

TECNICA DELLE COSTRUZIONI

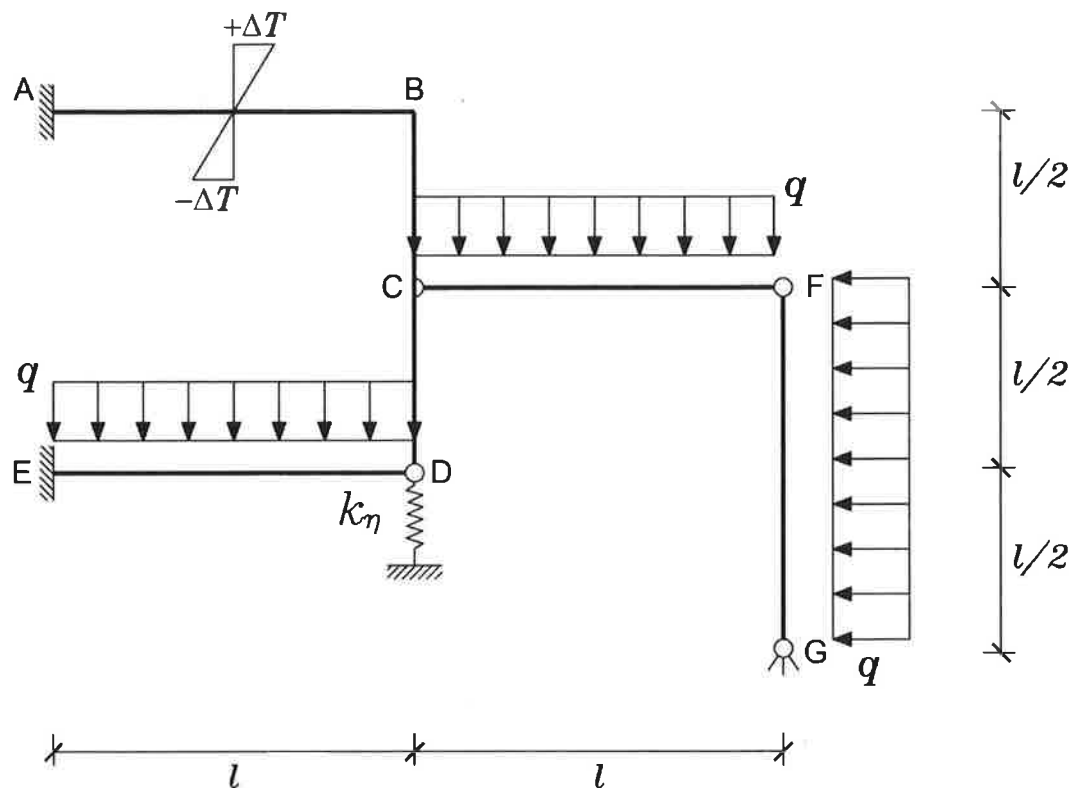
TEMA ESAME DEL 27 NOVEMBRE 2017

DOCENTE: PROF. FAUSTO MINELLI

ESERCITATORE: ING. LUCA FACCONI

DURATA: 2 ORE e 30 MIN.

Esercizio



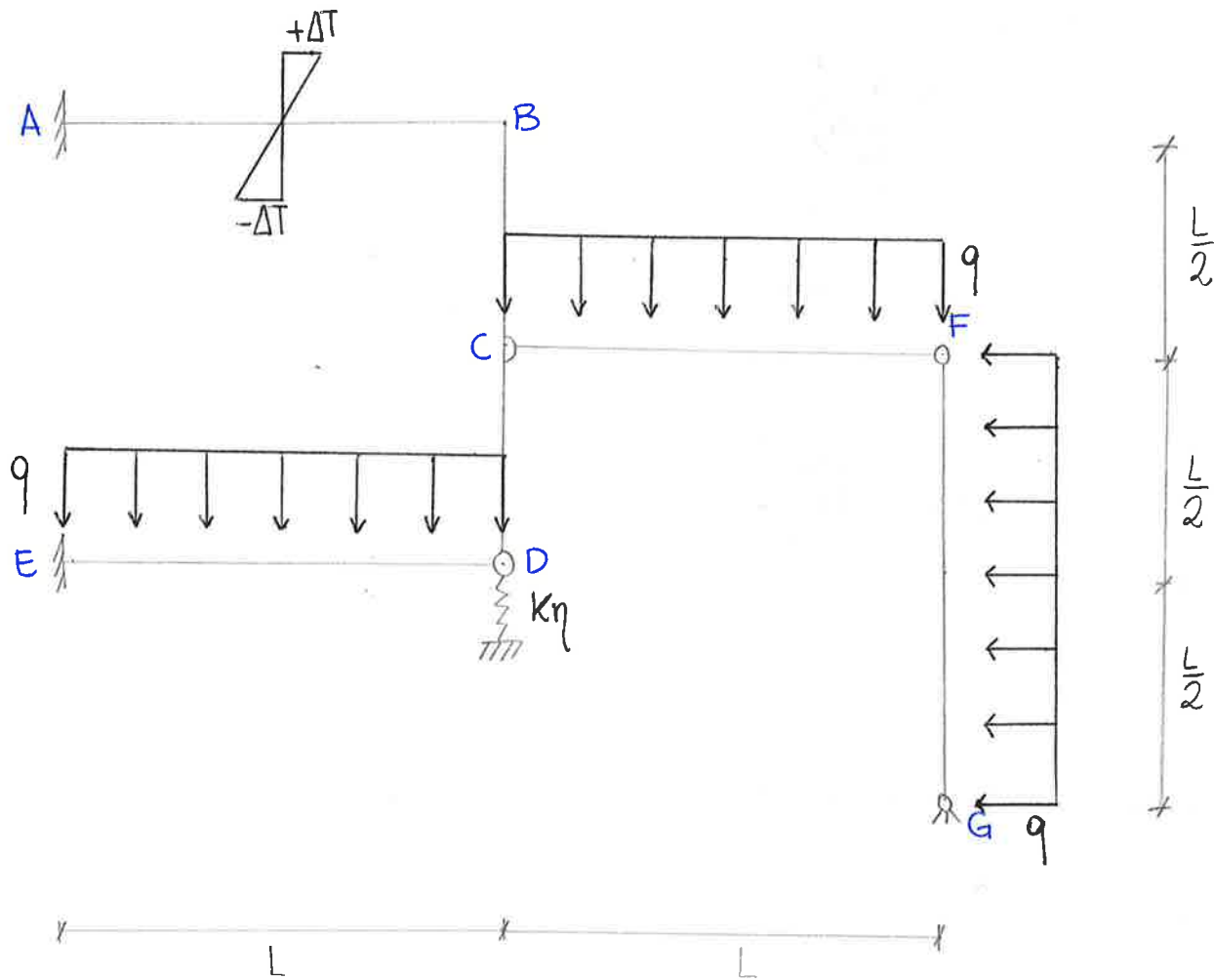
$$\frac{\alpha \Delta T}{H} = \frac{41 q l^2}{64 E J}$$

$$k_\eta = 23 \frac{E J}{l^3}$$

Dato il telaio in figura, si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME 24/11/2017

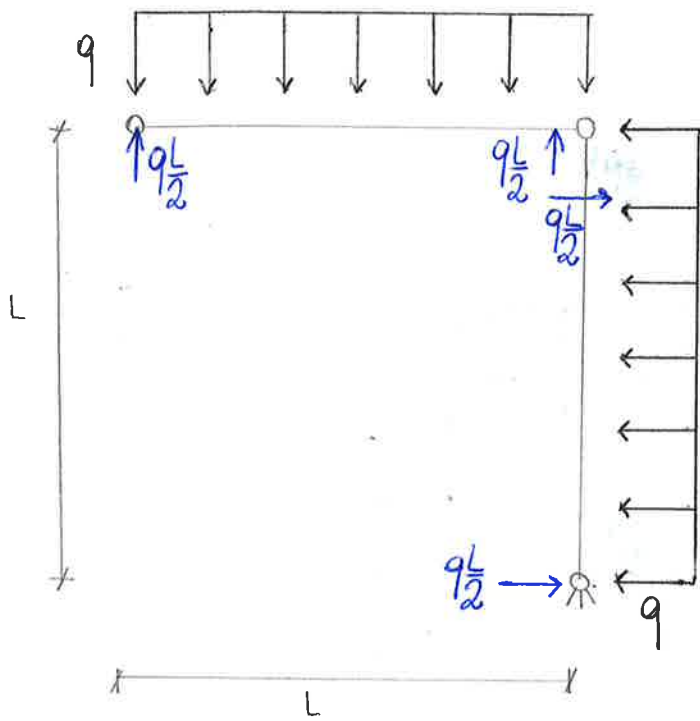


DATI:

$$\frac{\Delta T}{H} = \frac{41}{64} \frac{qL^2}{EJ}$$

$$k_\eta = 23 \frac{EJ}{L^3}$$

IN SI OSSERVA COME LE ASTE CF E FG SIANO UN'APPENDICE ISOSTATICA, PUO' ESSERE RISOLTA SEMPLICEMENTE CON GLI EQUILIBRI E PUO' ESSERE "STACCATA" DALLA STRUTTURA PRINCIPALE

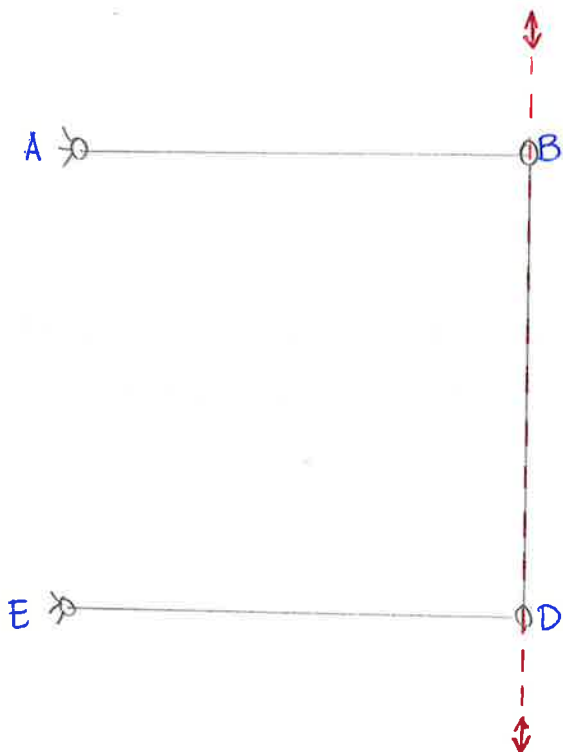


SI VALUTA QUINDI IL GRADO DI IPERSTATICITA' DELLA STRUTTURA

$$GdV = 3 + 3 + 2 = 8$$

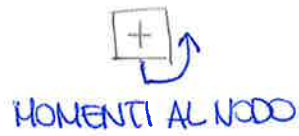
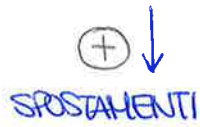
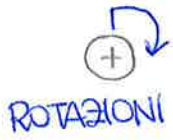
$$GdL = 3 \cdot 2 = 6$$

SI VALUTA SE IL TELAIIO È A NODI FISSI O SPOSTABILI



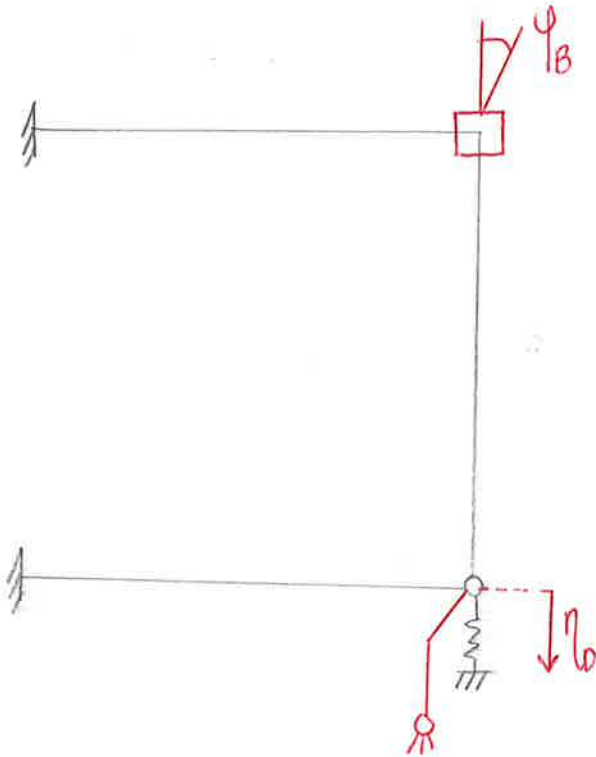
SI OSSERVA UN POSSIBILE CINEMATISMO PER L'ASTA BD
IL TELAIIO È A NODI SPOSTABILI

N CONVENZIONI ADOTTATE NELLA RISOLUZIONE DEL TELAI0



N RISOLVO IL TELAI0 CON IL METODO DEGLI SPOSTAMENTI
LE INCOGNITE SONO:

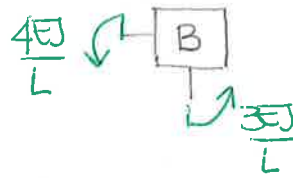
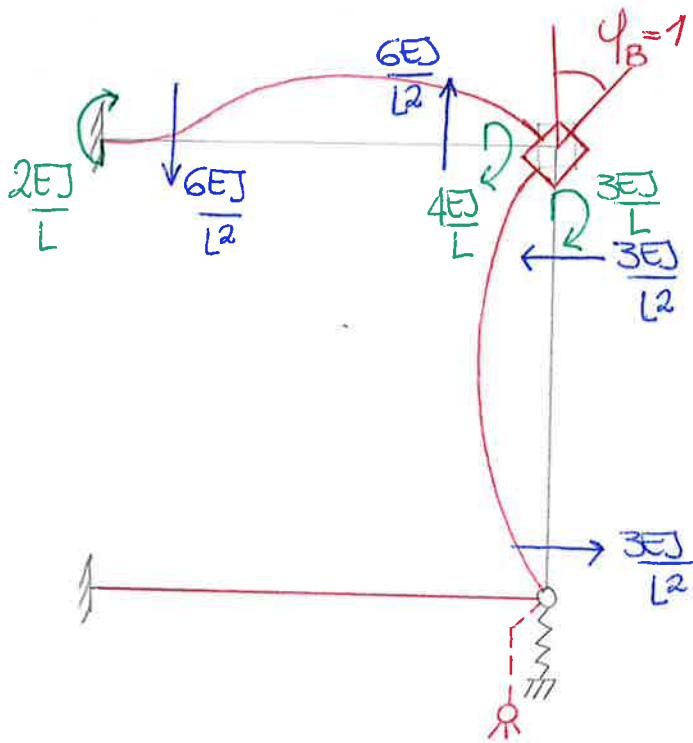
$$\begin{cases} \varphi_B \\ \eta_D \end{cases}$$



IL SISTEMA RISOLVENTE RISULTA ESSERE:

$$\begin{cases} \sum M_B = 0 \\ \sum R_D = 0 \end{cases} \quad \begin{cases} m_{BB} \cdot \varphi_B + m_{B\eta} \cdot \eta_D + m_{B0} = 0 \\ h_{\eta B} \cdot \eta + h_{\eta\eta} \cdot \eta_D + h_{\eta 0} = 0 \end{cases}$$

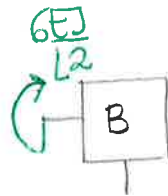
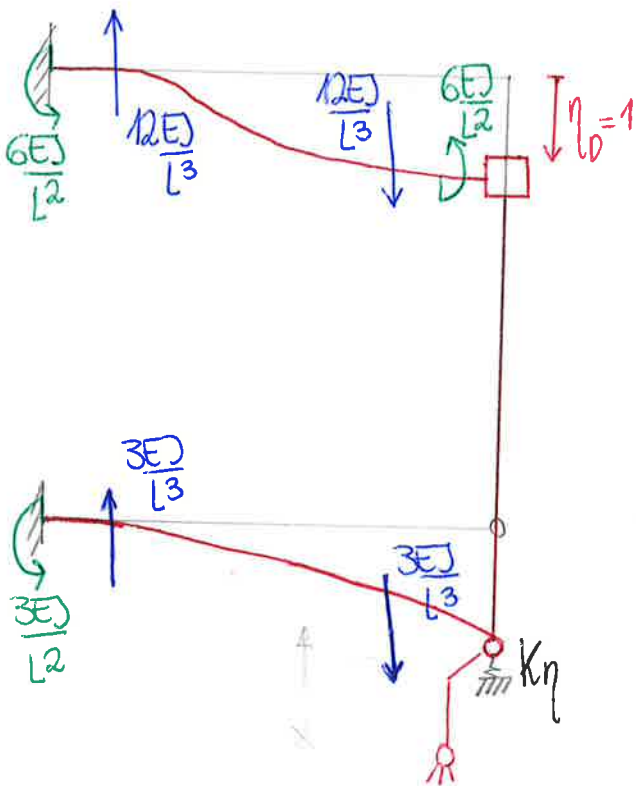
N CASO 1: $\psi_B = 1$; $\eta_D = 0$; $V_{P_{EXT}} = 0$



$$m_{BB} = \frac{4EJ}{L} + \frac{3EJ}{L} = \frac{7EJ}{L}$$

$$h_{\eta B} = \frac{6EJ}{L^2} \quad \text{COMPRESSIONE}$$

N CASO 2: $\psi_B = 0$; $\eta_D = 1$; $V_{P_{EXT}} = 0$



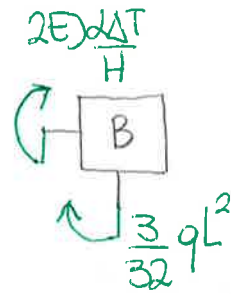
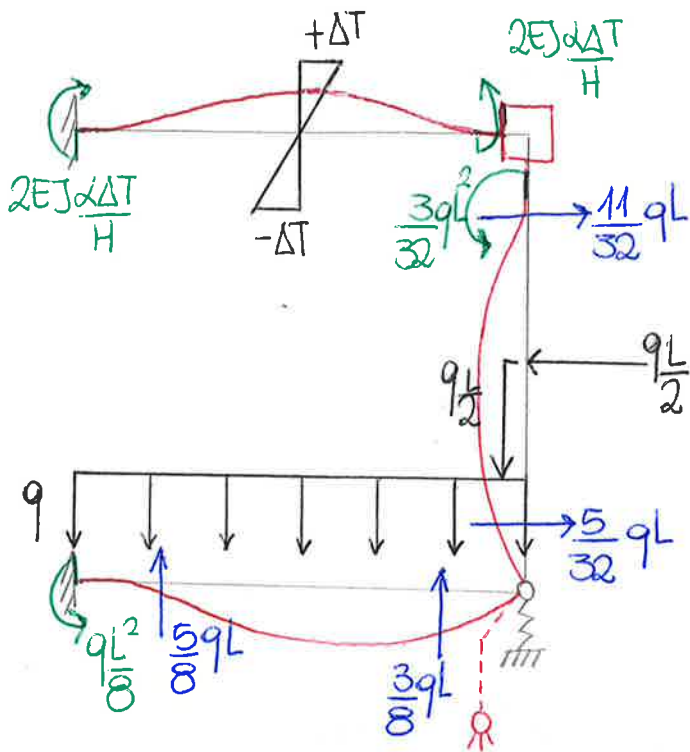
$$m_{B\eta} = -\frac{6EJ}{L^2}$$

$$h_{\eta\eta} = -\frac{12EJ}{L^3} - \frac{3EJ}{L^3} - K_{\eta} =$$

$$= -\frac{12EJ}{L^3} - \frac{3EJ}{L^3} - \frac{23EJ}{L^3} =$$

$$= -\frac{38EJ}{L^3}$$

N CASO 3: $\varphi_B = 0$; $\eta_0 = 0$; $V_{PEXT} \neq 0$



$$\begin{aligned}
 M_{B0} &= -2EJ\frac{\alpha\Delta T}{H} - \frac{3}{32}qL^2 \\
 &= -2EJ \cdot \frac{41}{64} \frac{qL^2}{EJ} - \frac{3}{32}qL^2 = \\
 &= -\frac{41}{32}qL^2 = -\frac{11}{8}qL^2
 \end{aligned}$$

$$h\eta_0 = \frac{qL}{2} + \frac{3}{8}qL = \frac{7}{8}qL$$

NSISTEMA RISOLVENTE :

$$\begin{cases} \frac{7EJ}{L} \cdot \varphi_B - \frac{6EJ}{L^2} \cdot \eta_0 - \frac{11}{8}qL^2 = 0 & (1) \end{cases}$$

$$\begin{cases} \frac{6EJ}{L^2} \cdot \varphi_B - \frac{38EJ}{L^3} \cdot \eta_0 + \frac{7}{8}qL = 0 & (2) \end{cases}$$

MOLTIPLICO (1) PER $(-\frac{6}{7L})$ E LA SOMMO A (2)

$$\frac{7EJ}{L} \cdot (-\frac{6}{7L}) \cdot \varphi_B - \frac{6EJ}{L^2} (-\frac{6}{7L}) \cdot \eta_0 - \frac{11}{8}qL^2 (-\frac{6}{7L}) = 0$$

$$\frac{6EJ}{L^2} \cdot \varphi_B + \frac{36EJ}{7L^3} \eta_0 + \frac{33}{28}qL = 0$$

$$\left[-\frac{6EJ}{L^2} + \frac{6EJ}{L^2} \right] \cdot \varphi_B + \left[\frac{36}{7} \frac{EJ}{L^3} - 38 \frac{EJ}{L^3} \right] \cdot \eta_D + \left[\frac{33}{28} qL + \frac{7}{8} qL \right] = 0$$

$$-\frac{230}{7} \frac{EJ}{L^3} \cdot \eta_D + \frac{230}{112} qL = 0 \longrightarrow \eta_D = \frac{230}{112} qL \cdot \frac{7}{230} \frac{L^3}{EJ} = \frac{1}{16} \frac{qL^4}{EJ}$$

SOSTITUISCO η_D NELLA (2)

$$\frac{6EJ}{L^2} \cdot \varphi_B - 38 \cdot \frac{EJ}{L^3} \cdot \frac{1}{16} \frac{qL^4}{EJ} + \frac{7}{8} qL = 0$$

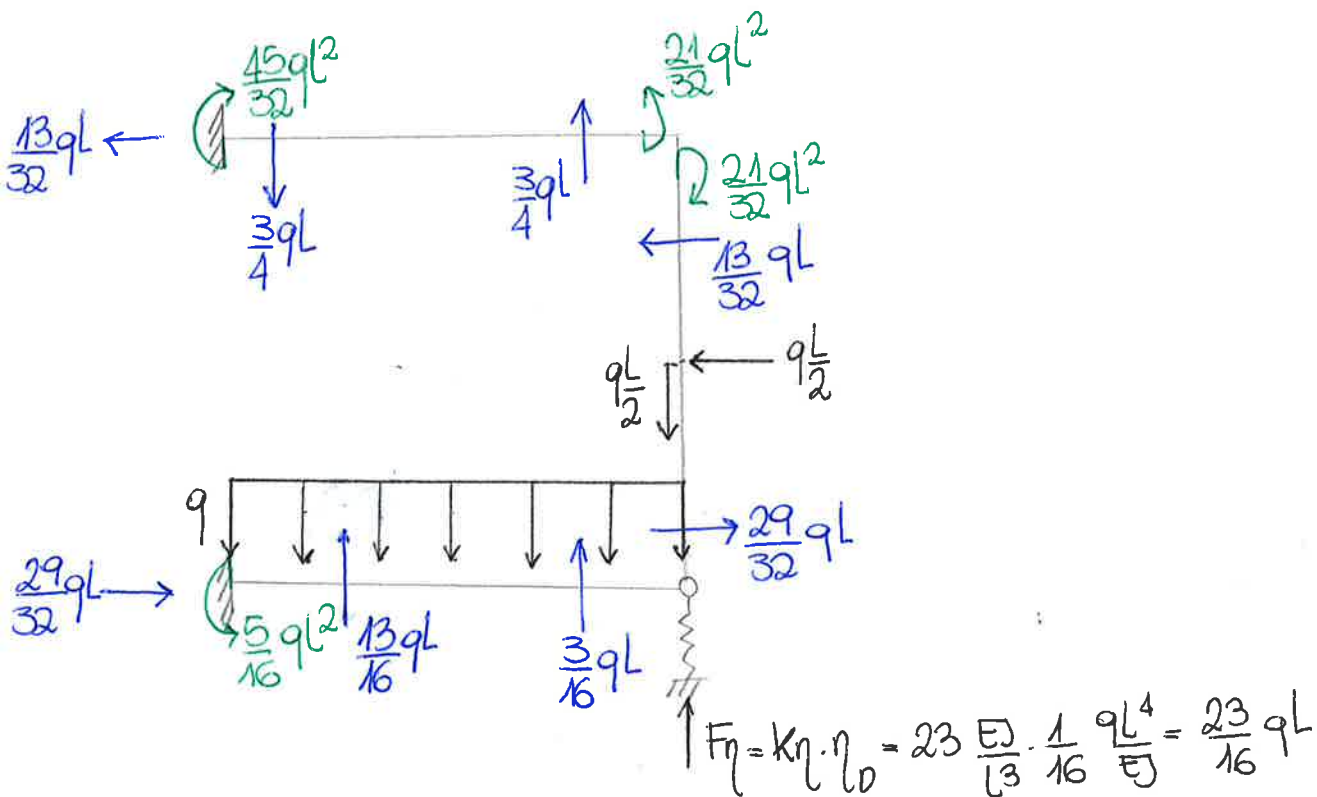
$$\frac{6EJ}{L^2} \varphi_B = \frac{38}{16} qL - \frac{7}{8} qL$$

$$\frac{6EJ}{L^2} \varphi_B = \frac{24}{16} qL \longrightarrow \varphi_B = \frac{24}{16} qL \cdot \frac{L^2}{6EJ} = \frac{1}{4} \frac{qL^3}{EJ}$$

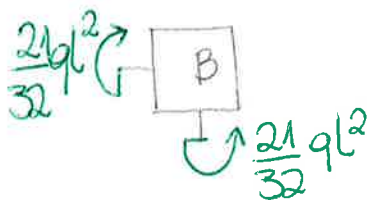
DA CUI IN CONCLUSIONE

$$\begin{cases} \varphi_B = \frac{1}{4} \frac{qL^3}{EJ} \\ \eta_D = \frac{1}{16} \frac{qL^4}{EJ} \end{cases}$$

IV CALCOLO DELLE AZIONI INTERNE



IV VERIFICA AL NODO B



IV VERIFICA EQUILIBRIO ALLA TRASLAZIONE VERTICALE : $\sum F_V = 0$

$$\uparrow + \frac{13}{16} qL - \frac{3}{4} qL - qL + \frac{23}{16} qL - \frac{qL}{2} = 0 \quad 0=0 \quad \text{ok!}$$

IV VERIFICA EQUILIBRIO ALLA TRASLAZIONE ORIZZONTALE $\sum F_H = 0$

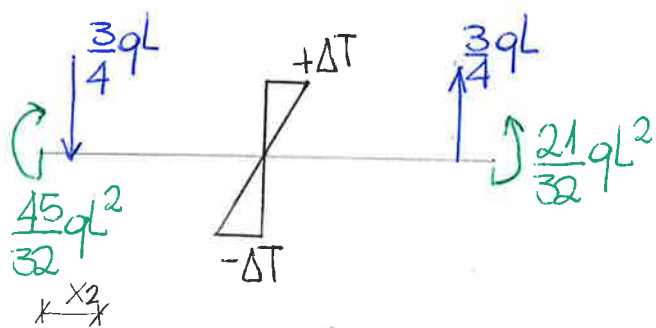
$$\rightarrow + \frac{29}{32} qL - \frac{13}{32} qL - \frac{qL}{2} = 0 \quad 0=0 \quad \text{ok!}$$

IV VERIFICA EQUILIBRIO ALLA ROTAZIONE $\sum M_E = 0$

$$\rightarrow + \frac{qL \cdot L}{2} - \frac{5}{16} qL^2 - \frac{23}{16} qL \cdot L - \frac{13}{32} qL \cdot L + \frac{45}{32} qL^2 + \frac{qL}{2} \cdot L - \frac{qL}{2} \cdot \frac{L}{2} = 0$$

$$0=0 \quad \text{ok!}$$

IV STUDIO DEFORMATA ASTA AB



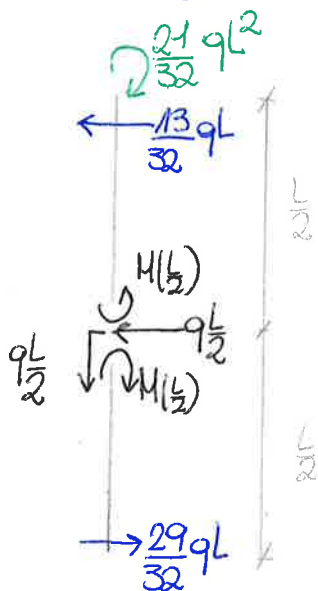
$$M(x) = \frac{45}{32} qL^2 - \frac{3}{4} qLx$$

$$y''(x) = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{H} = -\left(\frac{45}{32} qL^2 - \frac{3}{4} qLx\right) \cdot \frac{1}{EJ} + 2 \cdot \frac{41}{64} \frac{qL^2}{EJ}$$

$$y'' \cdot EJ = -\frac{45}{32} qL^2 + \frac{3}{4} qLx + \frac{41}{32} qL^2$$

$$y'' \cdot EJ = \frac{3}{4} qLx - \frac{4}{32} qL^2 \geq 0 \rightarrow x_2 \geq \frac{1}{6} L$$

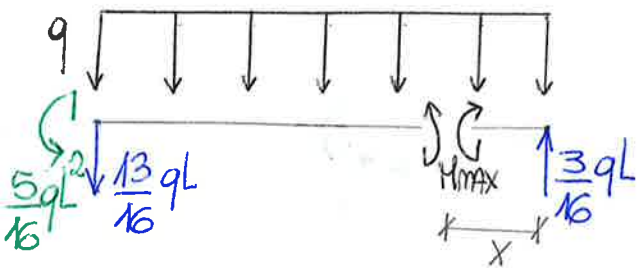
IV MOMENTO A $\frac{L}{2}$ SULL'ASTA BD



$$\rightarrow M\left(\frac{L}{2}\right) - \frac{29}{32} qL \cdot \frac{L}{2} = 0$$

$$M\left(\frac{L}{2}\right) = \frac{29}{64} qL^2$$

v MMAX ASTA DE



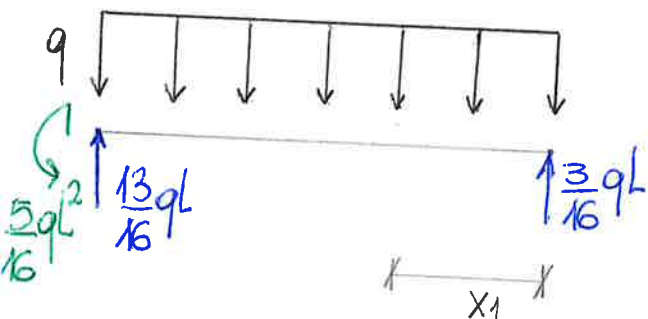
$$+2) M_{MAX} - \frac{3}{16} qL \cdot x + q \cdot x \cdot \frac{x}{2} = 0$$

$$M_{MAX} = \frac{3}{16} qLx - \frac{qx^2}{2}$$

$$\frac{dM_{MAX}}{dx} = 0 \rightarrow \frac{3}{16} qL - qx = 0 \rightarrow x = \frac{3}{16} L = 0,1875L$$

$$M_{MAX} = \frac{3}{16} qL \cdot \frac{3}{16} L - \left(\frac{3L}{16}\right)^2 \cdot \frac{q}{2} = \frac{9}{512} qL^2$$

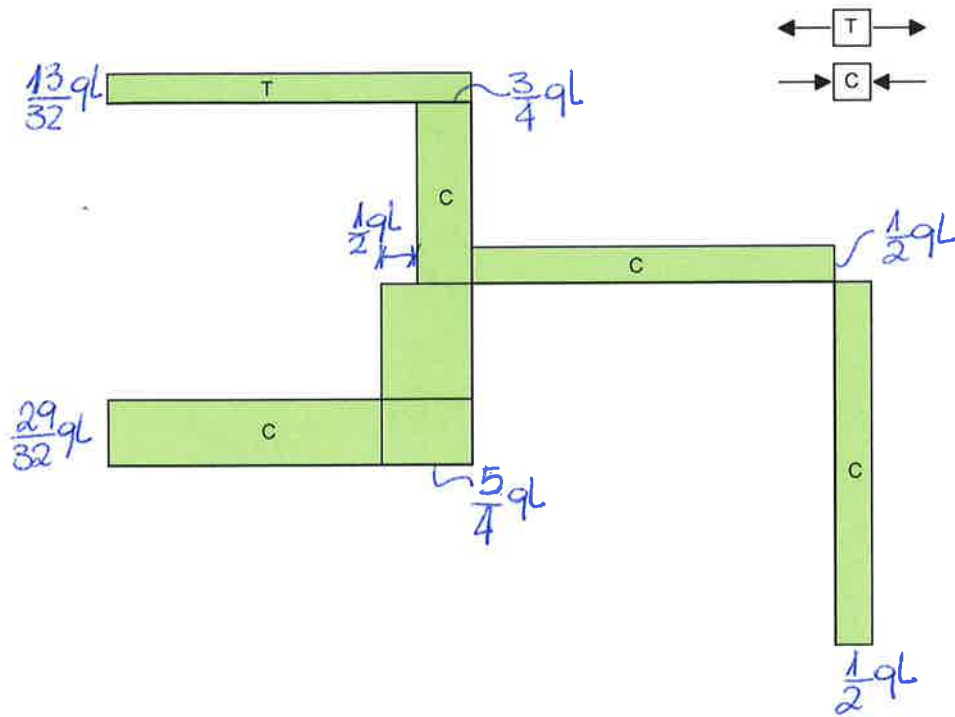
n FUESSO 1 ASTA DE



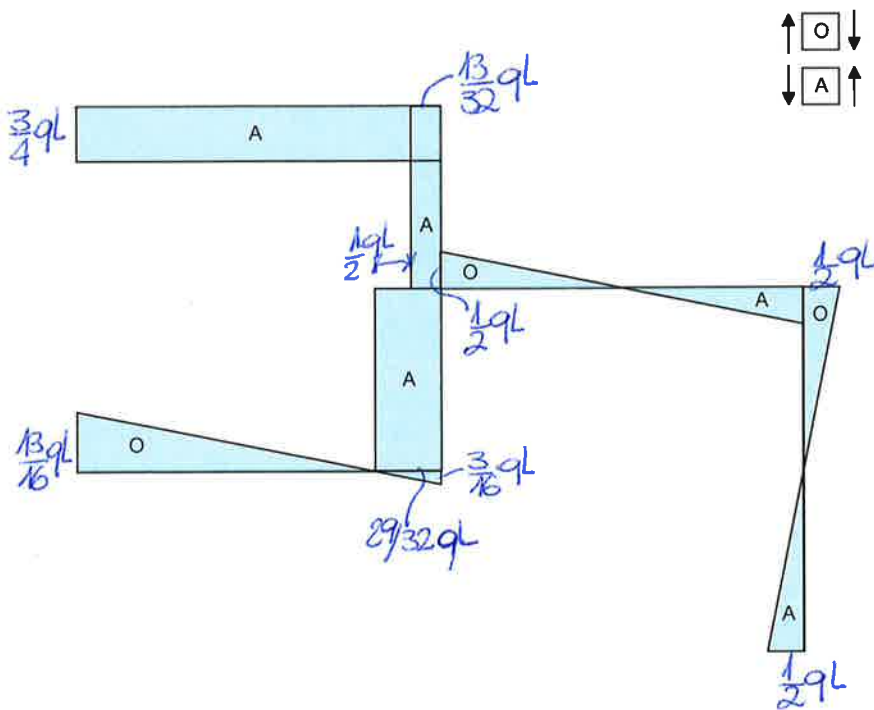
$$+2) M(x) = -\frac{3}{16} qLx + \frac{qx^2}{2} = 0$$

$$x \left(\frac{x}{2} - \frac{3L}{16} \right) = 0 \begin{cases} x = 0 \\ x_1 = \frac{3L}{8} = 0,375L \end{cases}$$

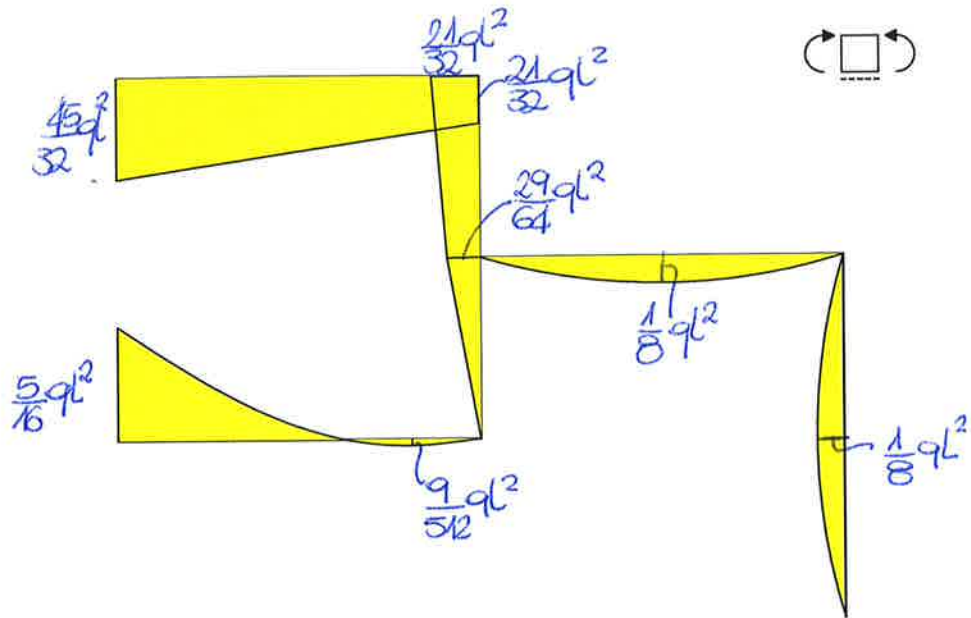
AZIONE ASSIALE



TAGLIO



MOMENTO FLETTENTE



DEFORMATA QUALITATIVA

