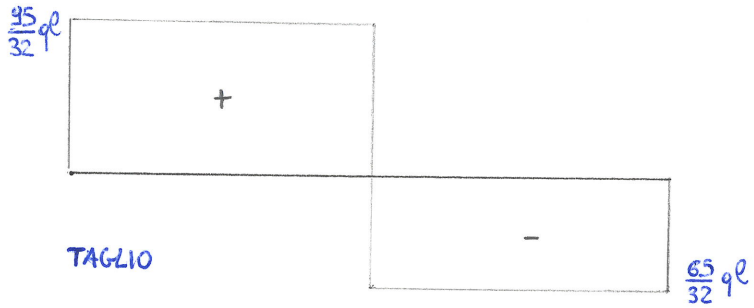
TAGLIO



MOMENTO

ASTA BD:

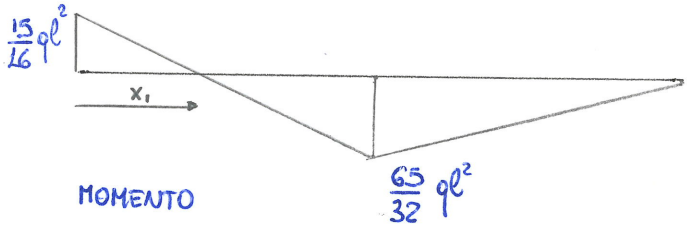




TAGLIO

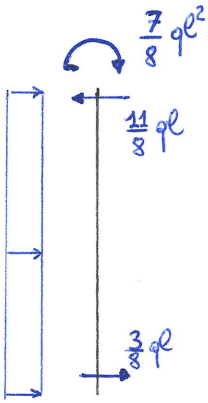
$\frac{15}{16} : X_1 = \frac{95}{32} : \ell$

$X_1 = \frac{6}{19}\ell$



MOMENTO

ASTA BC:

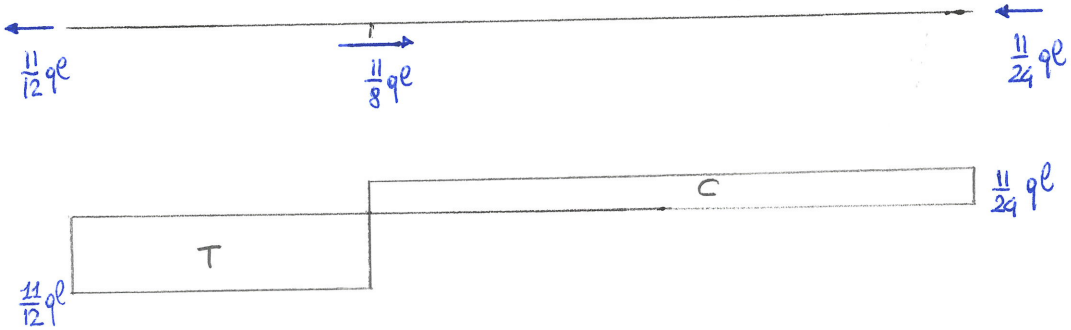


TAGLIO

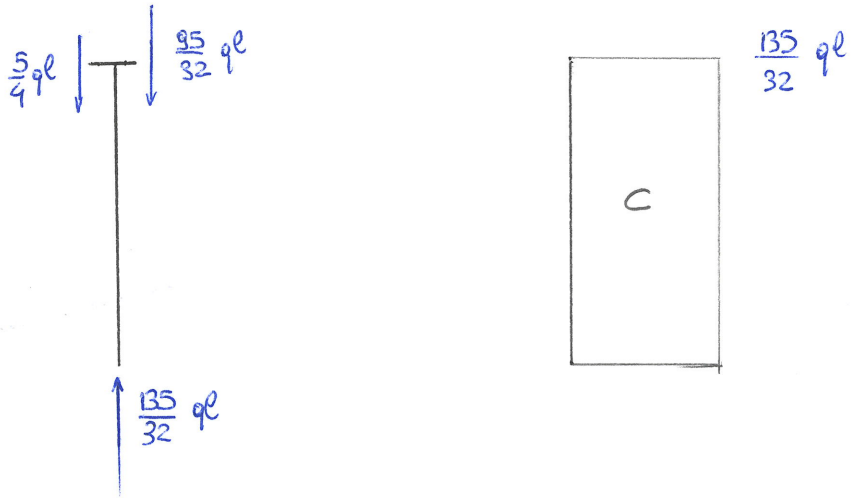
MOMENTO

Azioni assiali:

ASTE AB + BD

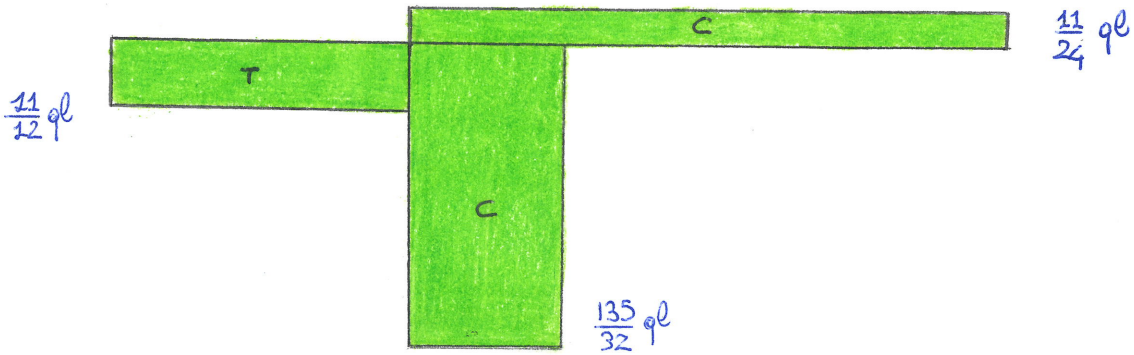


ASTA BC



- Grafici finali:

AZIONE ASSIALE



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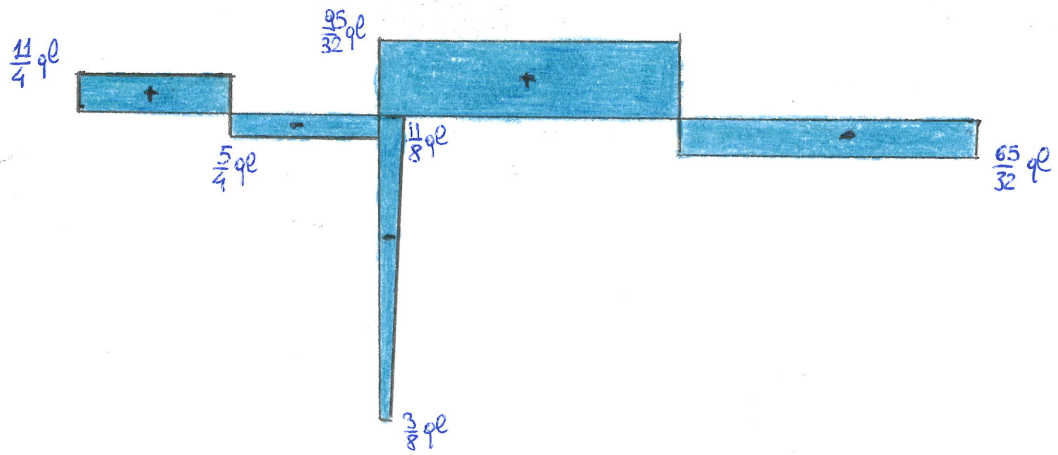
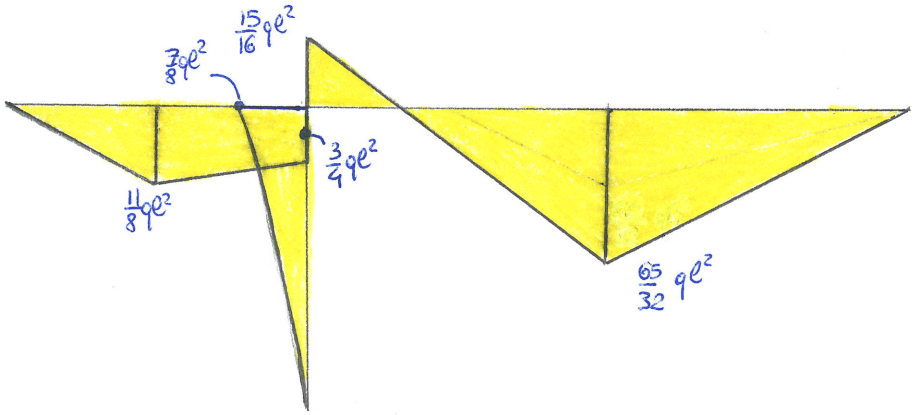


MOMENTO



DEFORMATA QUALITATIVA