

TECNICA DELLE COSTRUZIONI

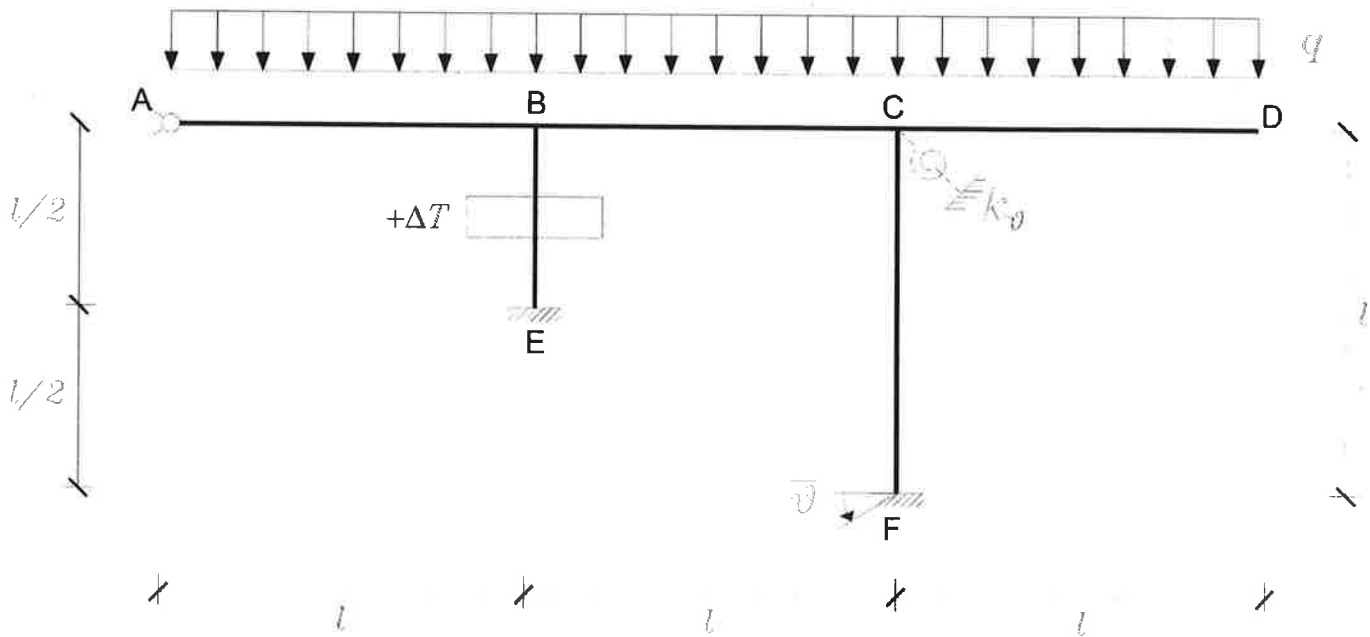
TEMA ESAME DEL 29 GENNAIO 2018

DOCENTE: PROF. FAUSTO MINELLI

ESERCITATORE: ING. LUCA FACCONI

DURATA: 2 ORE.

Esercizio



$$\alpha \Delta T = \frac{47 q l^3}{36 E J}$$

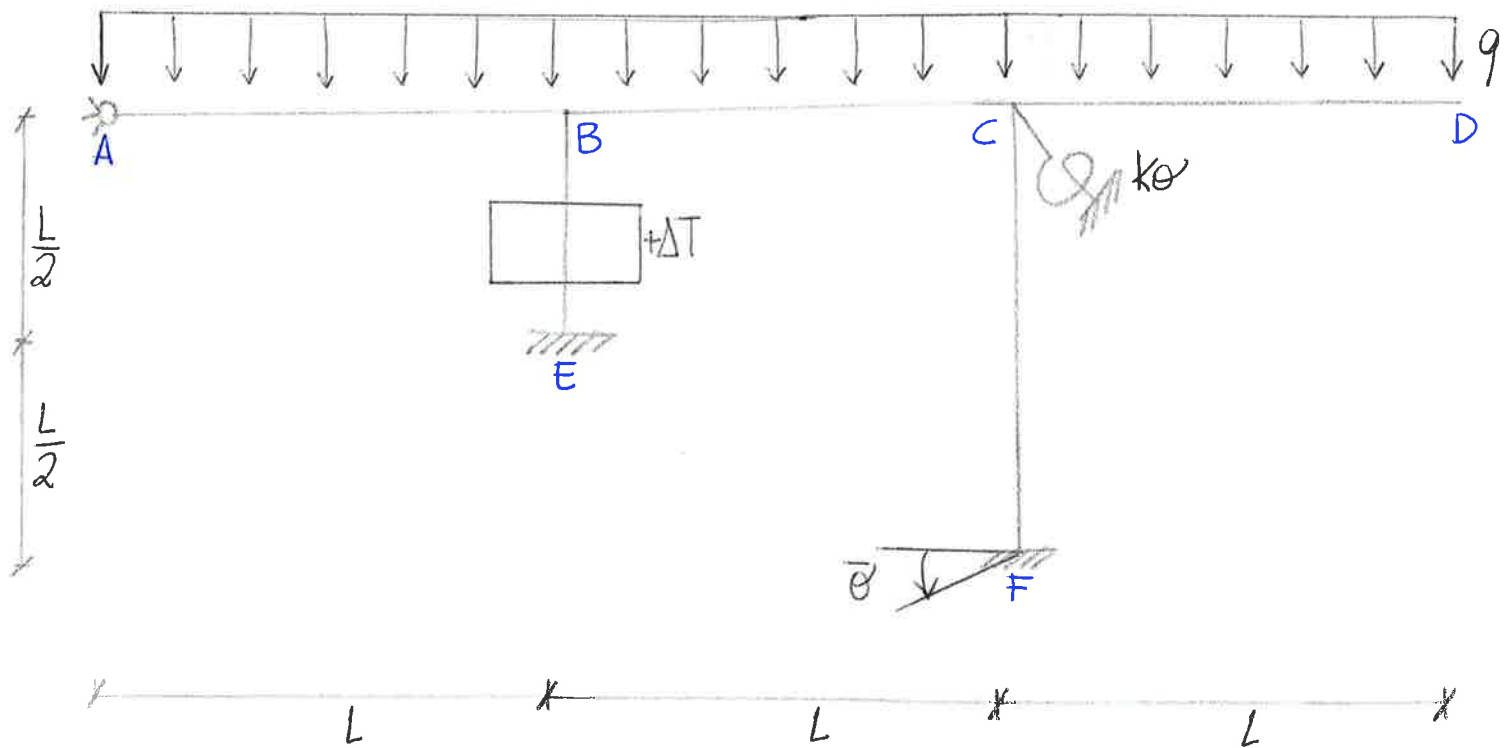
$$k_{\theta} = 6 \frac{E J}{l}$$

$$\bar{\theta} = \frac{1 q l^3}{4 E J}$$

Dato il telaio in figura, si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME 29/01/2018



DATI

$$\alpha \Delta T = \frac{47}{36} \frac{qL^3}{EJ}$$

$$k_{\theta} = 6 \frac{EJ}{L}$$

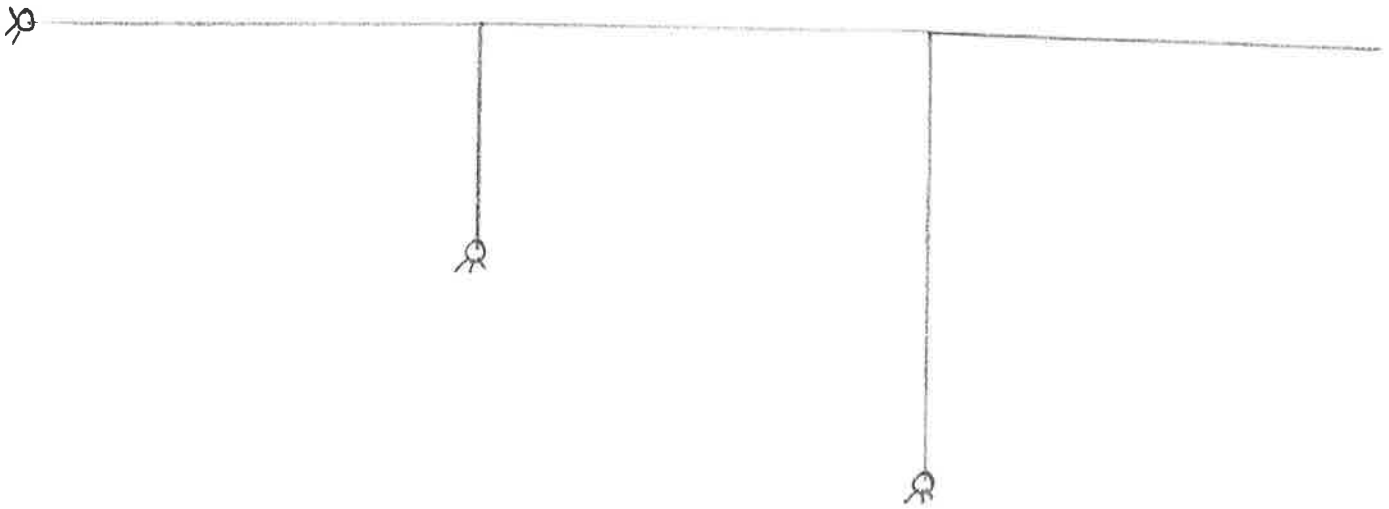
$$\bar{\theta} = \frac{1}{4} \frac{qL^3}{EJ}$$

SI VALUTA IL GRADO DI IPERSTATICITA' DELLA STRUTTURA

$$GdV = 3 + 3 + 2 + 1 = 9$$

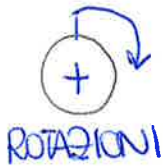
$$GdL = 1 \cdot 3 = 3$$

SI VALUTA SE IL TELAIIO È A NODI FISSI O SPOSTABILI



SI OSSERVA CHE NON È POSSIBILE NESSUN CINEMATISMO
IL TELAIIO È A NODI FISSI

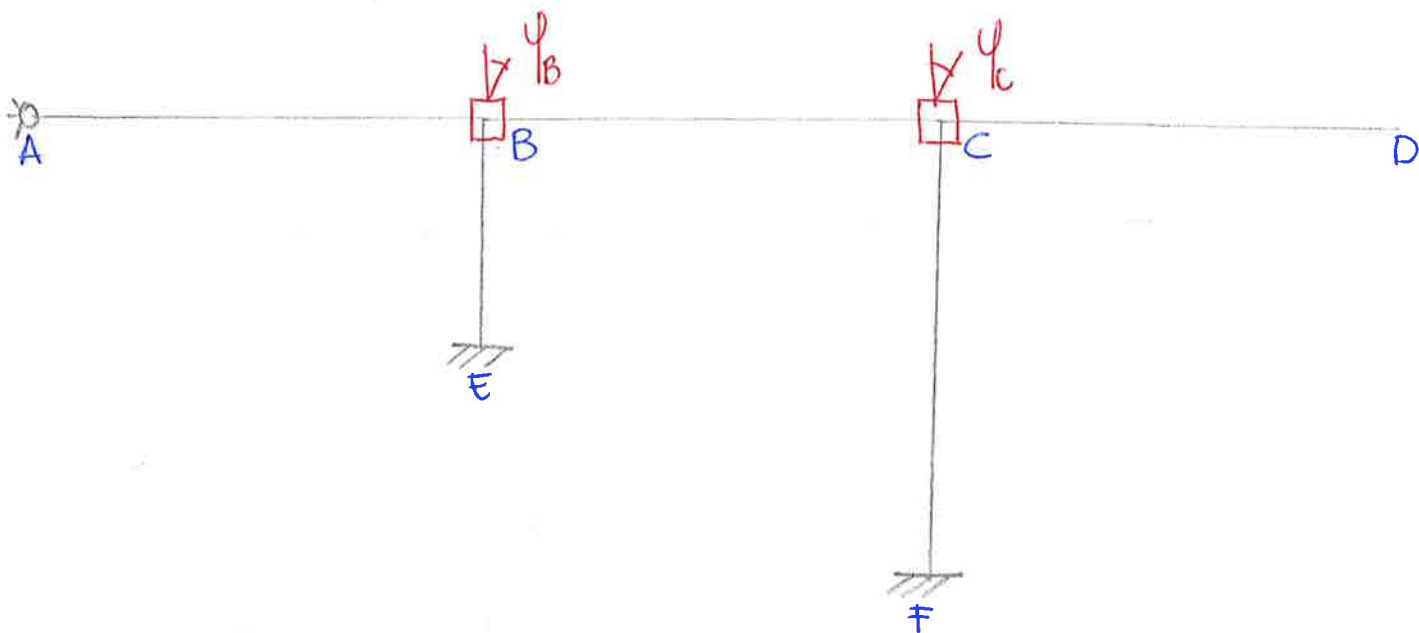
LE CONVENZIONI USATE NELLA RISOLUZIONE DEL TELAIIO



IL RISOLVO CON IL METODO DEGLI SPOSTAMENTI

LE INCOGNITE SONO:

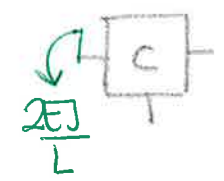
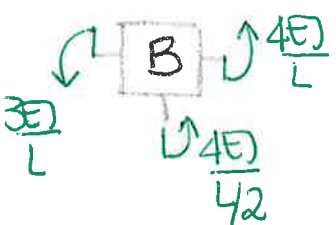
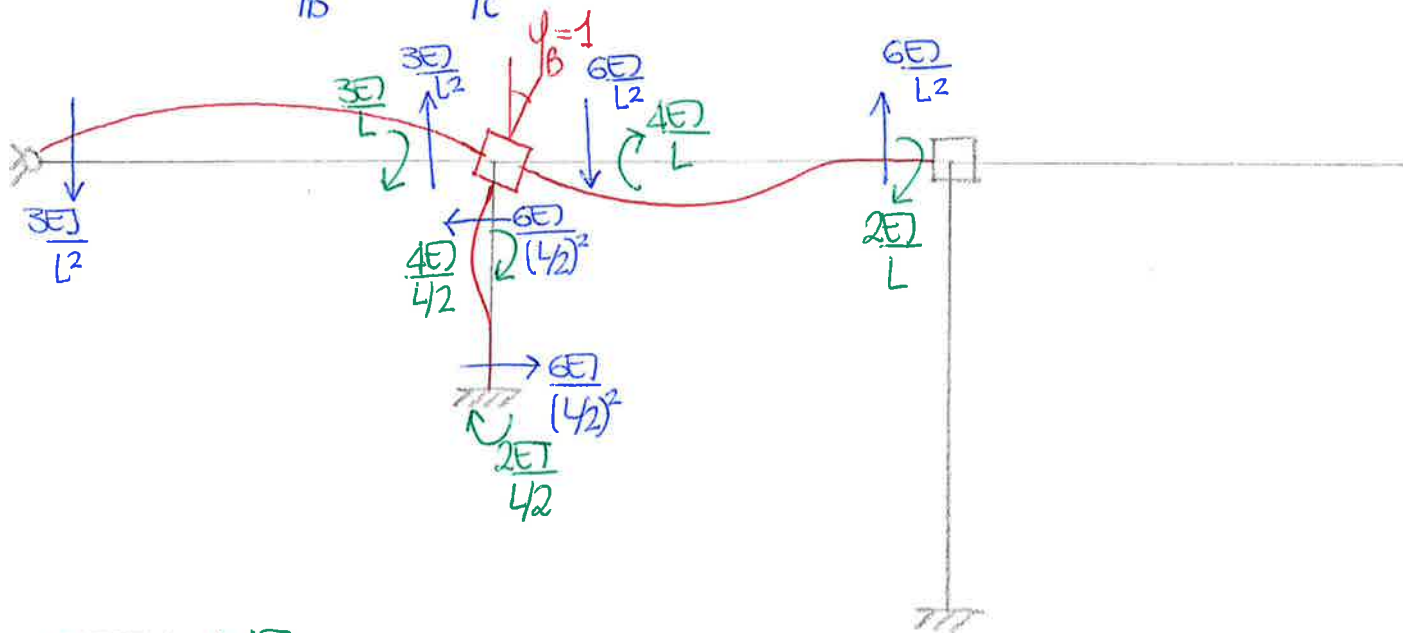
$$\begin{cases} \psi_B \\ \psi_C \end{cases}$$



IL SISTEMA RISOLUENTE RISULTA ESSERE:

$$\begin{cases} \sum M_B = 0 \\ \sum M_C = 0 \end{cases} \quad \begin{cases} M_{BB} \cdot \psi_B + M_{BC} \cdot \psi_C + M_{B0} = 0 \\ M_{CB} \cdot \psi_B + M_{CC} \cdot \psi_C + M_{C0} = 0 \end{cases}$$

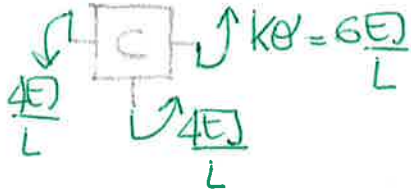
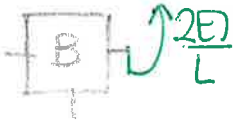
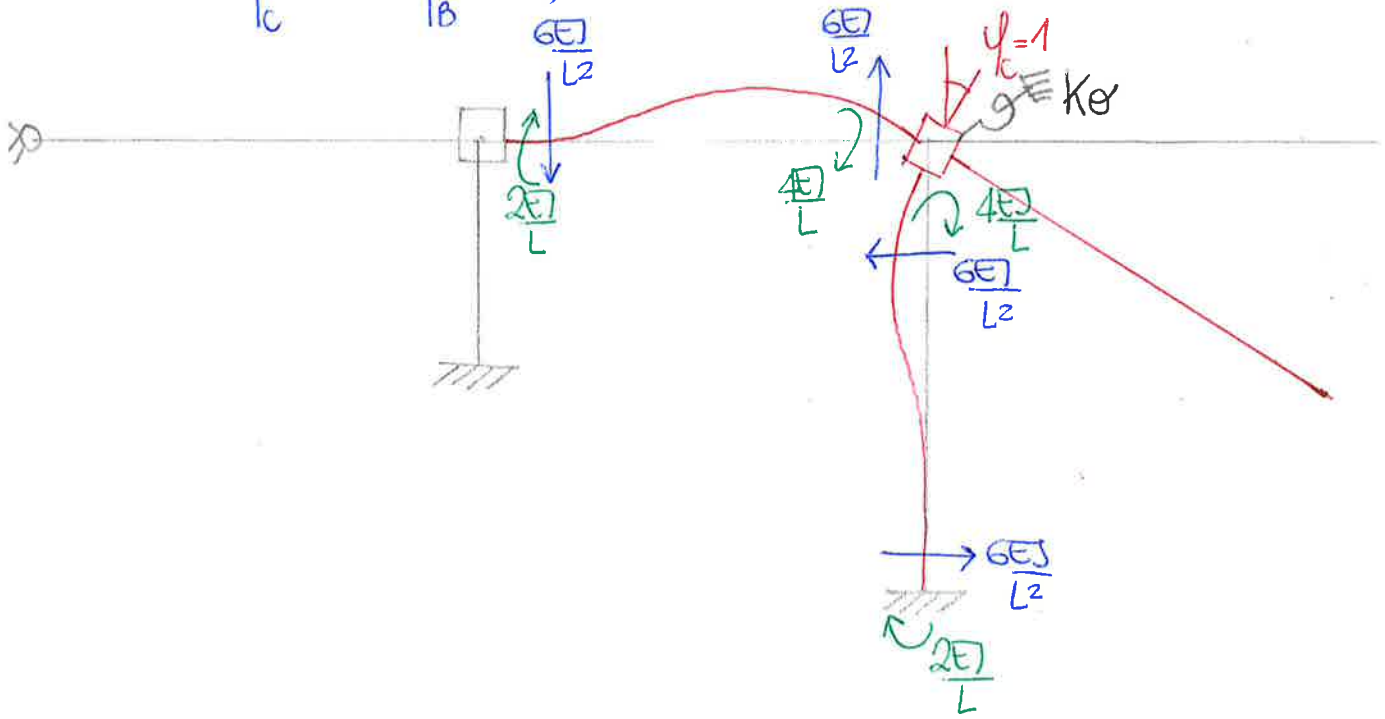
* CASO 1: $\psi_B = 1$; $\psi_C = 0$; $\forall P_{ex} = 0$



$$M_{BB} = \frac{3EJ}{L} + \frac{4EJ}{L} + \frac{8EJ}{L} = \frac{15EJ}{L}$$

$$M_{CB} = \frac{2EJ}{L}$$

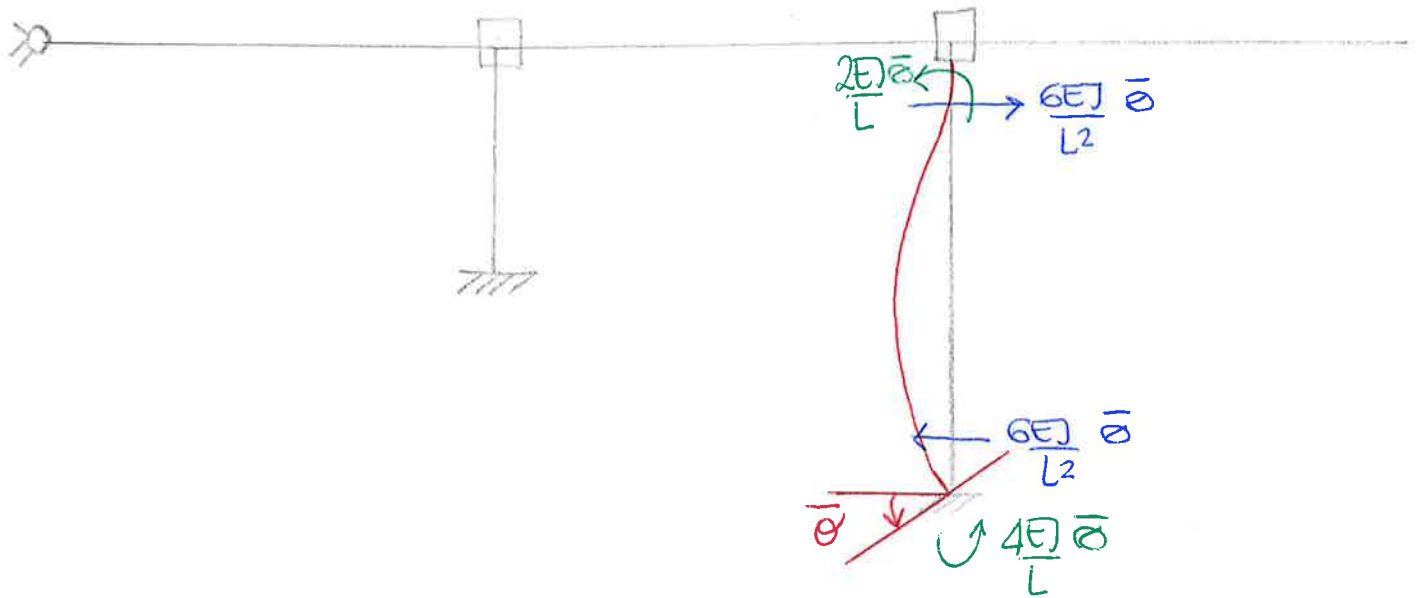
*CASO 2: $\psi_c = 1$; $\psi_B = 0$; $V_{Pex} = 0$



$$M_{BC} = \frac{2EJ}{L}$$

$$M_{CC} = \frac{4EJ}{L} + \frac{4EJ}{L} + \frac{6EJ}{L} = \frac{14EJ}{L}$$

* CASO 3: $\bar{\theta} \neq 0$

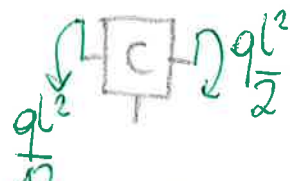
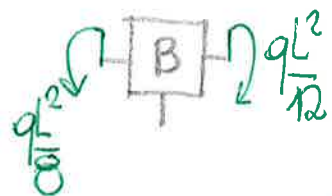
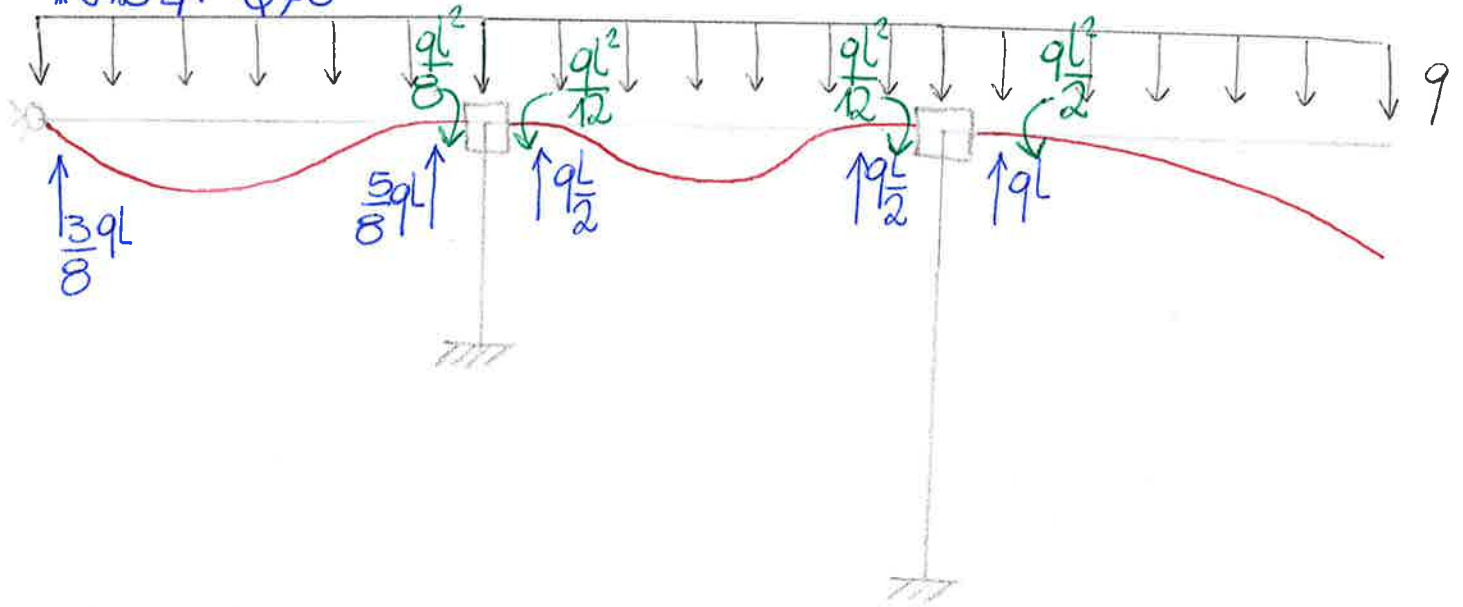


$$\uparrow \frac{2EI}{L} \bar{\theta}$$

$$M_{B0,1} = 0$$

$$M_{C0,1} = -\frac{2EI}{L} \cdot \bar{\theta} = -\frac{2EI}{L} \cdot \frac{1}{4} \frac{qL^3}{EI} = -\frac{1}{2} qL^2$$

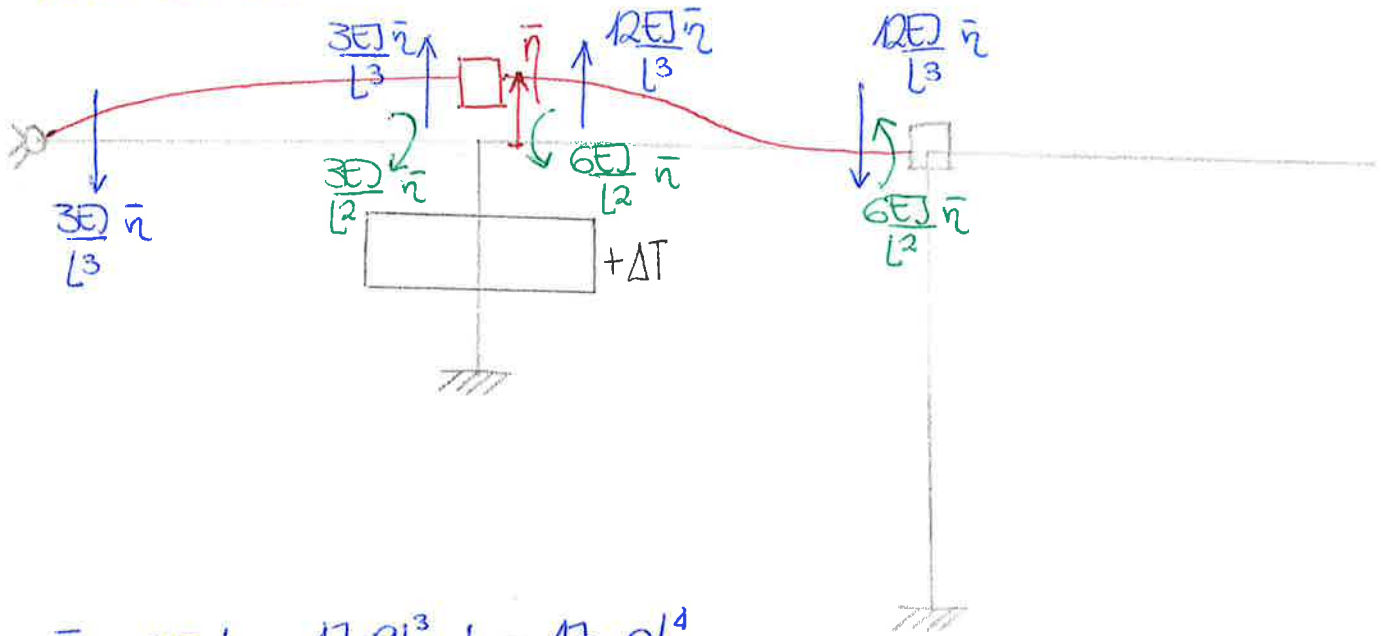
* CASO 4: $\Phi \neq 0$



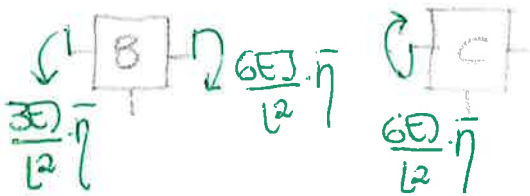
$$M_{B0,2} = \frac{1}{8} qL^2 - \frac{1}{12} qL^2 = \frac{1}{24} qL^2$$

$$M_{C0,2} = \frac{qL^2}{12} - \frac{qL^2}{2} = -\frac{5}{12} qL^2$$

*CASO 5: $\Delta T \neq 0$



$$\bar{\eta} = \alpha \Delta T \frac{L}{2} = \frac{47}{36} \frac{qL^3}{EJ} \cdot \frac{L}{2} = \frac{47}{72} \frac{qL^4}{EJ}$$



$$M_{B0,3} = 3 \frac{EJ}{L^2} \cdot \frac{47}{72} \frac{qL^4}{EJ} - 6 \frac{EJ}{L^2} \cdot \frac{47}{72} \frac{qL^4}{EJ} = \frac{47}{24} qL^2 - \frac{47}{12} qL^2 = -\frac{47}{24} qL^2$$

$$M_{C0,3} = -6 \frac{EJ}{L^2} \cdot \frac{47}{72} \frac{qL^4}{EJ} = -\frac{47}{12} qL^2$$

$$M_{B0} = \frac{qL^2}{24} - \frac{47}{24} qL^2 = -\frac{23}{12} qL^2$$

$$M_{C0} = -\frac{1}{2} qL^2 - \frac{5}{2} qL^2 - \frac{47}{12} qL^2 = -\frac{29}{6} qL^2$$

N SISTEMA RIDUENTE

$$\begin{cases} 15 \frac{EJ}{L} \cdot \varphi_B + 2 \frac{EJ}{L} \cdot \varphi_C - \frac{23}{12} qL^2 = 0 & (1) \end{cases}$$

$$\begin{cases} 2 \frac{EJ}{L} \cdot \varphi_B + 14 \frac{EJ}{L} \cdot \varphi_C - \frac{29}{6} qL^2 = 0 & (2) \end{cases}$$

MOLTIPLICO (1) PER $-\frac{2}{15}$ E LA SOMMO A (1)

$$15 \frac{EJ}{L} \cdot \left(-\frac{2}{15}\right) \cdot \varphi_B + 2 \frac{EJ}{L} \left(-\frac{2}{15}\right) \cdot \varphi_C - \frac{23}{12} \cdot \left(-\frac{2}{15}\right) \cdot ql^2 = 0$$

(+)

$$2 \frac{EJ}{L} \cdot \varphi_B + 14 \frac{EJ}{L} \varphi_C - \frac{29}{6} ql^2 = 0$$

$$\left[-\frac{2EJ}{L} + \frac{2EJ}{L} \right] \cdot \varphi_B + \left[-\frac{4}{15} \frac{EJ}{L} + 14 \frac{EJ}{L} \right] \cdot \varphi_C + \left[\frac{23}{90} - \frac{29}{6} \right] ql^2 = 0$$

$$\frac{206}{15} \frac{EJ}{L} \cdot \varphi_C - \frac{412}{90} ql^2 = 0$$

$$\varphi_C = \frac{412}{90} ql^2 \cdot \frac{15}{206} \frac{EJ}{L} \rightarrow \varphi_C = \frac{1}{3} \frac{ql^3}{EJ}$$

SOSTITUISCO φ_C NELLA (2)

$$2 \frac{EJ}{L} \cdot \varphi_B + 14 \frac{EJ}{L} \cdot \frac{1}{3} \frac{ql^3}{EJ} - \frac{29}{6} ql^2 = 0$$

$$2 \frac{EJ}{L} \varphi_B = \left(\frac{29}{6} - \frac{14}{3} \right) ql^2$$

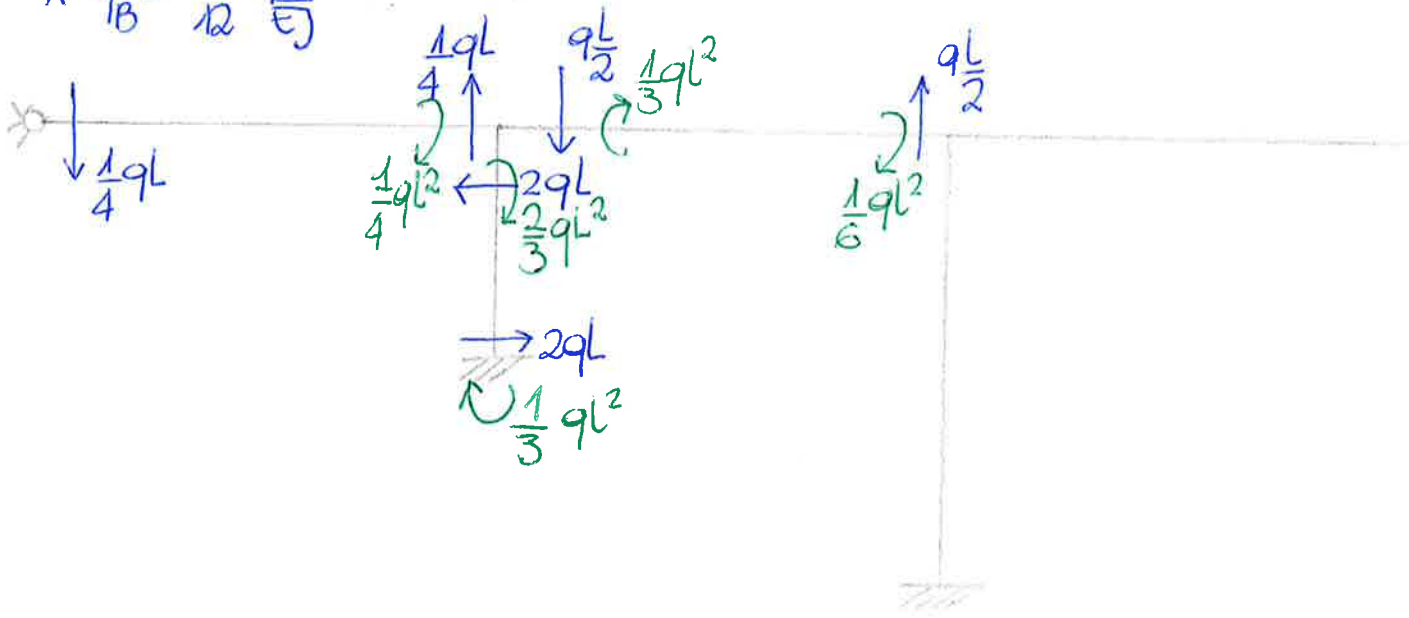
$$\varphi_B = \frac{L}{2EJ} \cdot \frac{1}{6} ql^2 \rightarrow \varphi_B = \frac{1}{12} \frac{ql^3}{EJ}$$

IN CONCLUSIONE

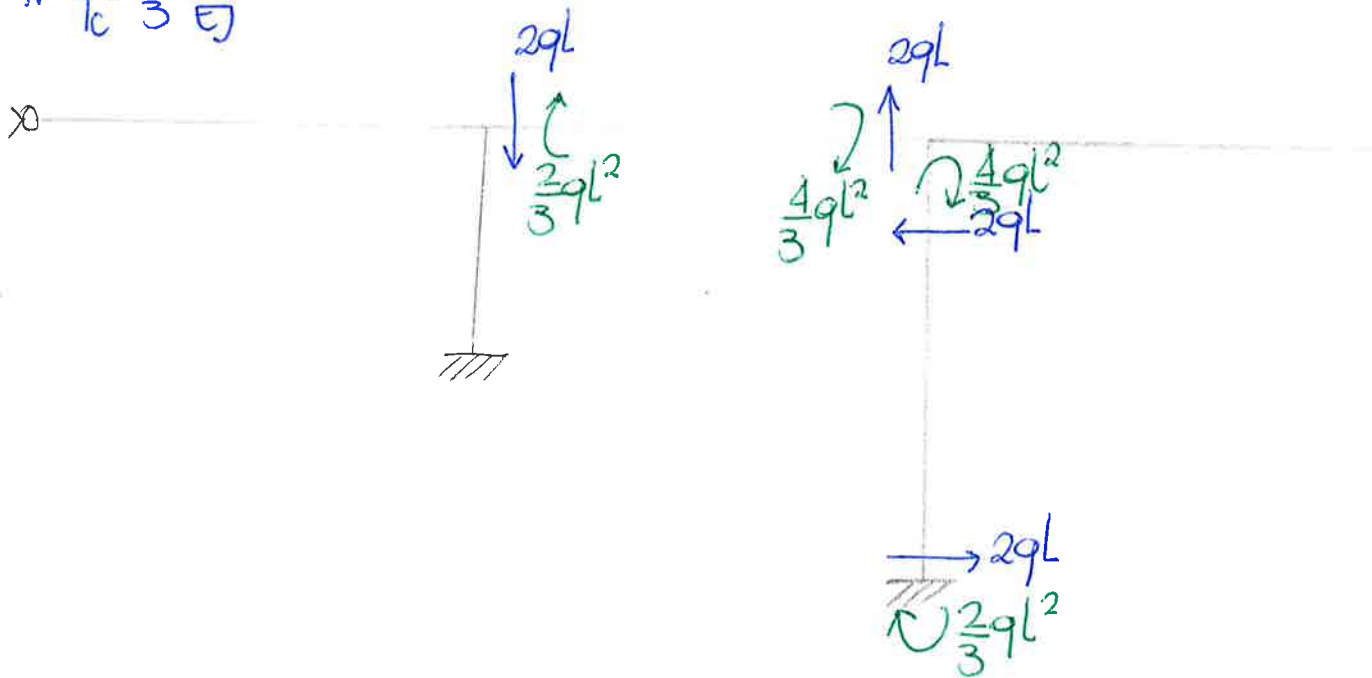
$$\left\{ \varphi_B = \frac{1}{12} \frac{ql^3}{EJ} \right.$$

$$\left. \varphi_C = \frac{1}{3} \frac{ql^3}{EJ} \right.$$

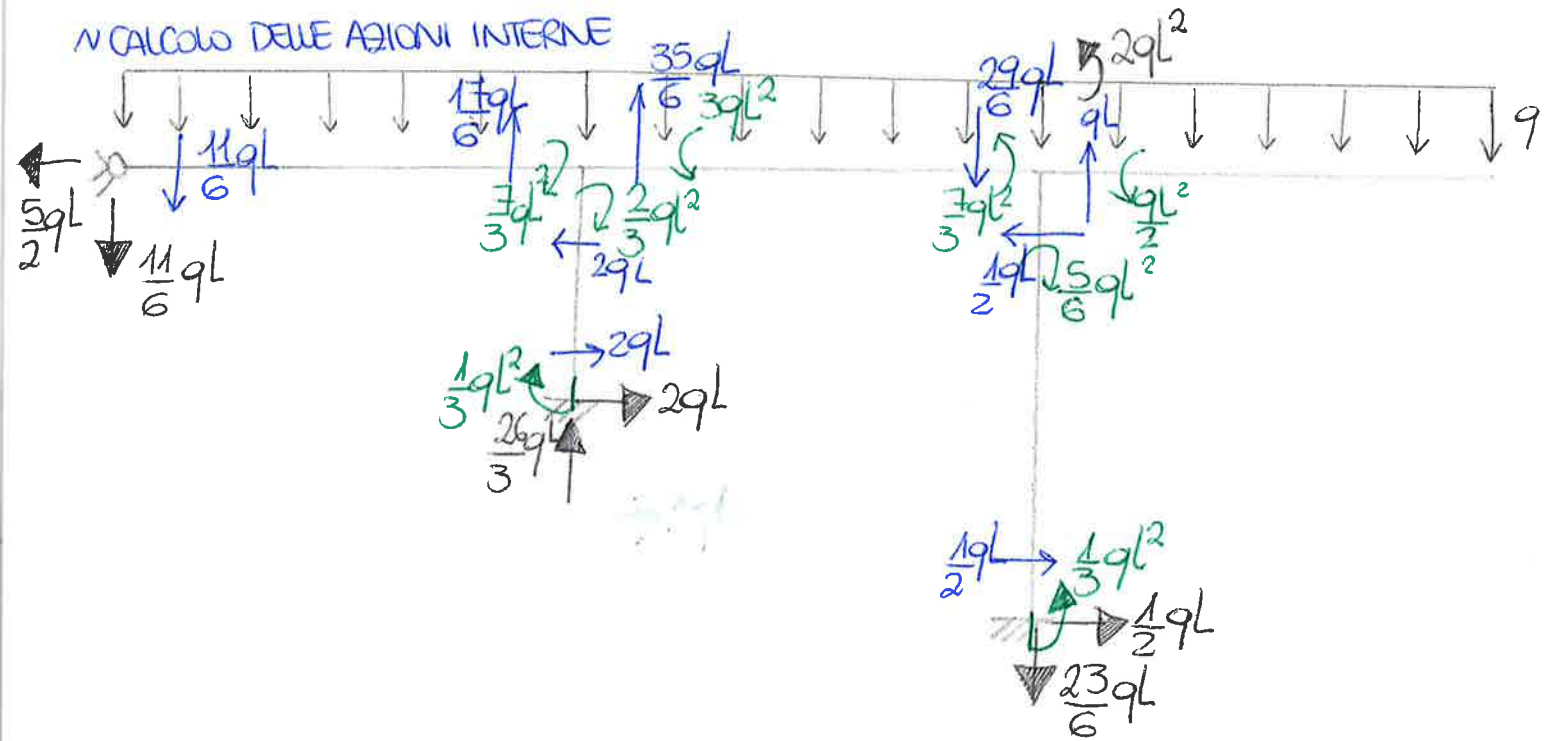
$$\approx \varphi_B = \frac{1}{12} \frac{ql^3}{EI}$$



$$\approx \varphi_C = \frac{1}{3} \frac{ql^3}{EI}$$



N CALCOLO DELLE AZIONI INTERNE



EQ: $\uparrow = 0$

$$-\frac{11}{6}qL + \frac{26}{3}qL - \frac{23}{6}qL - 3qL = 0 \quad \text{ok!}$$

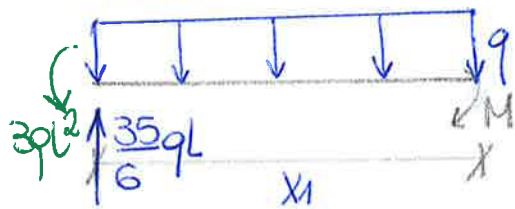
EQ: $\rightarrow = 0$

$$-\frac{5}{2}qL + 2qL + \frac{1}{2}qL = 0 \quad \text{ok!}$$

EQ: $\curvearrowright_A = 0$

$$-\frac{26}{3}qL \cdot L - 2qL \cdot \frac{L}{2} + \frac{23}{6}qL \cdot 2L - \frac{1}{2}qL \cdot L + q \cdot 3L \cdot \frac{3}{2}L - 2qL^2 + \frac{qL^2}{3} - \frac{qL^2}{3} = 0 \quad \text{ok!}$$

1 FLESO 1

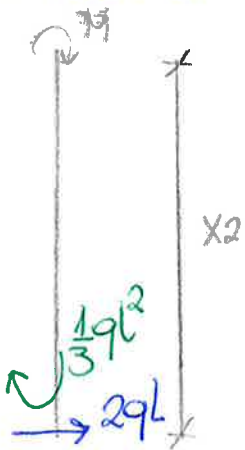


$$M = \frac{35}{6}ql \cdot x_1 - 3ql^2 - q \cdot x_1 \cdot \frac{x_1}{2} = 0$$

$$\frac{x^2}{2} - \frac{35}{6}L + 3ql^2 = 0$$

$$x = \frac{\frac{35}{3} \pm \sqrt{\left(\frac{35}{3}\right)^2 - 24}}{2} = \begin{cases} 0,54 \rightarrow x_1 \\ 11,13 \end{cases}$$

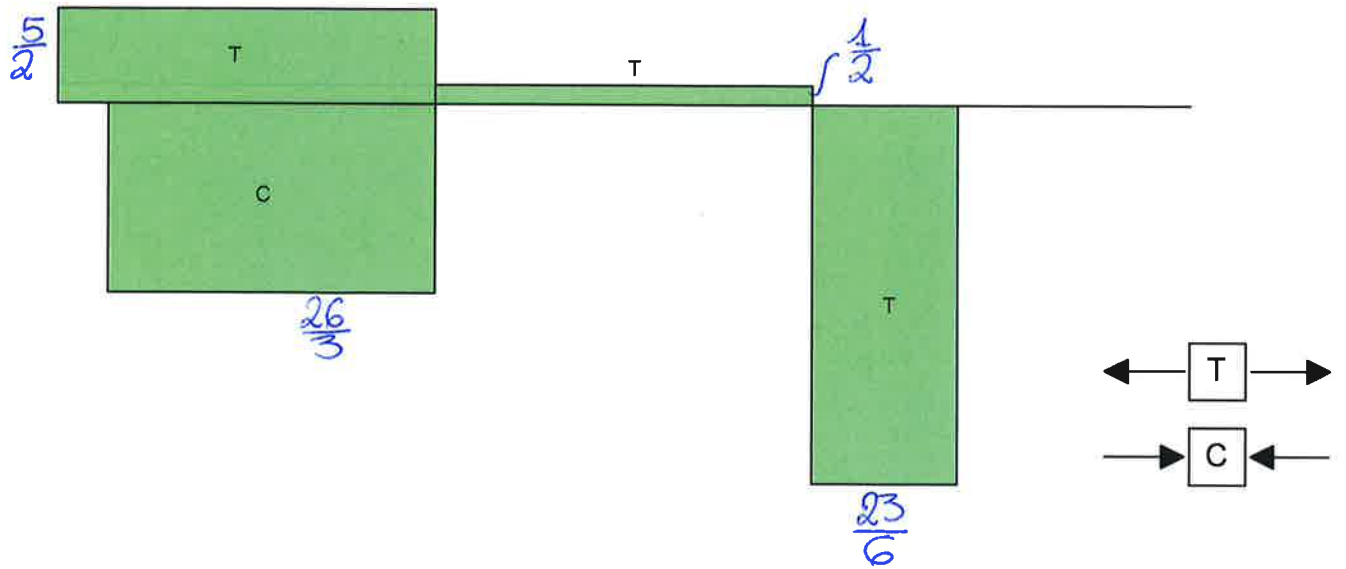
2 FLESO 2



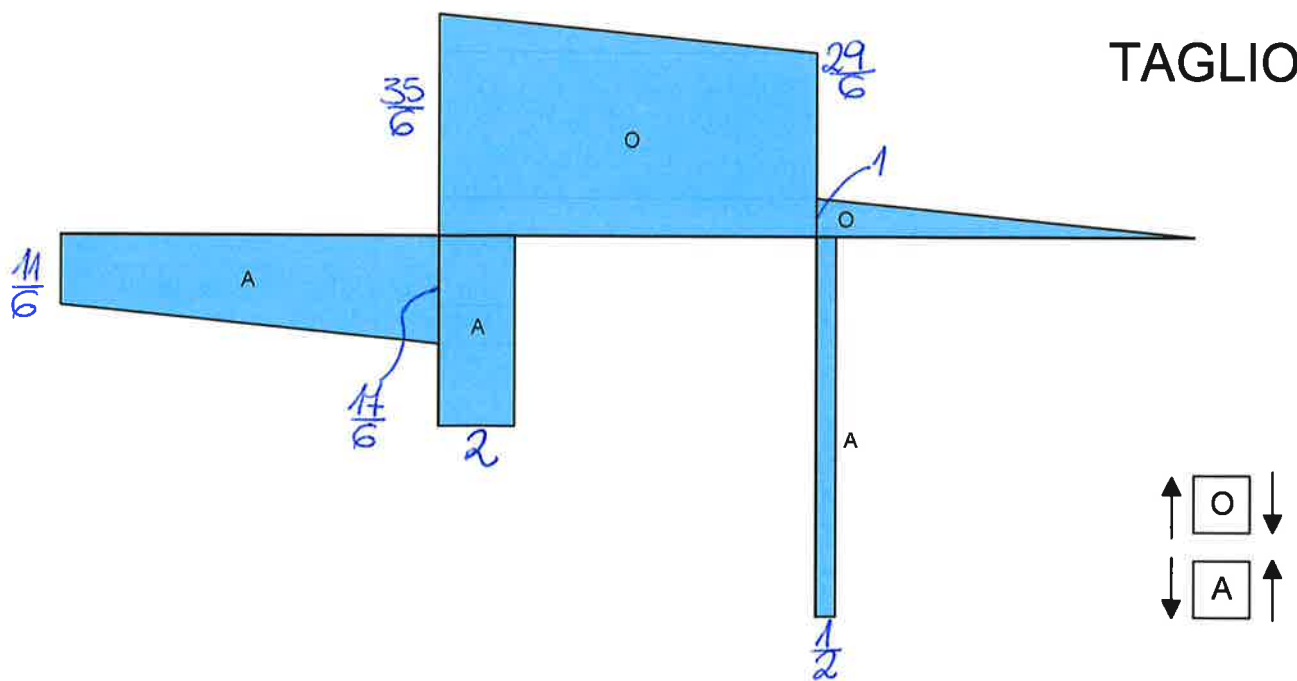
$$M = \frac{ql^2}{3} - 2ql \cdot x_2 = 0$$

$$2x_2 = \frac{L}{3} \rightarrow x_2 = \frac{L}{6} \approx 0,16L$$

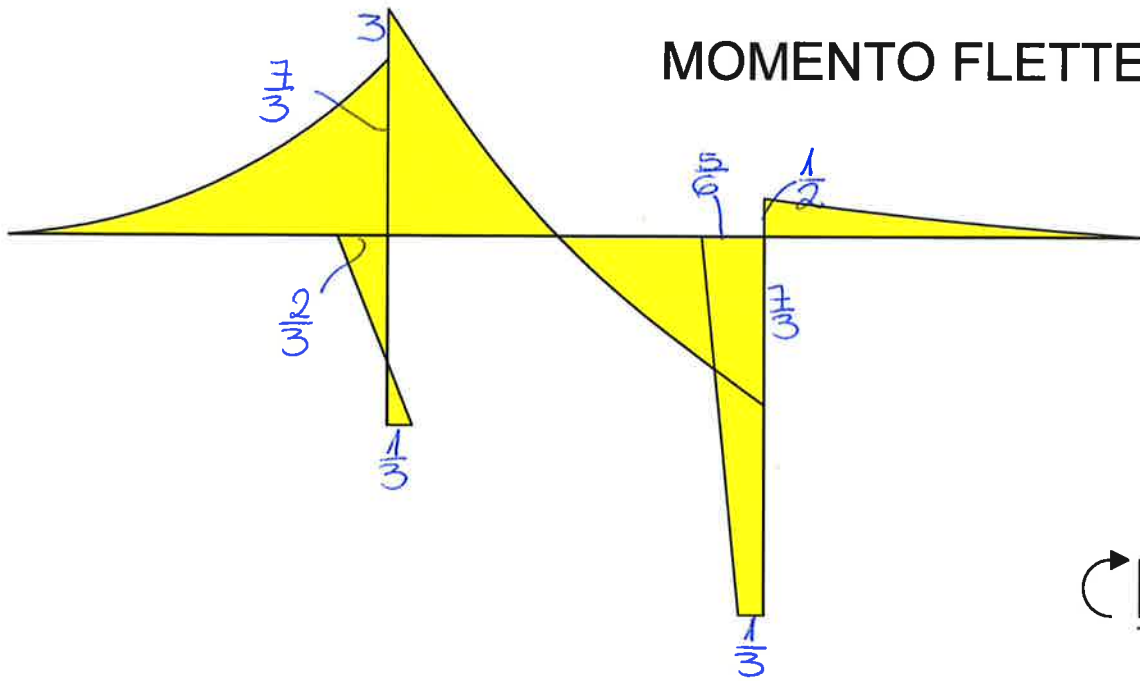
AZIONE ASSIALE



TAGLIO



MOMENTO FLETTENTE



DEFORMATA QUALITATIVA

