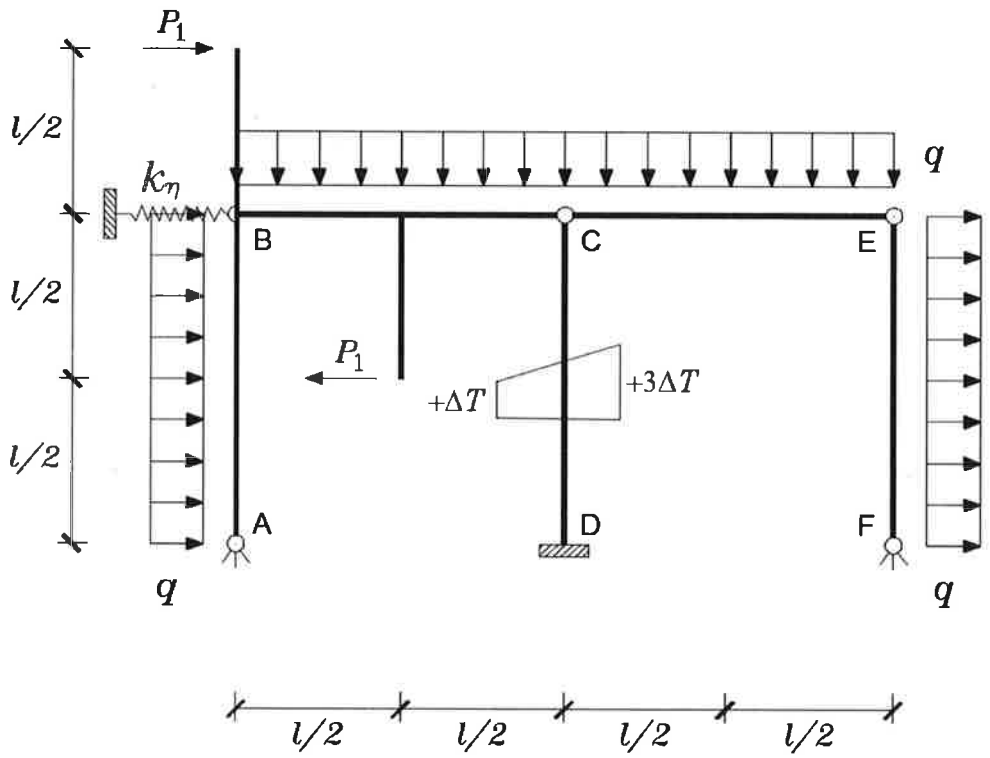
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 26 SETTEMBRE 2017

DOCENTE: **PROF. FAUSTO MINELLI** ESERCITATORE: **ING. LUCA FACCONI**

DURATA: **2 ORE**

Esercizio



$$\alpha\Delta T = \frac{1}{64}\frac{ql^3}{EJ} \qquad P_1 = \frac{27}{2}ql$$

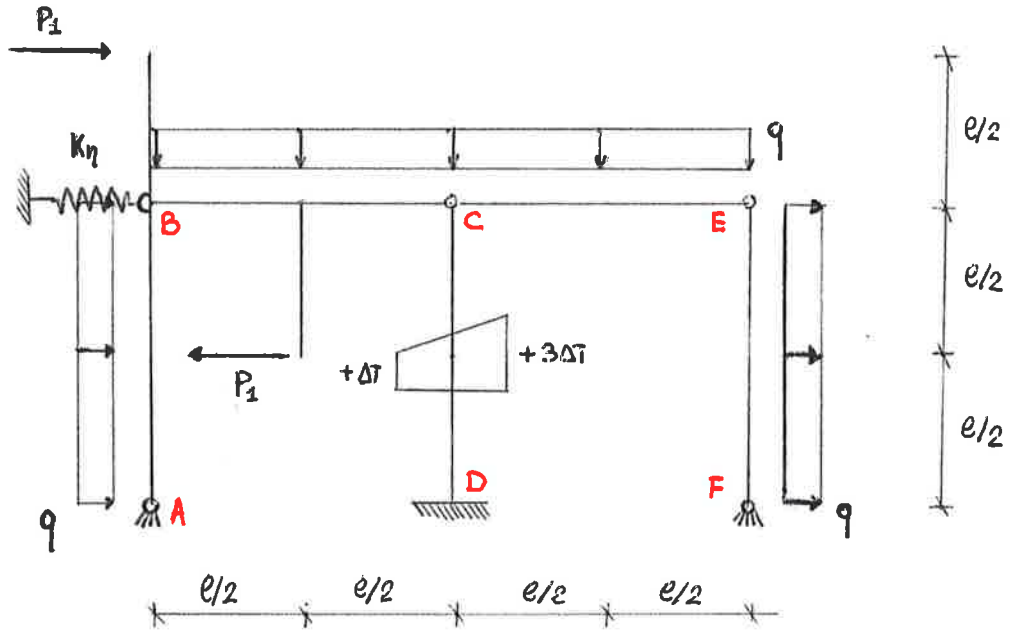
$$\frac{h}{l} = \frac{1}{14} \qquad k_\eta = \frac{99}{2}\frac{EJ}{l^3}$$

dove h rappresenta l'altezza delle aste.

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

* TEMA ESAME 26 / 09 / 2017



Dati:

$\alpha\Delta T = \frac{1}{64}\frac{qe^3}{EJ}$ $\qquad P_1 = \frac{27}{2}ql$

$\frac{h}{e} = \frac{1}{14}$ $\qquad K_\eta = \frac{99}{2}\frac{EJ}{e^3}$

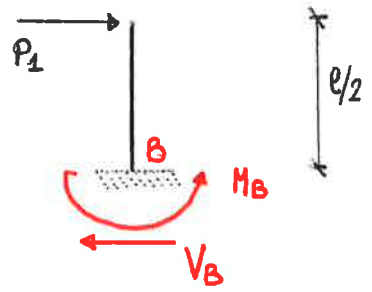
con h altezza delle aste.

- Si osserva come la struttura possa essere notevolmente semplificata riconoscendo 3 appendici isostatiche:

  - 2 MENSOLE: IN CORRISPONDENZA DEL NODO B E NELLA MEZZERIA DELL'ASTA BC

  - UN ARCO A TRE CERNIERE DATO DALLE ASTE $\overline{CE}$ E $\overline{EF}$

→ Mensola in corrispondenza del nodo B:

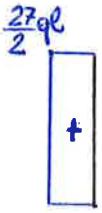


DALL' EQUILIBRIO SI PUÒ RICAVARE:

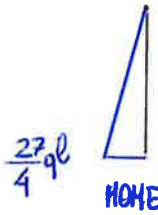
$$H_B = P_1 \cdot \frac{\ell}{2} = \frac{27}{4} q\ell^2$$

$$V_B = P_1 = \frac{27}{2} q\ell$$

AZIONE ASSIALE



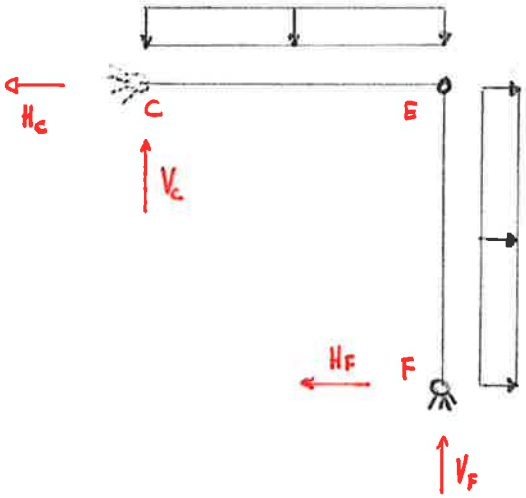
TAGLIO



MOMENTO

→ Analogamente per l'appendice posta nella mezzeria dell'asta BC.

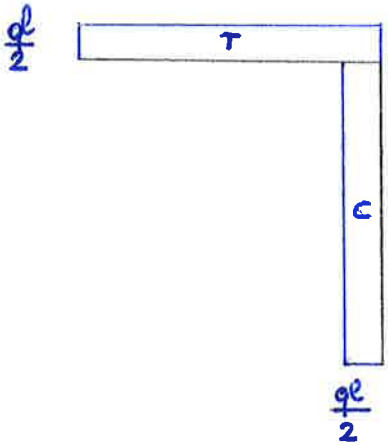
→ Arco a tre cerniere:

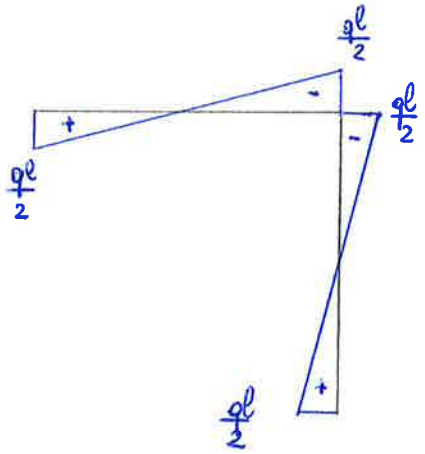


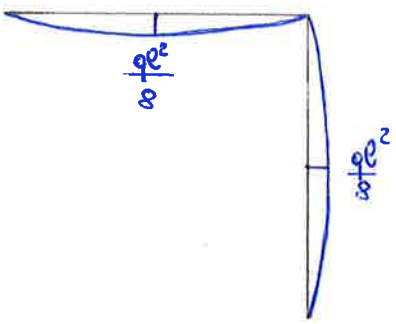
DALL' EQUILIBRIO SI PUÒ RICAVARE:

$$V_C = V_F = \frac{q\ell}{2}$$

$$H_C = H_F = \frac{q\ell}{2}$$





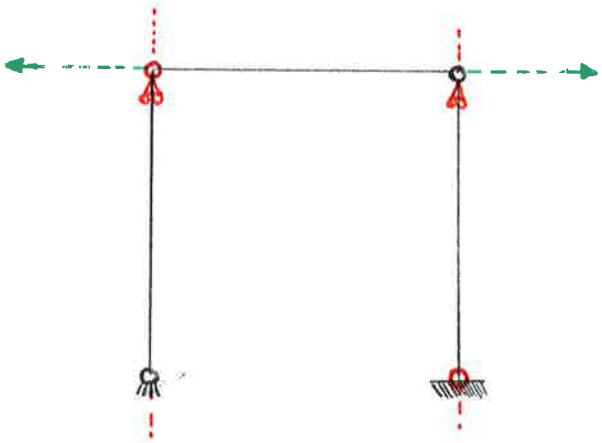


- Si valuta quindi il grado di iperstaticità della struttura:

$G.d.V = 2 + 4 + 3 + 2 + 2 = 13$

$G.d.L = 4 \cdot 3 = 12$

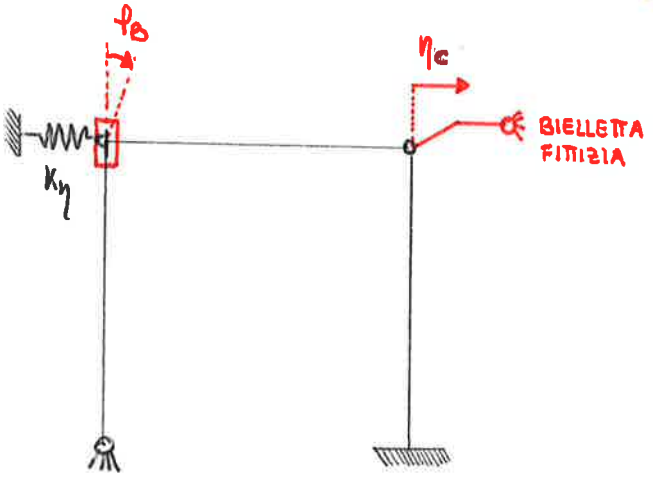
- Si valuta se il telaio sia a nodi fissi o spostabili:



➡ Si osserva chiaramente come la parte superiore del telaio possa presentare un possibile cinematismo.
Il telaio è quindi a nodi spostabili.

- Si decide di risolvere il telaio adottando il <u>metodo degli spostamenti</u>, le incognite sono:

$$\begin{cases} \varphi_B \\ \eta_D \end{cases}$$

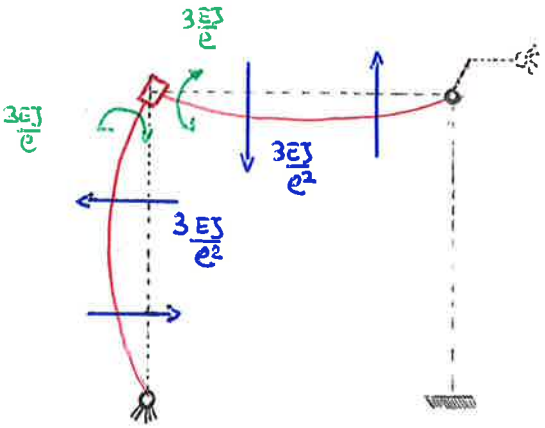


da cui il sistema risolvente (SISTEMA DELLE EQUAZIONI DI EQUILIBRIO):

$$\begin{cases} \Sigma H_B = 0 \\ \Sigma R_C = 0 \end{cases} \qquad \begin{cases} m_{BB}\,\varphi_B + m_{BC}\,\eta_C + m_{B0} = 0 \\ h_{CB}\,\varphi_B + h_{CC}\,\eta_C + h_{C0} = 0 \end{cases}$$

- Convenzioni adottate nella risoluzione:

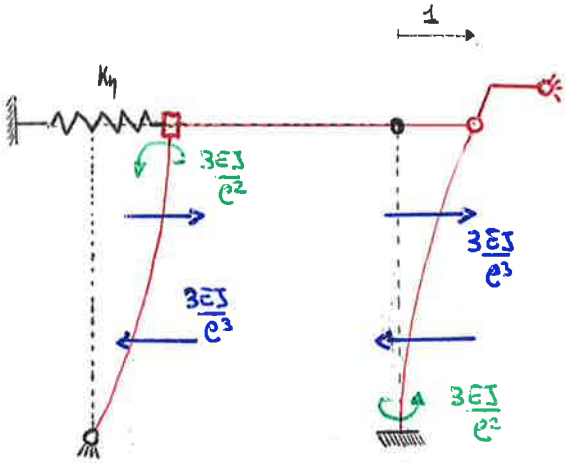
* CASO 1: $\beta_B = 1$; $\eta_C = 0$; $\forall P_{ext} = 0$



$$m_{BB} = + \frac{6EJ}{\ell}$$

$$h_{CB} = + \frac{3EJ}{\ell^2}$$

* CASO 2: $\beta_B = 0$; $\eta_C = 1$; $\forall P_{ext} = 0$

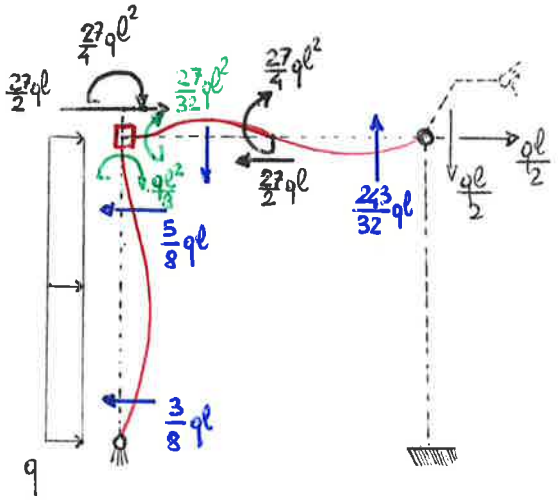


$$m_{BC} = - \frac{3EJ}{\ell^2}$$

$$h_{CC} = - \frac{6EJ}{\ell^3} - K_\eta \cdot 1$$

$$= - \frac{111}{2} \frac{EJ}{\ell^3}$$

* CASO 3 : $\varphi_B = 0$; $\eta_C = 0$; $q \neq 0$; $\frac{ql}{2} \neq 0$; $\frac{27}{2} ql \neq 0$; $\frac{27}{4} ql^2 \neq 0$

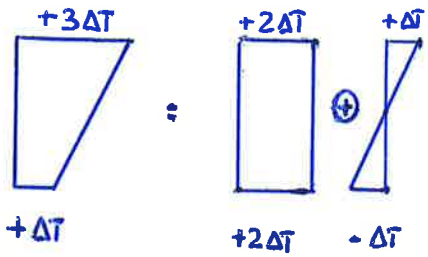


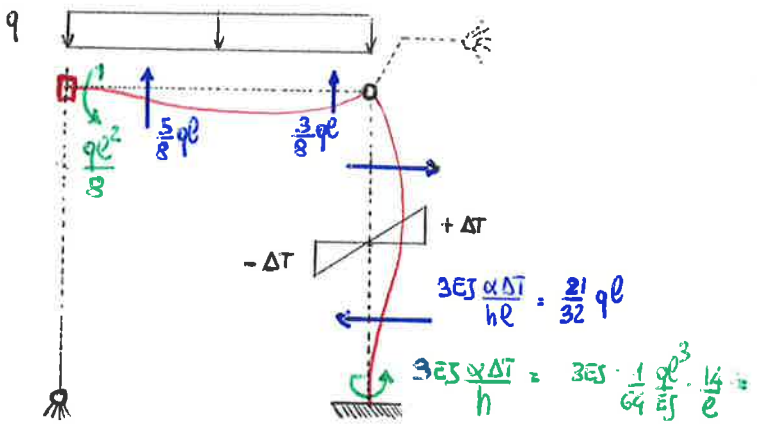
$$m^1_{BO} = +\frac{ql^2}{8} + \frac{27}{32}ql^2 - \frac{27}{4}ql^2 = -\frac{185}{32}ql^2$$

$$h^1_{CO} = +\frac{5}{8}ql - \frac{27}{2}ql + \frac{27}{2}ql + \frac{ql}{2} = +\frac{9}{8}ql$$

* CASO 4 : $\varphi_B = 0$; $\eta_C = 0$; $q \neq 0$; $\underbrace{\Delta T \neq 0}$

OSSERVAZIONE : IL CARICO TERMICO TRAPEZOIDALE DEVE ESSERE SCOMPOSTO IN DUE CONTRIBUTI :

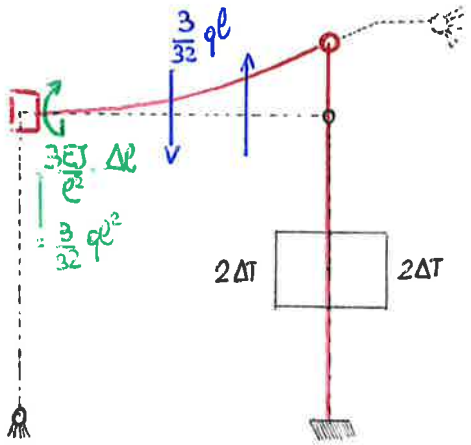




$$m^2_{BO} = -\frac{ql^2}{8}$$

$$h^2_{CO} = -\frac{21}{32}ql$$

* CASO 5: $\varphi_B = 0$; $\eta_C = 0$; $\Delta T \neq 0$



$\Delta\ell = 2\alpha\Delta T \cdot \ell$
$= 2 \cdot \frac{1}{64} \frac{q\ell^3}{EJ} \cdot \ell$
$= \frac{1}{32} \frac{q\ell^4}{EJ}$

$M_{BO}^3 = + \frac{3}{32} q\ell^2$

$h_{CO}^3 = 0$

Sistema risolvente:

$$\begin{cases} 6\frac{EJ}{\ell}\varphi_B - 3\frac{EJ}{\ell^2}\eta_C - \frac{185}{32}q\ell^2 - \frac{q\ell^2}{8} + \frac{3}{32}q\ell^2 = 0 \\ +3\frac{EJ}{\ell^2}\varphi_B - \frac{111}{2}\frac{EJ}{\ell^3}\eta_C + \frac{9}{8}q\ell - \frac{21}{32}q\ell = 0 \end{cases}$$

$$\begin{cases} 6\frac{EJ}{\ell}\varphi_B - 3\frac{EJ}{\ell^2}\eta_C - \frac{93}{16}q\ell^2 = 0 \quad 1) \\ 3\frac{EJ}{\ell^2}\varphi_B - \frac{111}{2}\frac{EJ}{\ell^3}\eta_C + \frac{15}{32}q\ell = 0 \quad 2) \end{cases}$$

moltiplico la 2) per -2ℓ e sommo alla 1):

$$-3\frac{EJ}{\ell^2}\eta_C + 111\frac{EJ}{\ell^2}\eta_C - \frac{93}{16}q\ell^2 - \frac{15}{16}q\ell^2 = 0$$

$$\rightarrow 108\frac{EJ}{\ell^2}\eta_C = \frac{27}{4}q\ell^2 \rightarrow \eta_C = +\frac{1}{16}\frac{q\ell^4}{EJ}$$

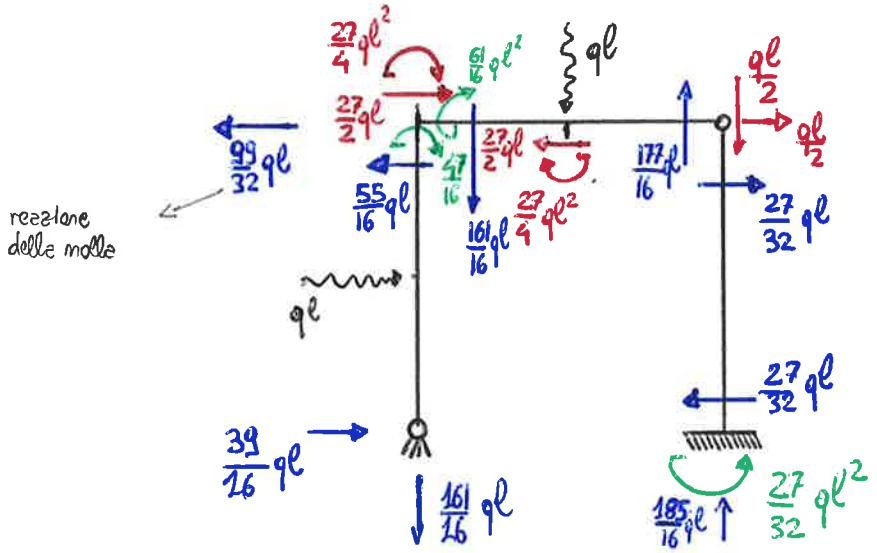
sostituisco η_C in 1):

$$6\frac{EJ}{\ell}\varphi_B - 3\frac{EJ}{\ell^2}\cdot\left(\frac{1}{16}\frac{q\ell^4}{EJ}\right) - \frac{93}{16}q\ell^2 = 0 \rightarrow \varphi_B = +\frac{q\ell^3}{EJ}$$

Da cui in conclusione:

$$\begin{cases} f_B = + \dfrac{q\ell^3}{EJ} \\ \eta_C = + \dfrac{1}{16} \dfrac{q\ell^4}{EJ} \end{cases}$$

- Si passa quindi al calcolo delle azioni interne:



VERIFICA EQUILIBRIO ALLA TRASLAZIONE ORIZZONTALE: $\Sigma F_H = 0$

$$\frac{39}{16}q\ell - \frac{99}{32}q\ell + \frac{27}{2}q\ell - \frac{27}{2}q\ell + \frac{q\ell}{2} - \frac{27}{32}q\ell + q\ell \stackrel{?}{=} 0 \quad \checkmark$$

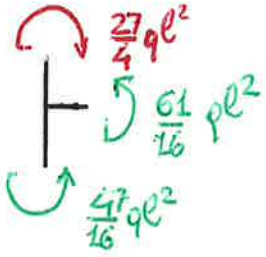
VERIFICA EQUILIBRIO ALLA TRASLAZIONE VERTICALE: $\Sigma F_V = 0$

$$\frac{161}{16}q\ell + q\ell - \frac{185}{16}q\ell + \frac{q\ell}{2} \stackrel{?}{=} 0 \quad \checkmark$$

VERIFICA DELL' EQUILIBRIO GLOBALE NEL NODO C: $\Sigma M_C = 0$

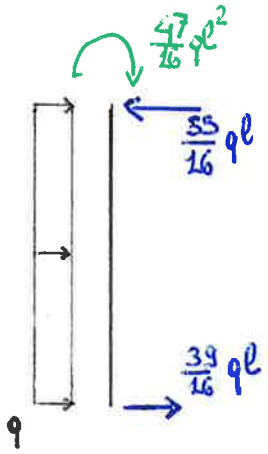
$$-\frac{161}{16}q\ell^2 - \frac{39}{16}q\ell^2 - \frac{q\ell^2}{2} + \frac{27}{4}q\ell^2 + \frac{27}{4}q\ell^2 - \frac{q\ell^2}{2} - \frac{27}{32}q\ell^2 + \frac{27}{32}q\ell^2 \stackrel{?}{=} 0 \quad \checkmark$$

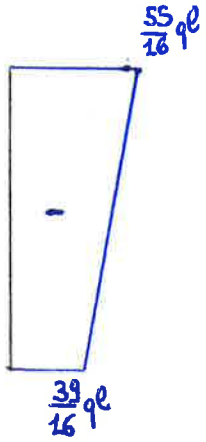
VERIFICA DELL' EQUILIBRIO LOCALE NEL NODO B:



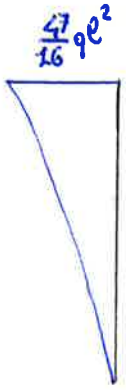
$$\frac{27}{4}q\ell^2 + \frac{61}{16}q\ell^2 + \frac{47}{16}q\ell^2 \stackrel{?}{=} 0 \quad \checkmark$$

ASTA AB



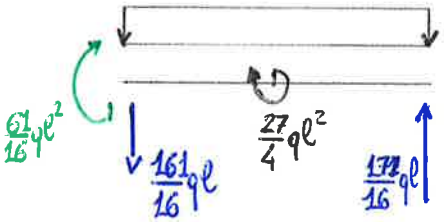


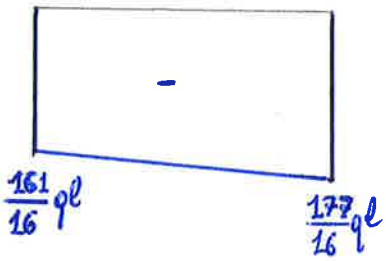
TAGLIO



MOMENTO

ASTA BC





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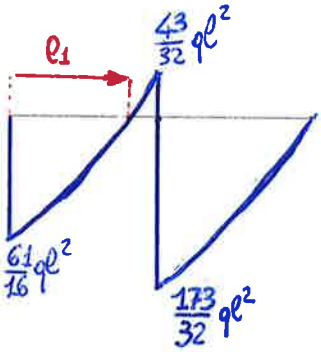


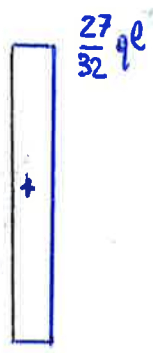
MOMENTO

$l_1 = ?$

$$M(x) = \frac{61}{16}ql^2 - \frac{161}{16}qlx - \frac{qx^2}{2} = 0 \longrightarrow x^2 + \frac{161}{8}lx - \frac{61}{8}l^2 = 0$$

$$x = 0.37\,l \longrightarrow \underline{l_1 = 0.37\,l}$$

ASTA CD



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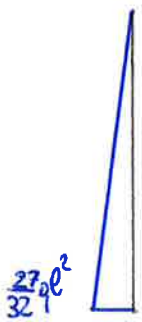
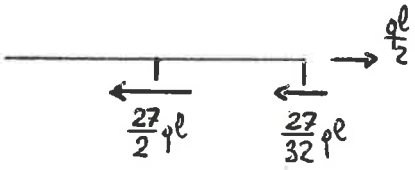
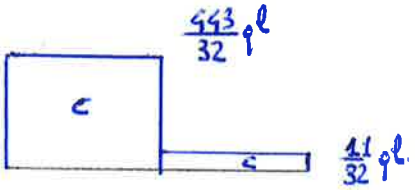


MOMENTO

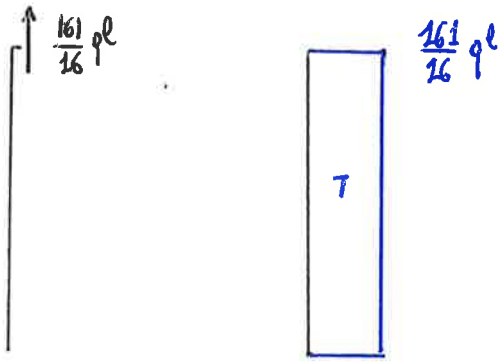
Azioni assiali

ASTA BC

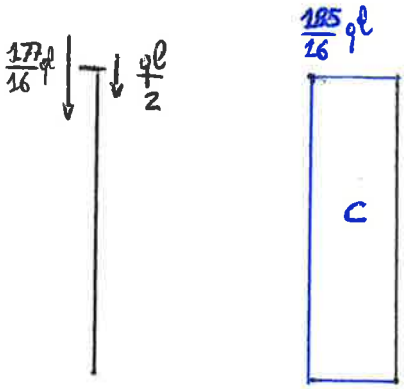




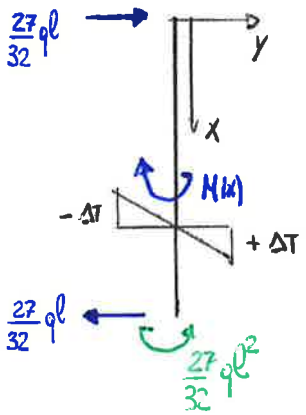
ASTA AB



ASTA CD



- Studio della deformata per l'asta CD:



$H(x) = ?$

$H(x) = -\frac{27}{32}qlx$

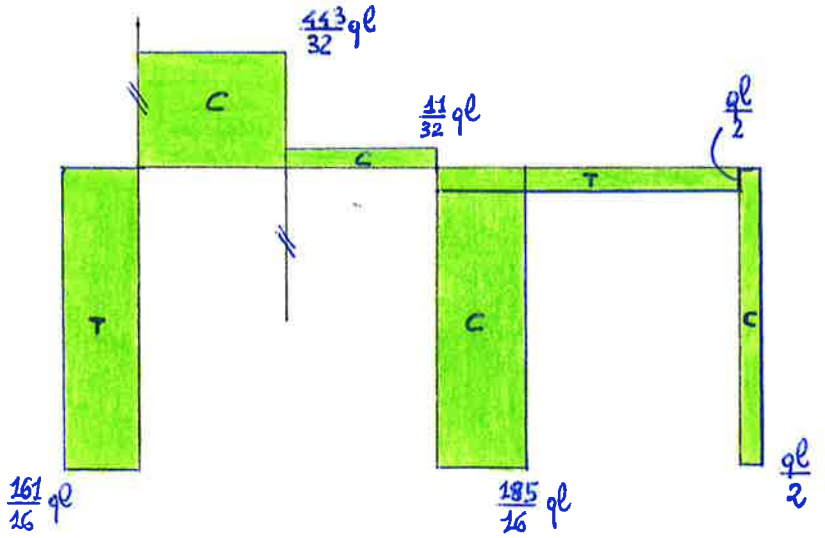
$$y''(x) = -\frac{H(x)}{EJ} \pm 2\frac{\alpha\Delta T}{h}$$

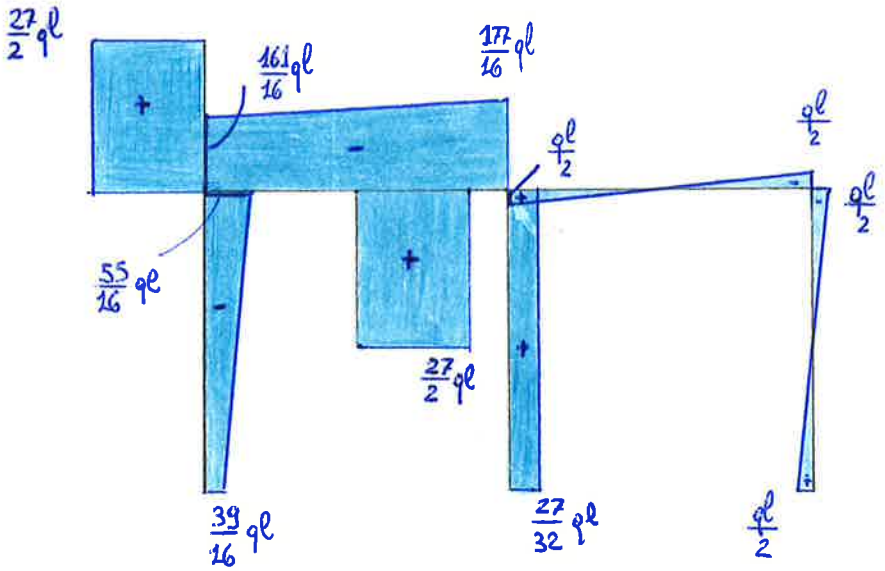
$$= \frac{27}{32}\frac{ql}{EJ}x - 2\cdot\frac{1}{64}\frac{ql^3}{EJ}\cdot\frac{14}{e} = \frac{27}{32}\frac{ql}{EJ}x - \frac{7}{16}\frac{ql^2}{EJ}$$

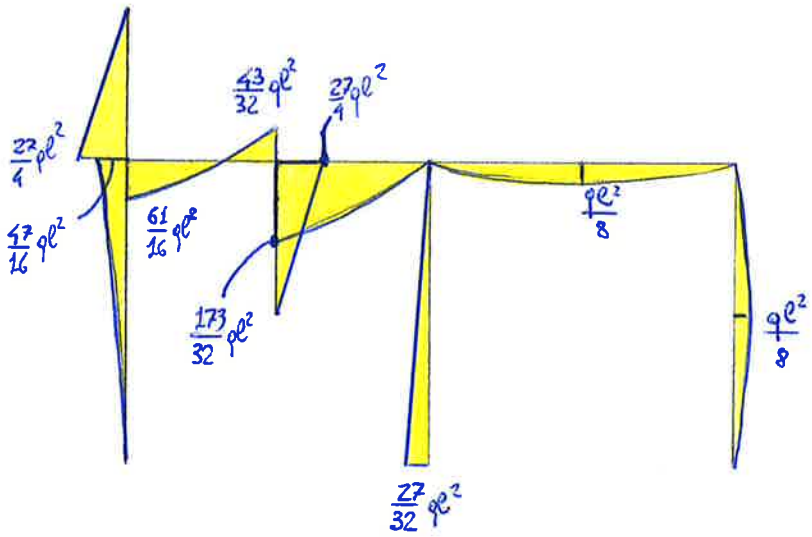
quando $y''(x) \gtrless 0 \longrightarrow \frac{27}{32}\frac{ql}{EJ}x > \frac{7}{16}\frac{ql^2}{EJ} \longrightarrow x > \frac{14}{27}e$

$\Rightarrow \forall x \in [0;\ 14/27\,e]\quad y'' \leq 0$

- Grafici finali:







- Deformata qualitativa: