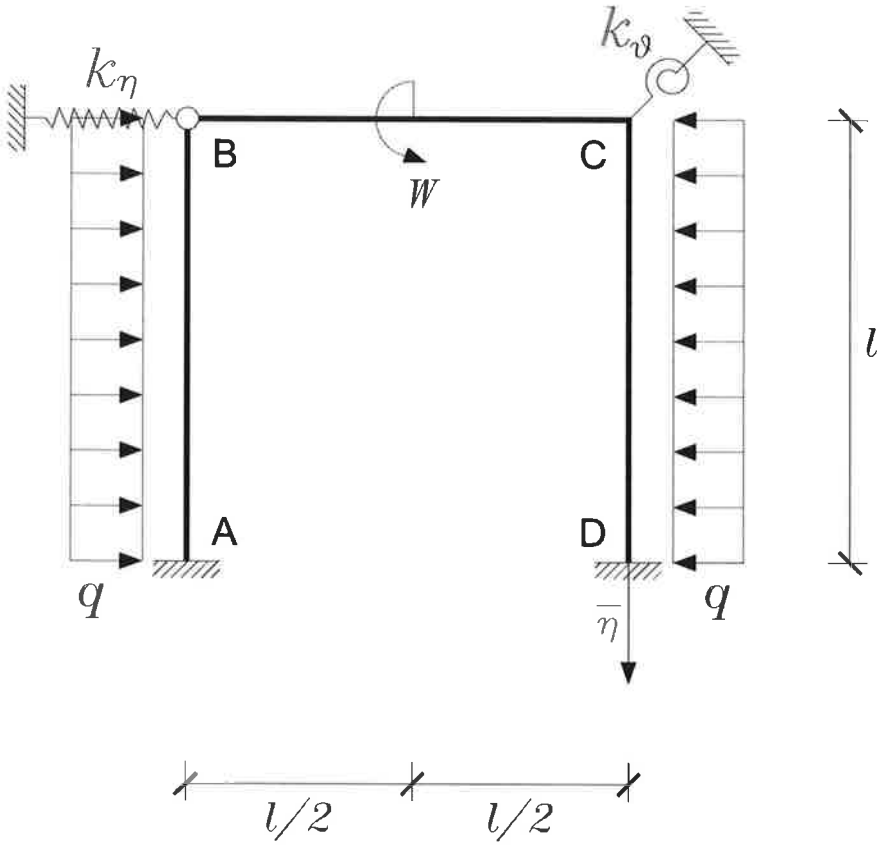
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 15 GENNAIO 2018

DOCENTE: **PROF. FAUSTO MINELLI** ESERCITATORE: **ING. LUCA FACCONI**

DURATA: **2 ORE.**

Esercizio



$$W = ql^2$$

$$k_\eta = 18\frac{EJ}{l^3}$$

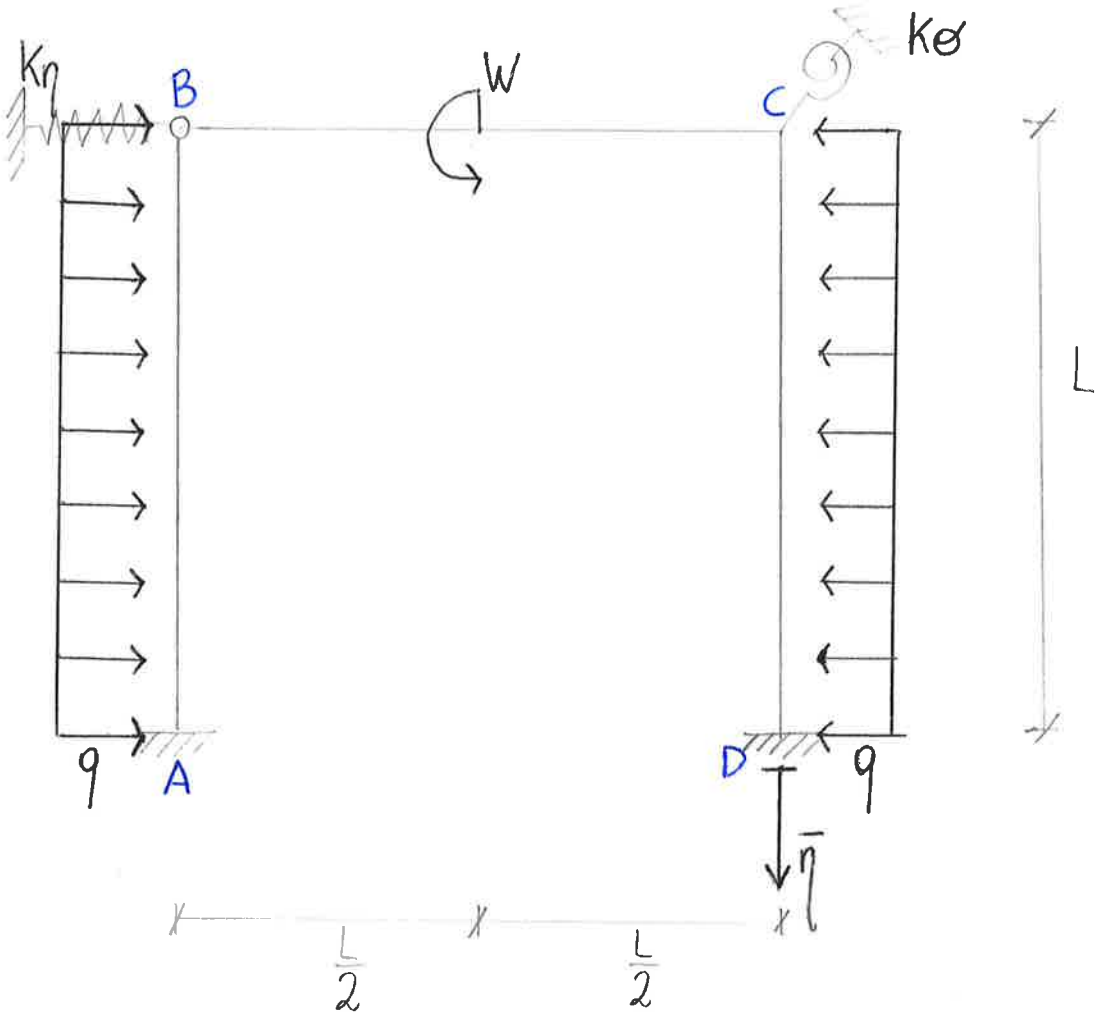
$$\bar{\eta} = \frac{1}{2}\frac{ql^4}{EJ}$$

$$k_\theta = \frac{5}{6}\frac{EJ}{l}$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME 15/01/2018



DATI

$W = qL^2$

$K\eta = 18\,\frac{EJ}{L^3}$

$\bar{\eta} = \frac{1}{2}\frac{qL^4}{EJ}$

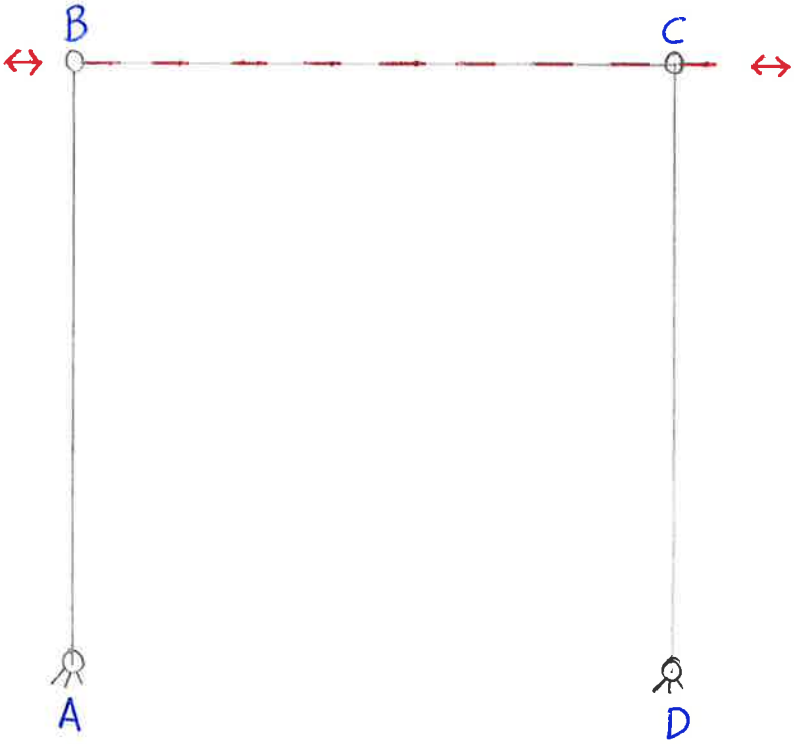
$K\vartheta = \frac{5}{6}\,\frac{EJ}{L}$

N SI VALUTA IL GRADO DI IPERSTATICITA' DELLA STRUTTURA

GdV = 3+2+3 = 8

GdL = 2.3 = 6

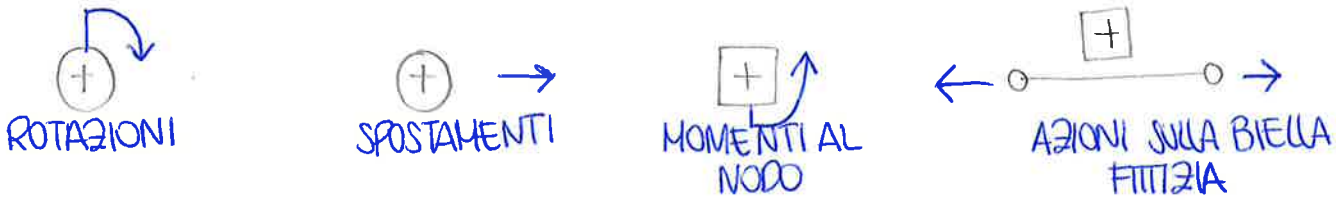
N SI VALUTA SE IL TELAIO È A NODI FISSI O SPOSTABILI



SI OSSERVA UN POSSIBILE CINEMATISMO PER L'ASTA BC

IL TELAIO È A NODI SPOSTABILI

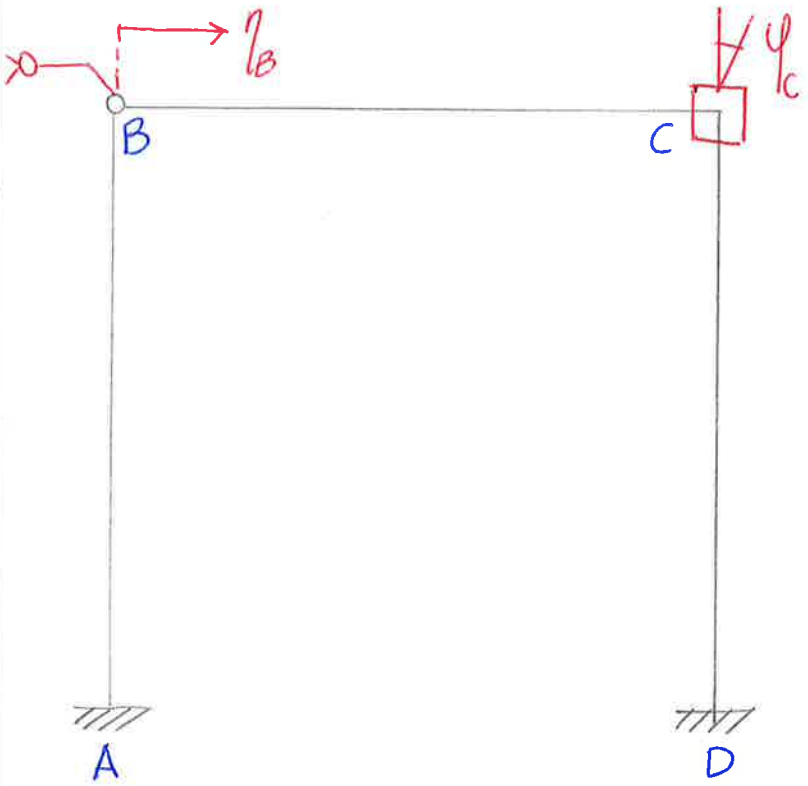
N CONVENZIONI USATE NELLA RISOLUZIONE DEL TELAIO



N RISOLVO IL TELAIO CON IL METODO DEGLI SPOSTAMENTI

LE INCOGNITE SONO:

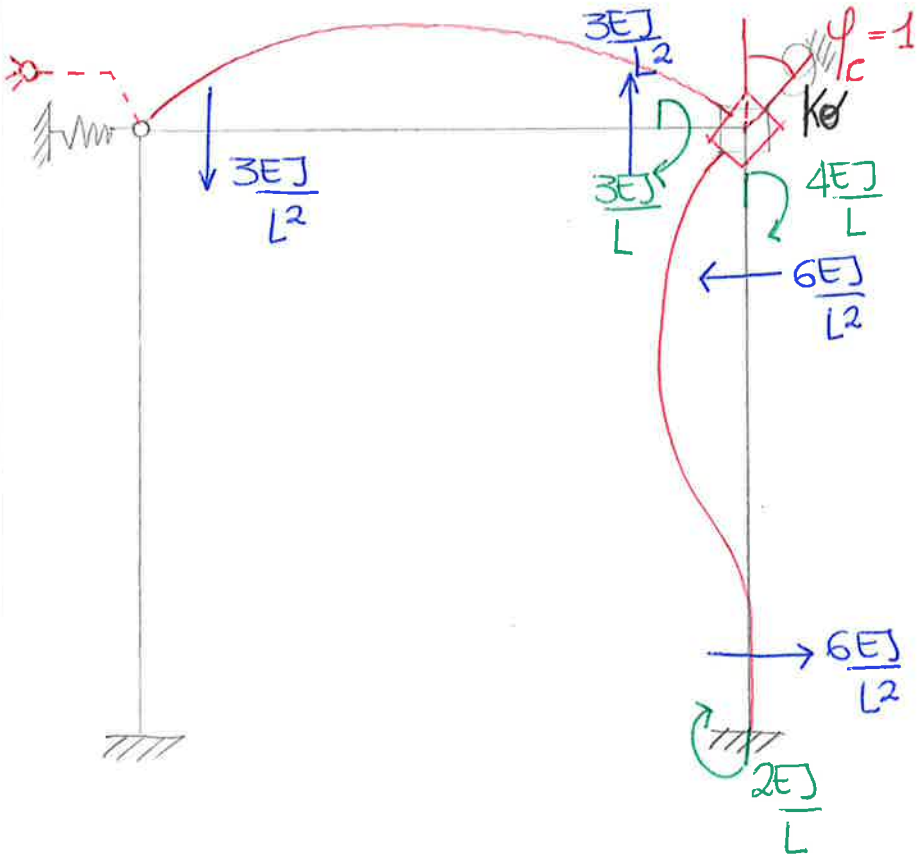
$$\begin{cases} \varphi_C \\ \eta_B \end{cases}$$

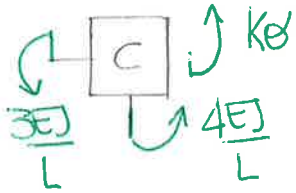


IL SISTEMA RISOLVENTE RISULTA ESSERE :

$$\begin{cases} \Sigma M_C = 0 \\ \Sigma R_B = 0 \end{cases} \qquad \begin{cases} m_{cc} \cdot \varphi_C + m_{c\eta} \cdot \eta_B + m_{c0} = 0 \\ h_{\eta c} \cdot \varphi_C + h_{\eta\eta} \cdot \eta_B + h_{\eta 0} = 0 \end{cases}$$

N CASO 1 $\quad \varphi_C = 1 \; ; \; \eta_B = 0 \; ; \; \forall P_{EXT} = 0$

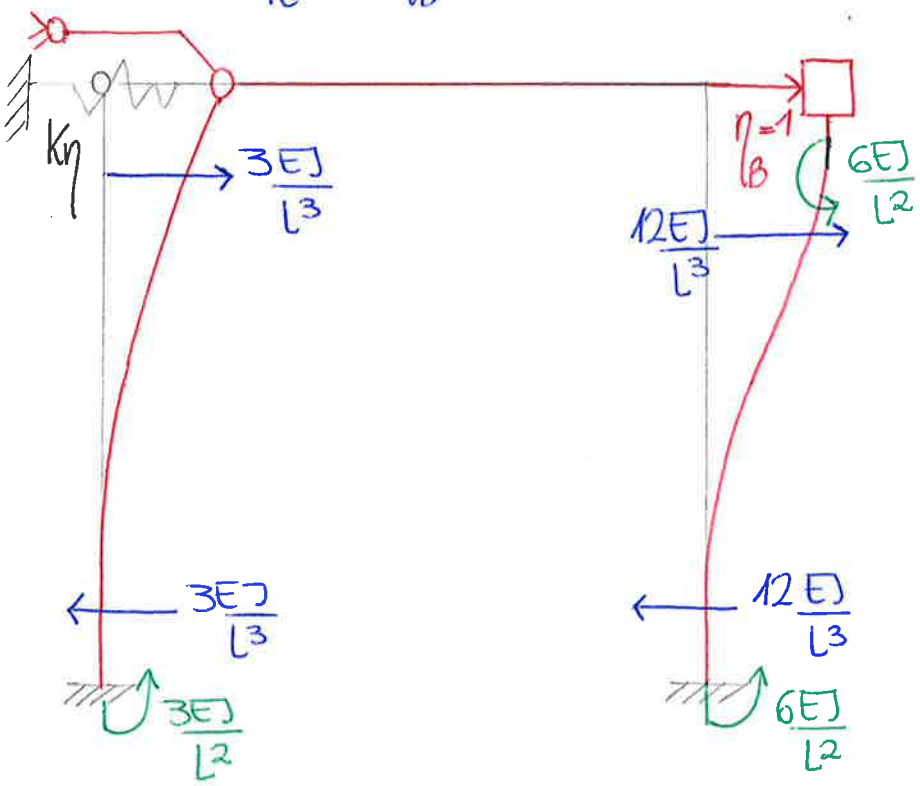


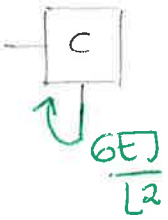


$$m_{cc} = \frac{4EJ}{L} + \frac{3EJ}{L} + \frac{5}{6}\frac{EJ}{L} = \frac{47}{6}\frac{EJ}{L}$$

$$h_\eta = \frac{6EJ}{L^2}$$

N CASO 2: $\varphi_C = 0$; $\eta_B = 1$; $\forall P_{ext} = 0$

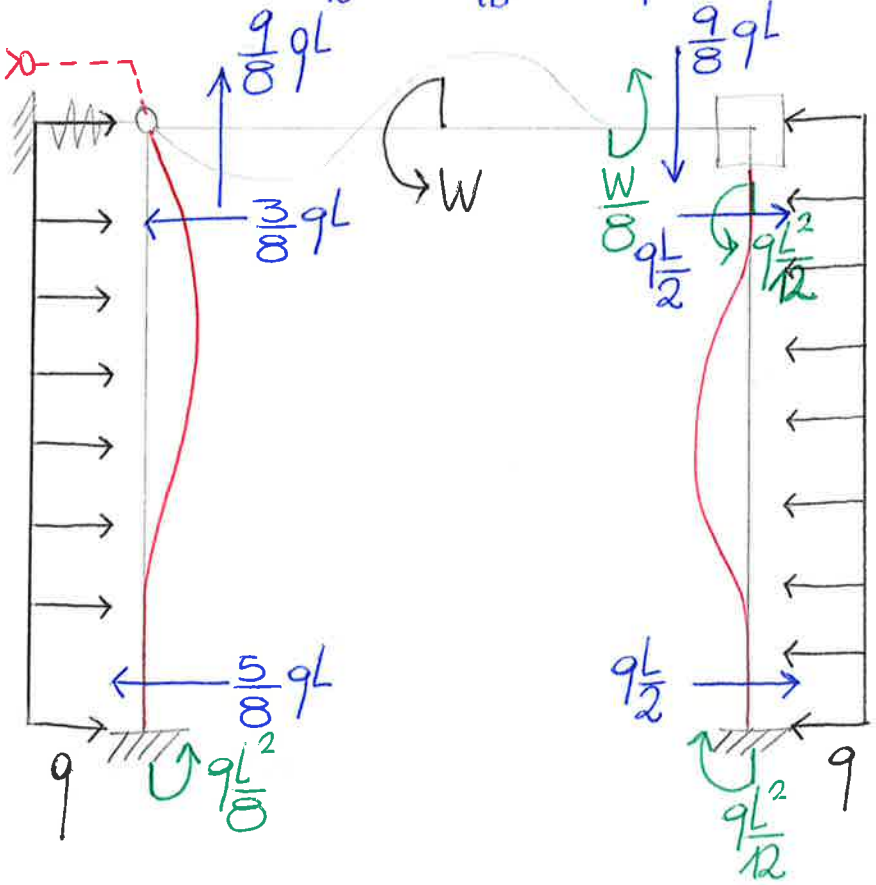


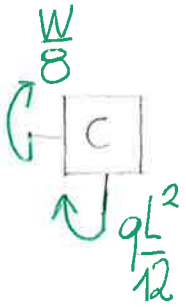


$$m_{C\eta} = -\frac{6EJ}{L^2}$$

$$h_{\eta\eta} = -\frac{3EJ}{L^3} - \frac{12EJ}{L^3} - \frac{18EJ}{L^2} = -\frac{33EJ}{L^3}$$

N CASO 3: $\varphi_C = 0$; $\eta_B = 0$; $q \neq 0$; $W \neq 0$; $\bar{\eta} = 0$

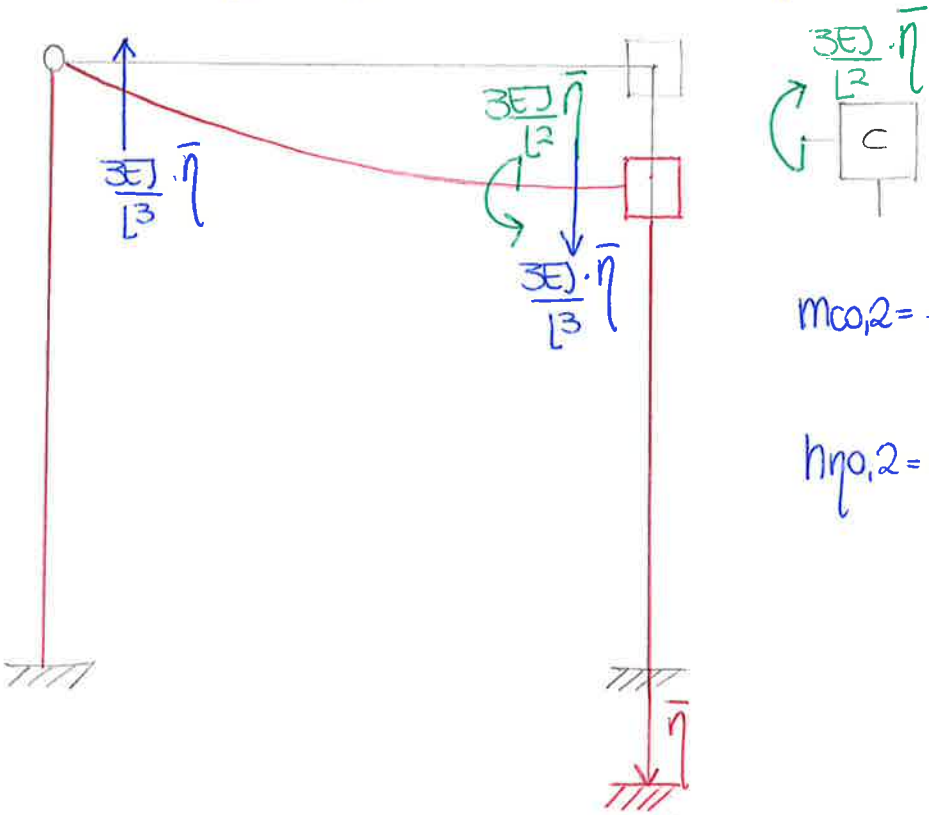




$$m_{C0,1} = -\frac{1}{8}qL^2 - \frac{1}{12}qL^2 = -\frac{5}{24}qL^2$$

$$h_{\eta 0,1} = \frac{3}{8}qL - \frac{qL}{2} = -\frac{1}{8}qL$$

N CASO 4: $\varphi_C = 0$; $\eta_B = 0$; $q = 0$; $W = 0$; $\bar{\eta} \neq 0$



$$m_{C0,2} = -\frac{3EJ}{L^2}\cdot\frac{1}{2}q\frac{L^4}{EJ} = -\frac{3}{2}qL^2$$

$$h_{P0,2} = 0$$

$$m_{C0} = m_{C0,1} + m_{C0,2} = -\frac{5}{24}qL^2 - \frac{3}{2}qL^2 = -\frac{41}{24}qL^2$$

$$h_{P0} = h_{P0,1} + h_{P0,2} = -\frac{1}{8}qL$$

N SISTEMA RISOLVENTE

$$\begin{cases} \frac{47}{6}\frac{EJ}{L}\cdot\varphi_C - 6\frac{EJ}{L^2}\eta_B - \frac{41}{24}qL^2 = 0 & (1) \\ 6\frac{EJ}{L^2}\cdot\varphi_C - 33\frac{EJ}{L^3}\cdot\eta_B - \frac{1}{8}qL = 0 & (2) \end{cases}$$

MOLTIPLICO (2) PER $\left(-\frac{6}{33}L\right)$ E LA SOMMO A (1)

$$6\frac{EJ}{L^2}\cdot\left(-\frac{6}{33}L\right)\cdot\varphi_C - 33\frac{EJ}{L^3}\left(-\frac{6}{33}L\right)\cdot\eta_B - \frac{1}{8}qL\cdot\left(-\frac{6}{33}L\right) = 0$$

$\oplus$

$$\frac{47}{6}\frac{EJ}{L}\cdot\varphi_C - 6\frac{EJ}{L^2}\cdot\eta_B - \frac{41}{24}qL^2 = 0$$

$$\left[\frac{47}{6}\frac{EJ}{L}+\left(-\frac{36}{33}\frac{EJ}{L}\right)\right]\cdot\varphi_C+\left[\frac{6EJ}{L^2}+\frac{6EJ}{L^2}\right]\cdot\eta_B+\left[-\frac{41}{24}qL^2+\frac{3}{132}qL^2\right]=0$$

$$-\frac{445}{66}\frac{EJ}{L}\varphi_C-\frac{445}{264}qL^2=0$$

$$\rightarrow \varphi_C=\frac{445}{264}qL^2\cdot\frac{66}{445}\frac{L}{EJ}=\frac{1}{4}\frac{qL^3}{EJ}$$

SOSTITUISCO φ_C NELLA ②

$$6\frac{EJ}{L^2}\cdot\frac{1}{4}\frac{qL^3}{EJ}-33\frac{EJ}{L^3}\cdot\eta_B-\frac{1}{8}qL=0$$

$$\frac{3}{2}qL-33\frac{EJ}{L^3}\eta_B-\frac{1}{8}qL=0$$

$$-33\frac{EJ}{L^3}\eta_B=\frac{1}{8}qL-\frac{3}{2}qL$$

$$\rightarrow \eta_B=\frac{11}{8}qL\cdot\frac{1}{33}\frac{L^3}{EJ}=\frac{1}{24}\frac{qL^4}{EJ}$$

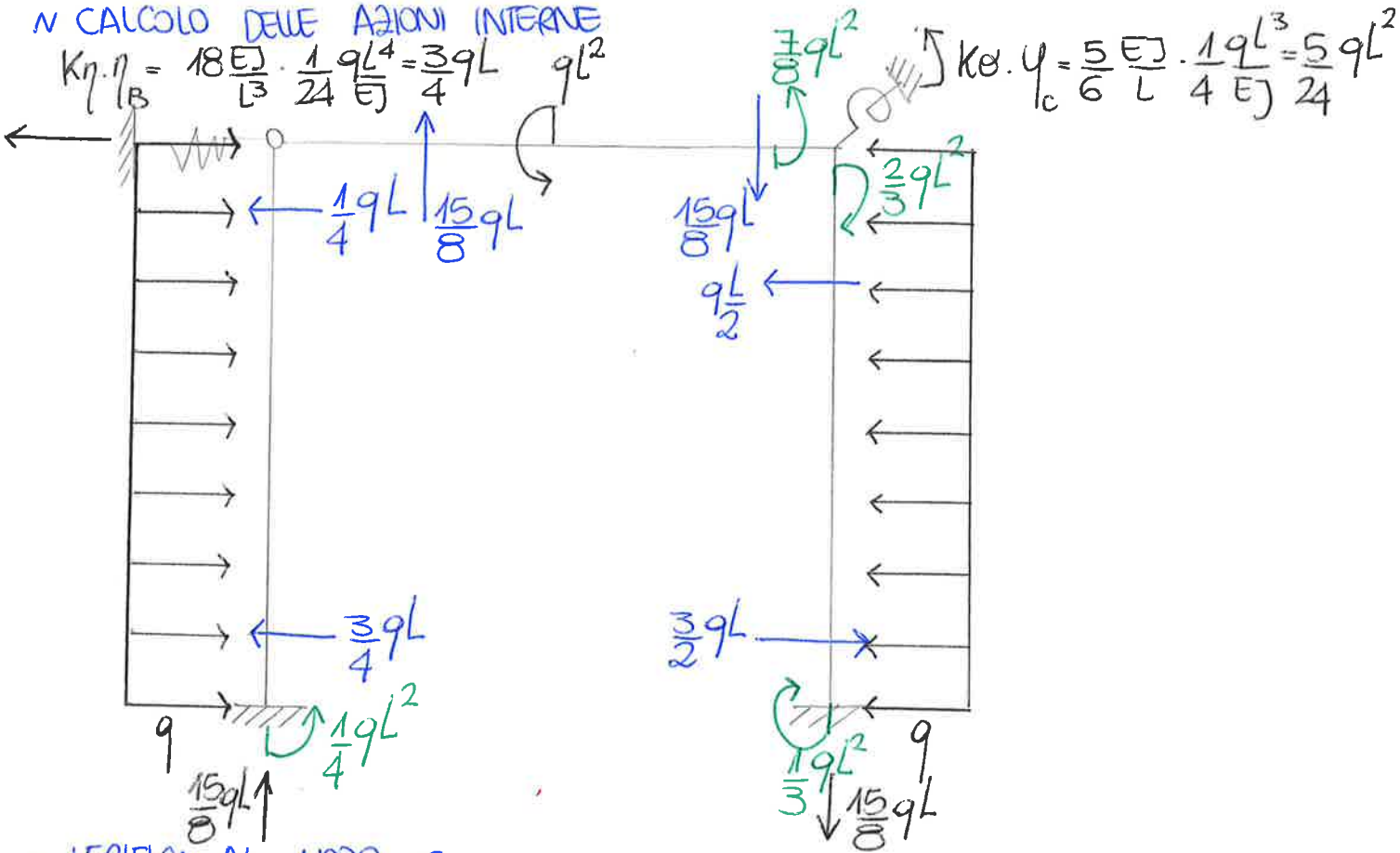
DA CUI IN CONCLUSIONE

$$\begin{cases}\varphi_C=\frac{1}{4}\frac{qL^3}{EJ}\\ \eta_B=\frac{1}{24}\frac{qL^4}{EJ}\end{cases}$$

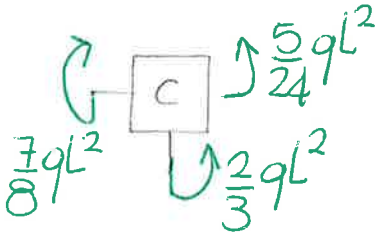
N CALCOLO DELLE AZIONI INTERNE

$K\eta \cdot \eta_B = \frac{18EJ}{L^3} \cdot \frac{1}{24}\frac{qL^4}{EJ} = \frac{3}{4}qL$ $\quad qL^2$

$K\vartheta \cdot \vartheta_C = \frac{5}{6}\frac{EJ}{L} \cdot \frac{1}{4}\frac{qL^3}{EJ} = \frac{5}{24}qL^2$



N VERIFICA AL NODO C



$\frac{2}{3}qL^2 - \frac{7}{8}qL^2 + \frac{5}{24}qL^2 = 0$ ok!

N VERIFICA EQUILIBRIO ALLA TRASLAZIONE VERTICALE $\Sigma F_V = 0$

$\uparrow + \quad \frac{15}{8}qL - \frac{15}{8}qL = 0 \qquad 0 = 0$ ok!

N VERIFICA EQUILIBRIO ALLA TRASLAZIONE ORIZZONTALE $\Sigma F_H = 0$

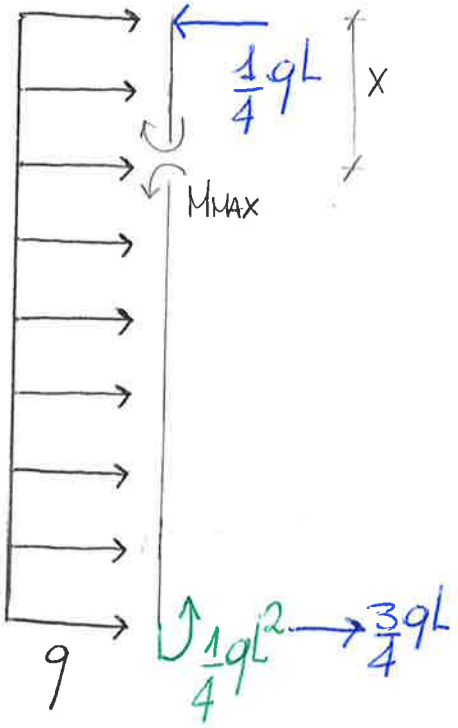
$\pm \rightarrow \quad -\frac{3}{4}qL + qL - \frac{3}{4}qL + \frac{3}{2}qL - qL = 0 \qquad 0 = 0$ ok!

N VERIFICA EQUILIBRIO ALLA ROTAZIONE $\Sigma M = 0$

$+\circlearrowleft \quad -\frac{1}{4}qL^2 + qL\frac{L}{2} - \frac{3}{4}qL \cdot L - qL^2 - \frac{5}{24}qL^2 + \frac{1}{3}qL^2 - qL\frac{L}{2} + \frac{15}{8}qL \cdot L = 0$

$0 = 0$ ok!

N MMAX ASTA AB

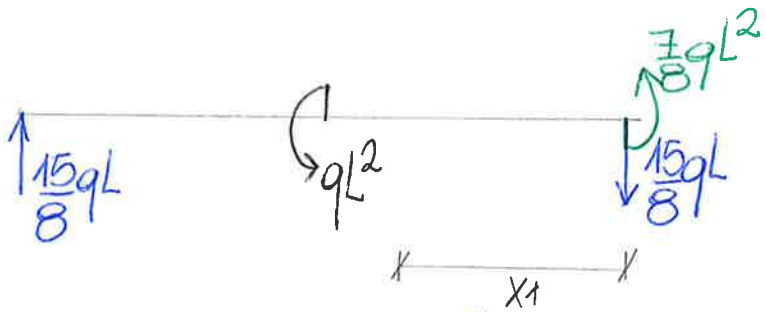


$$+\circlearrowleft \quad M_{MAX} - \frac{1}{4}qL\cdot x + q\cdot x\cdot\frac{x}{2} = 0 \longrightarrow M_{MAX}\left(x=\frac{L}{4}\right) = +\frac{1}{4}qL\cdot\frac{L}{4} - q\cdot\frac{L}{4}\cdot\frac{L}{8} = \frac{1}{32}qL^2$$

$$M_{MAX} = \frac{1}{4}qLx - q\frac{x^2}{2} = 0$$

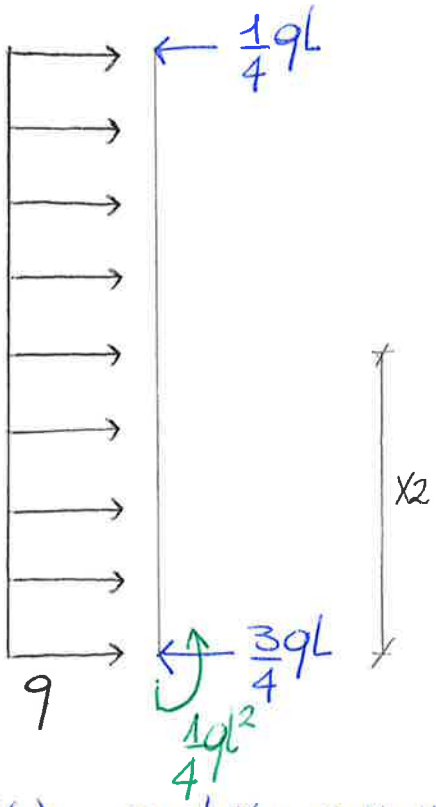
$$\frac{dM_{MAX}}{dx} = 0 \longrightarrow \frac{1}{4}qL - qx = 0 \longrightarrow x = \frac{1}{4}L = 0{,}25\,L$$

N FLESSO 1 ASTA BC



$$+\circlearrowleft \quad M(x) = \frac{15}{8}qL\cdot x_1 - \frac{7}{8}qL^2 = 0 \longrightarrow x_1 = \frac{7}{15}L \simeq 0{,}47\,L$$

N FLESSO 2 ASTA AB



$$M(x) = \frac{3}{4}qL \cdot X_2 - q \cdot X_2 \cdot \frac{X_2}{2} - \frac{1}{4}qL^2 = 0$$

$$-q\frac{X_2^2}{2} + \frac{3}{4}qLX_2 - \frac{1}{4}qL^2 = 0$$

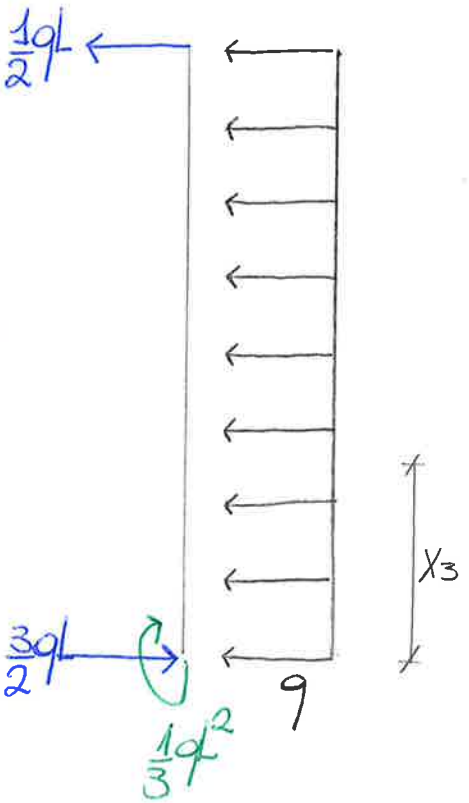
$$X_2^2 - \frac{3}{2}LX_2 + \frac{L^2}{2} = 0$$

$$X = \frac{\frac{3}{2}L \pm \sqrt{\frac{9}{4}L^2 - 2L^2}}{2}$$

$X = L$

$X_2 = \frac{L}{2} = 0{,}5L$

N FLESSO 3 ASTA CD



$$M(x) = \frac{1}{3}qL^2 - \frac{3}{2}qL \cdot X_3 + q X_3 \cdot \frac{X_3}{2} = 0$$

$$q\frac{X_3^2}{2} - \frac{3}{2}qL X_3 + \frac{1}{3}qL^2 = 0$$

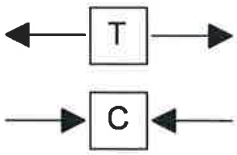
$$X_3^2 - 3L X_3 + \frac{2}{3}L^2 = 0$$

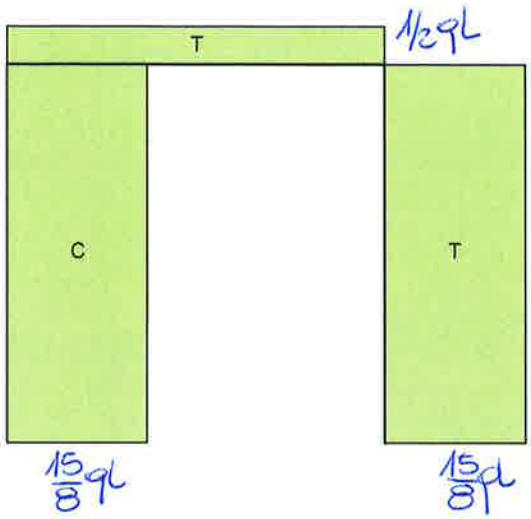
$$X = \frac{3L \pm \sqrt{9L^2 - \frac{8}{3}L^2}}{2}$$

$$X = \frac{9+\sqrt{57}}{6}L$$

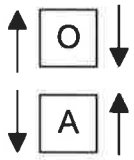
$$X_3 = \frac{9-\sqrt{57}}{6}L \simeq 0{,}24\,L$$

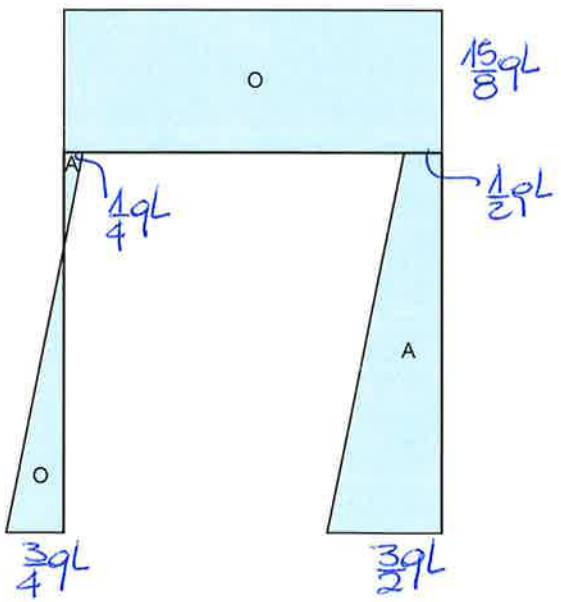
AZIONE ASSIALE



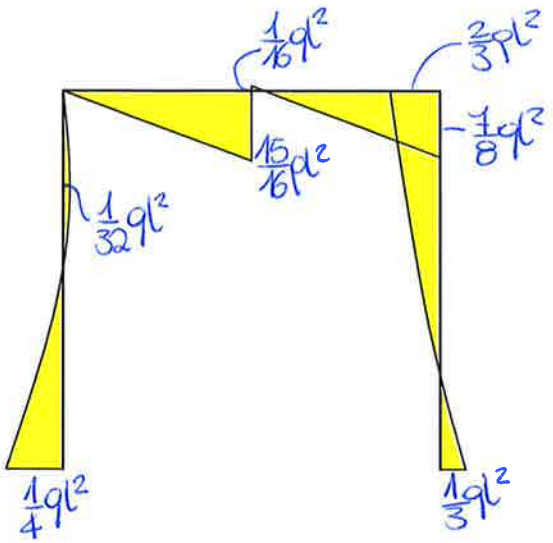


TAGLIO





MOMENTO FLETTENTE



DEFORMATA QUALITATIVA