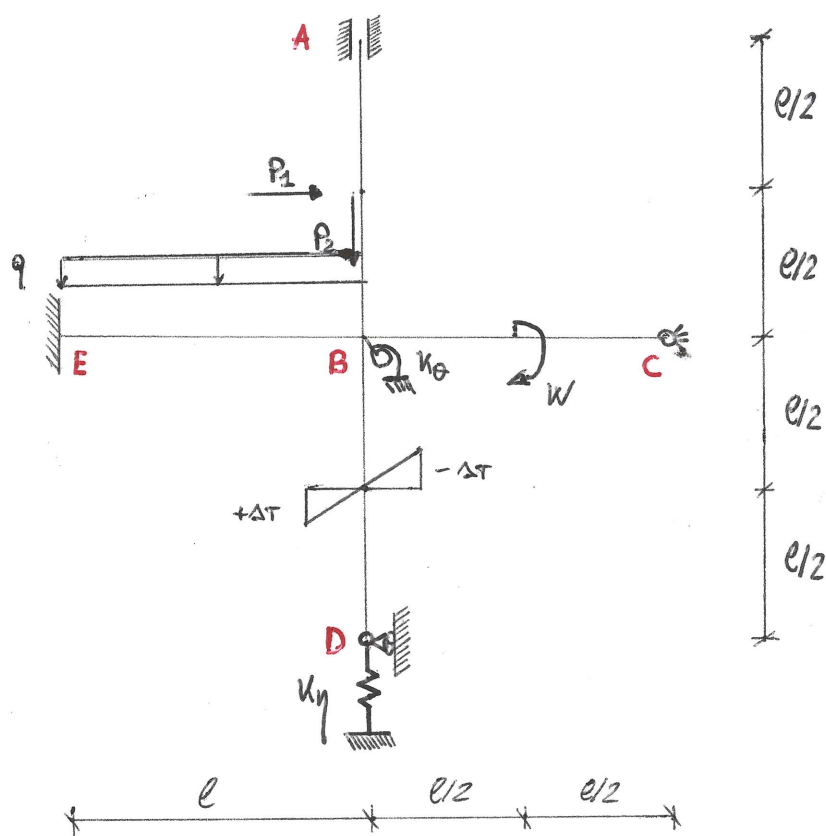




Dati:

$$k_{\theta} = \frac{1}{8} \frac{EJ}{e}$$

$$P_1 = 2ql$$

$$k_{\eta} = \frac{21}{2} \frac{EJ}{e^3}$$

$$P_2 = 4ql$$

$$\frac{\alpha \Delta T}{h} = 3 \frac{ql^2}{EJ}$$

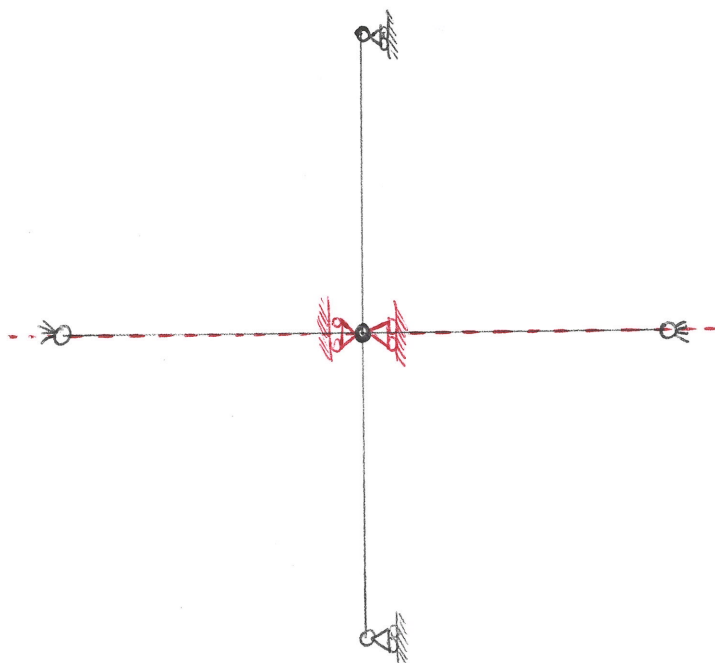
$$W = 2ql^2$$

• Si vuole il grado di iperstaticità della struttura:

$$G.d.V = 2 + 2 + 1 + 3 = 8$$

$$G.d.L = 3 \times 1 = 3$$

- Si valuta se il telaio sia a nodi fissi o spostabili;



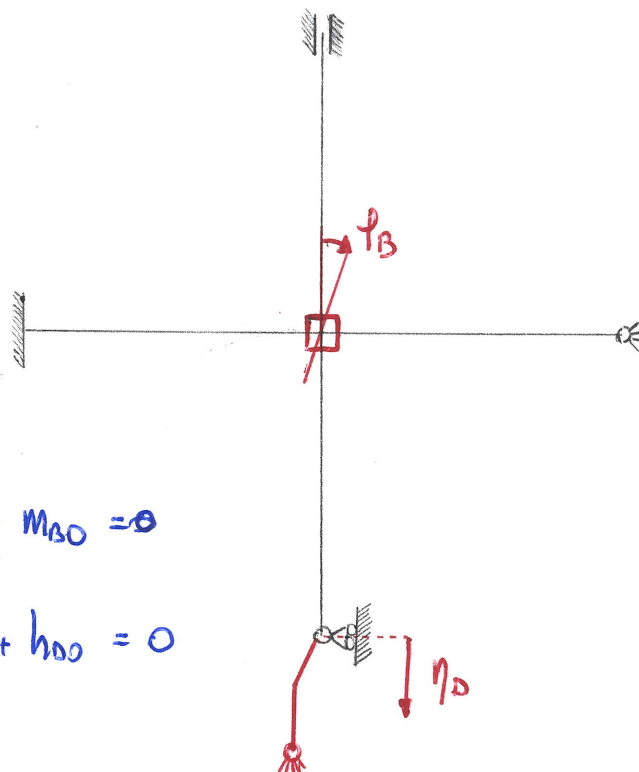
⇒ Si osserva un possibile cinetismo per l'asta ASD. Il telaio è a nodi spostabili.

- Risolvo il telaio adottando il metodo degli spostamenti.
Le incognite sono:

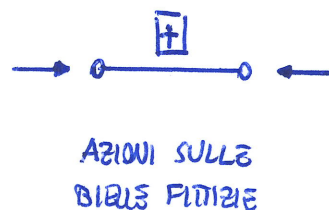
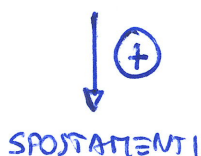
$$\begin{cases} P_B \\ \eta_D \end{cases}$$

quindi il sistema risolvente risulta essere:

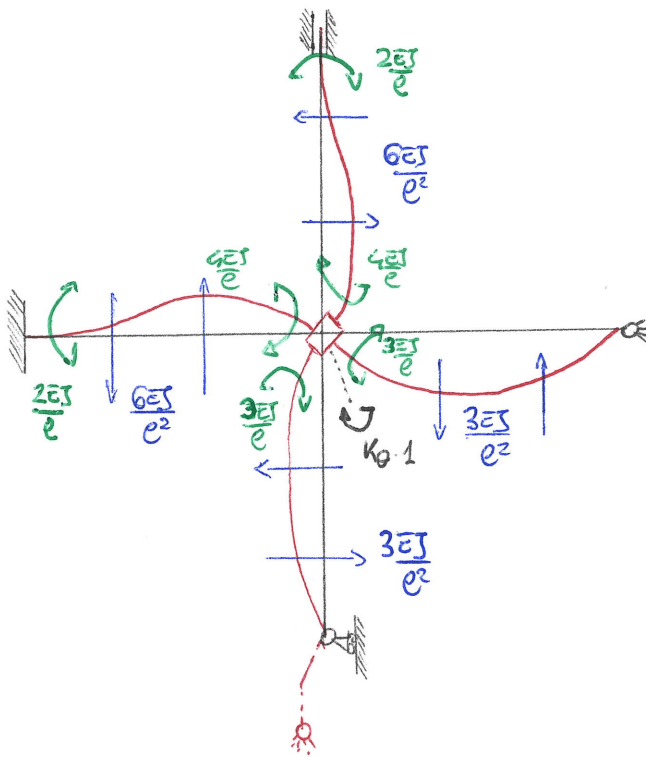
$$\begin{cases} \sum M_B = 0 \\ \sum R_D = 0 \end{cases} \Rightarrow \begin{cases} m_{BB} P_B + m_{BD} \eta_D + m_{BD} = 0 \\ h_{DB} P_B + h_{DD} \eta_D + h_{DD} = 0 \end{cases}$$



- Convenzioni adottate nella soluzione del telaio:



* CASO 1 : $f_B = 1$; $\eta_D = 0$; $\forall P_{ext} = 0$

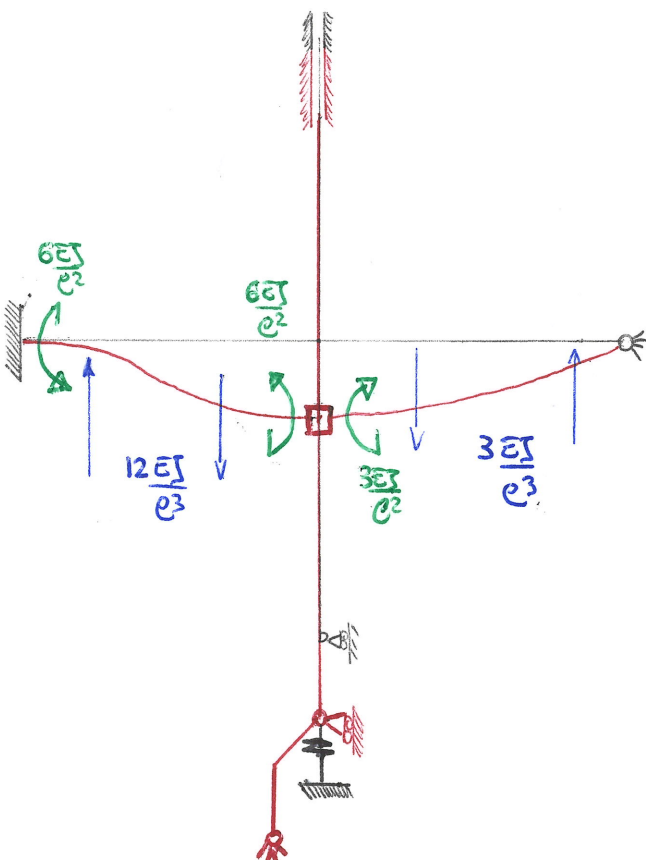


$$M_{BB} = + 14 \frac{EJ}{e} + K\theta$$

$$= + \frac{113}{8} \frac{EJ}{e}$$

$$h_{DB} = + 6 \frac{EJ}{e^2} - 3 \frac{EJ}{e^2} + 3 \frac{EJ}{e^2}$$

* CASO 2 : $f_B = 0$; $\eta_D = 1$; $\forall P_{ext} = 0$



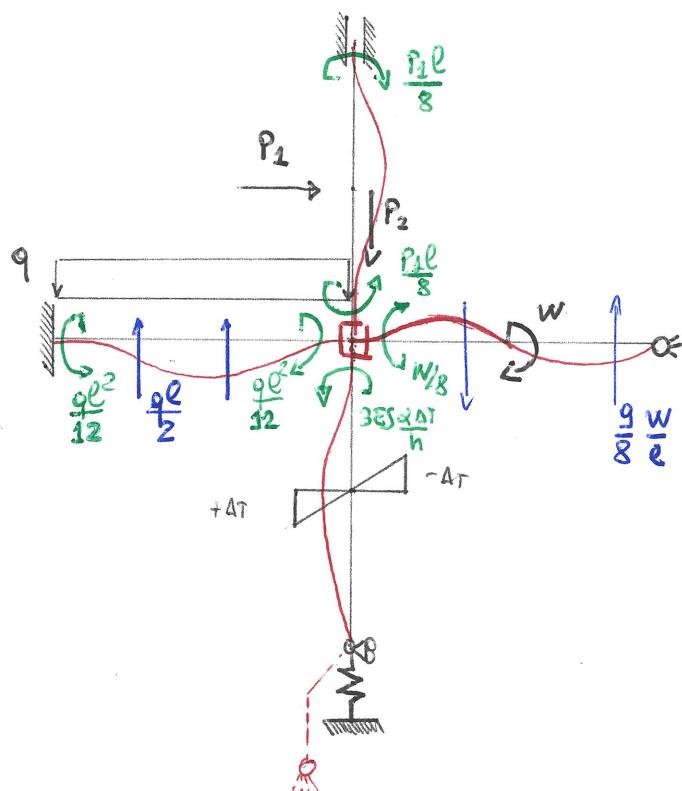
$$M_{BD} = + 3 \frac{EJ}{e^2} - \frac{6EJ}{e^2} = - 3 \frac{EJ}{e^2}$$

$$h_{DD} = - K_\eta \cdot 1 - 12 \frac{EJ}{e^3} - 3 \frac{EJ}{e^3}$$

$$= - \frac{21}{2} \frac{EJ}{e^3} - 12 \frac{EJ}{e^3} - 3 \frac{EJ}{e^3}$$

$$= - \frac{51}{2} \frac{EJ}{e^3}$$

* CASO 3: $p_B = 0$; $\eta_D = 0$; $\forall p_{ext} \neq 0$



$$m_{B0} = -\frac{p_1 l}{8} + \frac{W}{8} - 3EJ \frac{\alpha \Delta T}{h} + \frac{q l^2}{12}$$

$$= -\frac{q l^2}{4} + \frac{q l^2}{4} - 3 q l^2 + \frac{q l^2}{12}$$

$$= -\frac{107}{12} q l^2$$

$$h_{B0} = +\frac{q l}{2} + \frac{q}{8} \frac{W}{e} + p_2$$

$$= \frac{q l}{2} - \frac{3}{8} 2 q l + 4 q l$$

$$= \frac{9}{4} q l$$

Sistema Risolvente:

$$\begin{cases} +\frac{113}{8} \frac{EJ}{e} p_B - 3 \frac{EJ}{e^2} \eta_D - \frac{107}{12} q l^2 = 0 & 1) \\ +3 \frac{EJ}{e^2} p_B - \frac{51}{2} \frac{EJ}{e^3} \eta_D + \frac{9}{4} q l = 0 & 2) \end{cases}$$

moltiplico l'equazione 2) per $\left(\frac{2}{17} e\right)$ e la sottraggo alla 1):

$$\left[\frac{113}{8} \frac{EJ}{e} - \frac{6}{17} \frac{EJ}{e} \right] p_B - \frac{107}{12} q l^2 - \frac{9}{34} q l^2 = 0$$

$$\rightarrow \frac{1873}{136} \frac{EJ}{e} p_B - \frac{1873}{204} q l^2 = 0 \quad \rightarrow p_B = \frac{2}{3} \frac{q l^3}{EJ}$$

sostituisco p_B nella 2):

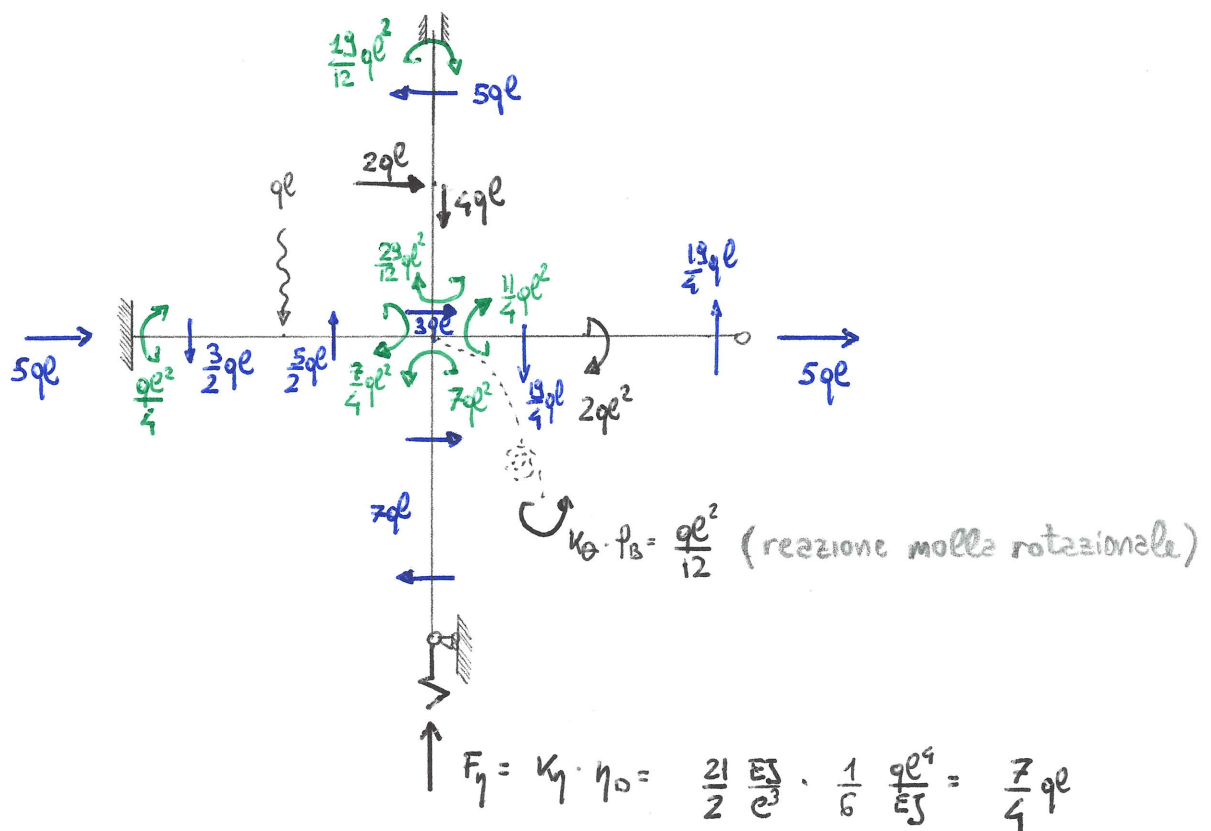
$$3 \frac{EJ}{e^2} \cdot \frac{2}{3} \frac{q l^3}{EJ} - \frac{51}{2} \frac{EJ}{e^3} \eta_D + \frac{9}{4} q l = 0$$

$$\rightarrow -\frac{51}{2} \frac{EJ}{e^3} \eta_D + \frac{17}{4} q l = 0 \quad \rightarrow \eta_D = \frac{1}{6} \frac{q l^4}{EJ}$$

Da cui in conclusione:

$$\begin{cases} p_B = \frac{2}{3} \frac{q l^3}{EJ} \\ \eta_D = \frac{1}{6} \frac{q l^4}{EJ} \end{cases}$$

• Si passa quindi al calcolo delle azioni interne:



Verifica dell'equilibrio alla traslazione orizzontale: $\sum F_H = 0$

$$5ql + 5ql + 2ql - 5ql - 7ql \stackrel{?}{=} 0 \quad 0 = 0 \quad \checkmark$$

Verifica dell'equilibrio alla traslazione verticale: $\sum F_V = 0$

$$-\frac{3}{2}ql - ql + \frac{7}{4}ql + \frac{19}{4}ql - 4ql \stackrel{?}{=} 0 \quad 0 = 0 \quad \checkmark$$

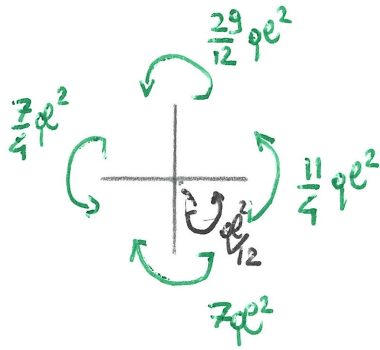
Verifica dell'equilibrio alla rotazione:

$$\sum M_B = 0$$

$$\frac{19}{12} qe^2 - 5qe^2 + qe^2 - \frac{qe^2}{12} + 7qe^2 + \frac{qe^2}{4} - \frac{3}{2} qe^2 - \frac{19}{4} qe^2 + 2qe^2 - \frac{qe^2}{2} = 0$$

$$0 = 0 \quad \checkmark$$

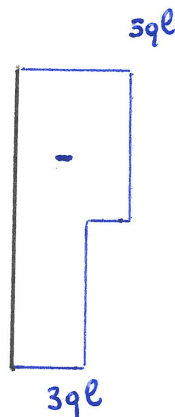
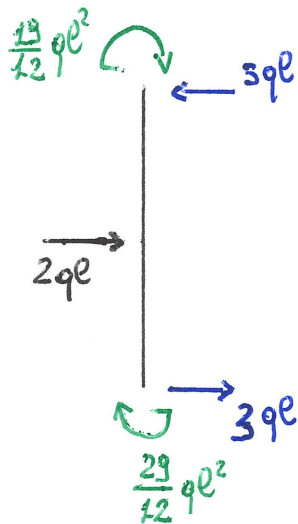
Verifica dell'equilibrio nel nodo B (LACRÈ):



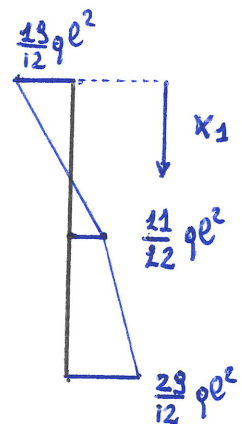
$$7qe^2 - \frac{7}{4} qe^2 - \frac{29}{12} qe^2 - \frac{11}{4} qe^2 - \frac{qe^2}{12} = 0$$

$$0 = 0 \quad \checkmark$$

ASTA AB:



TALIO

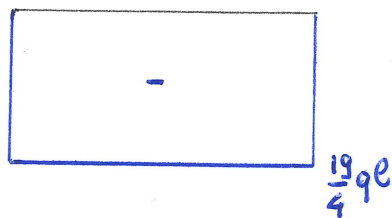
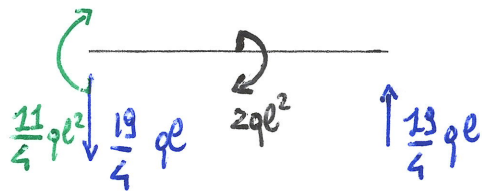


MOMENTO

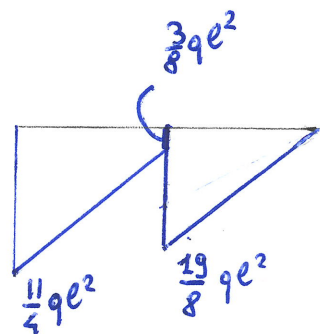
$$x_1 \longrightarrow \frac{5}{2} qe^2 : \frac{e}{2} = \frac{19}{12} qe^2 : x_1$$

$$x_1 = \frac{19}{60} e \approx 0,3167 e$$

ASTA BC

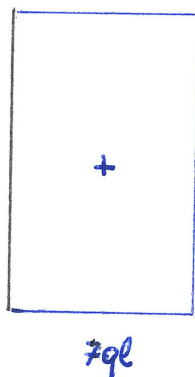
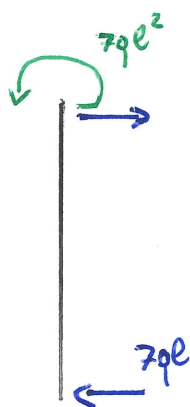


TAGLIO

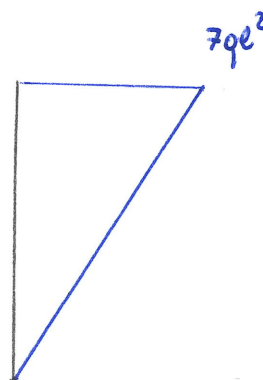


MOMENTO

ASTA BD

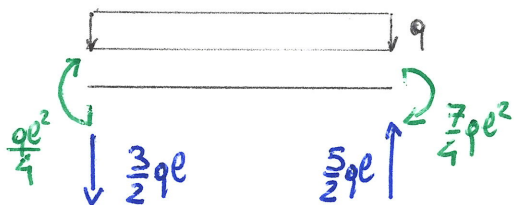


TAGLIO

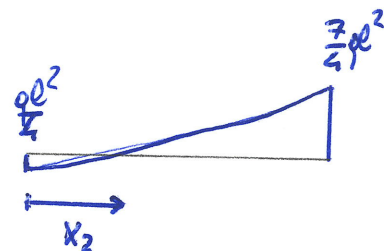


MOMENTO

ASTA BE



TAGLIO



MOMENTO

$x_2 \rightarrow$

$$M(x) = \frac{qe^2}{4} - \frac{3}{2} qlx - \frac{qx^2}{2}$$

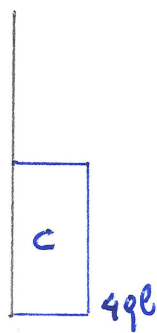
per $M(x) = 0$

$$\Rightarrow x^2 + 3ex - \frac{e^2}{2} = 0$$

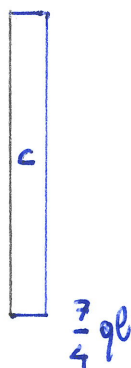
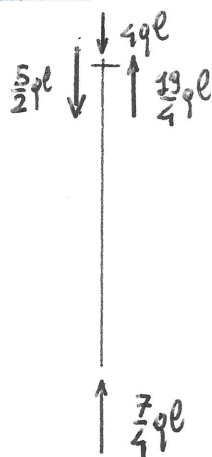
$$\rightarrow x_2 \approx 0,1583 e$$

Azioni Assiali:

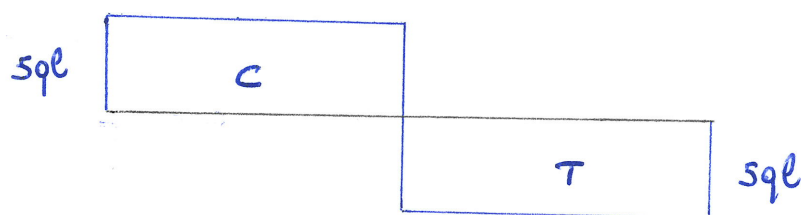
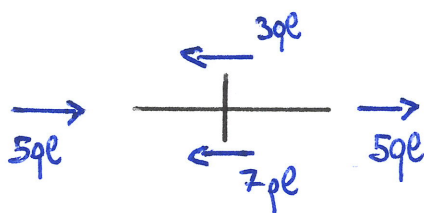
ASTA AB:



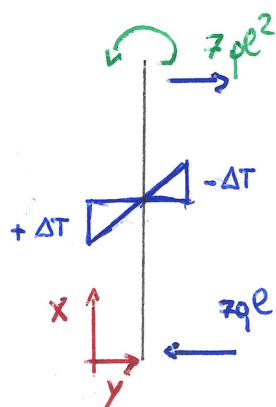
ASTA BD:



ASTA BC + BE



• Studio della deformata per l'asta BD:



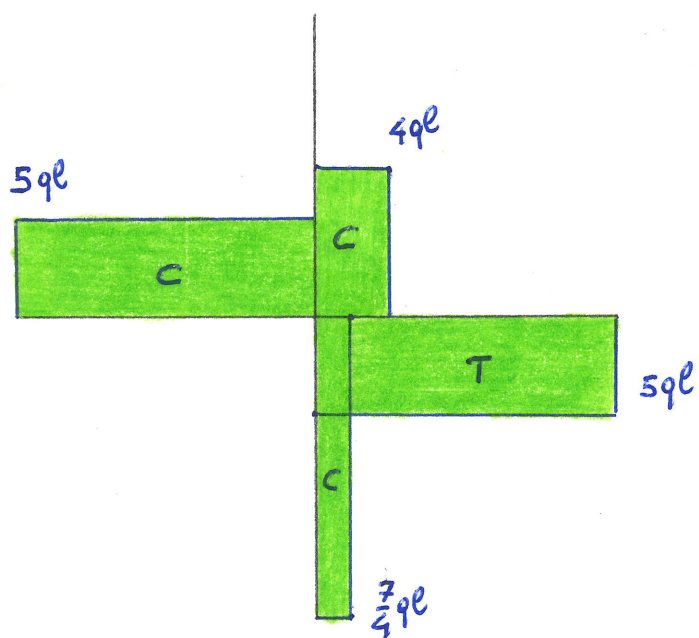
$$M(x) = 7qlx$$

$$y''(x) = -\frac{M}{EI} + 2\frac{\alpha\Delta T}{h} = -\frac{7ql}{EI}x + \frac{6ql^2}{EI}$$

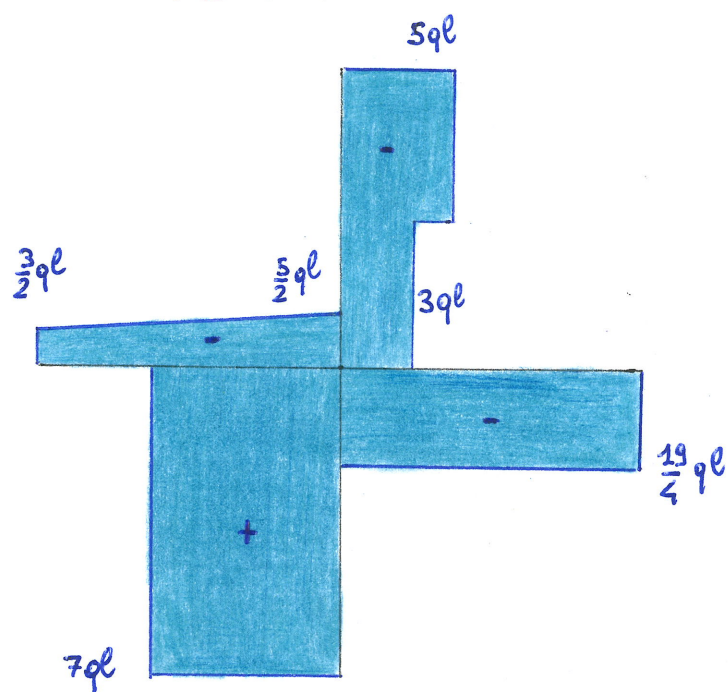
$$\text{per } y''(x) > 0 \rightarrow x_3 < \frac{6}{7}l$$

• Grafici finali:

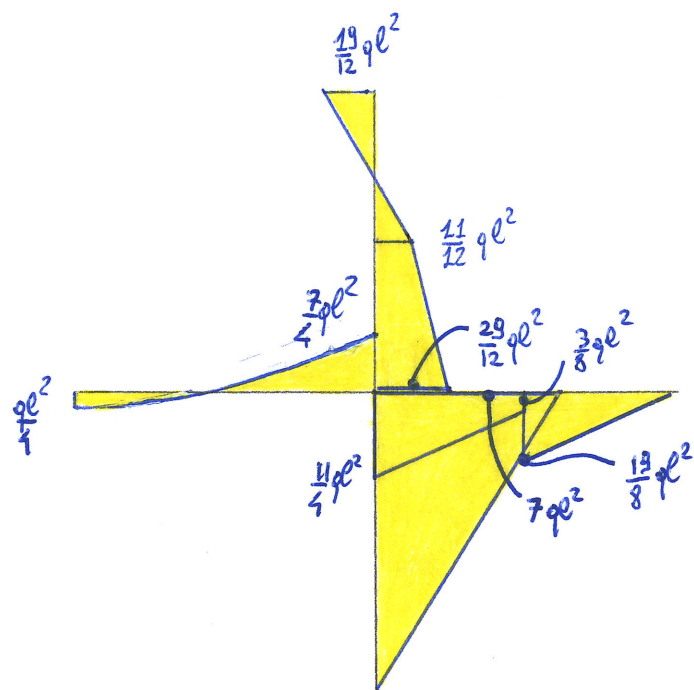
AZIONE ASSIALE



TAGLIO



MOMENTO



DEFORMATA QUALITATIVA

