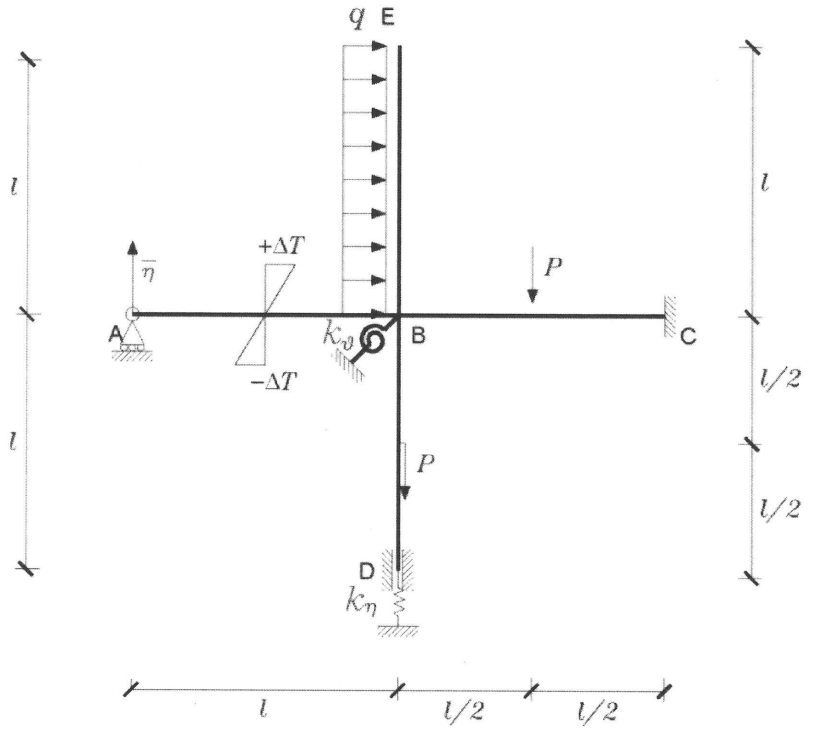
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 26 MARZO 2018

DOCENTE: **PROF. FAUSTO MINELLI** ESERCITATORE: **ING. LUCA FACCONI**

DURATA: **2 ORE.**

Esercizio



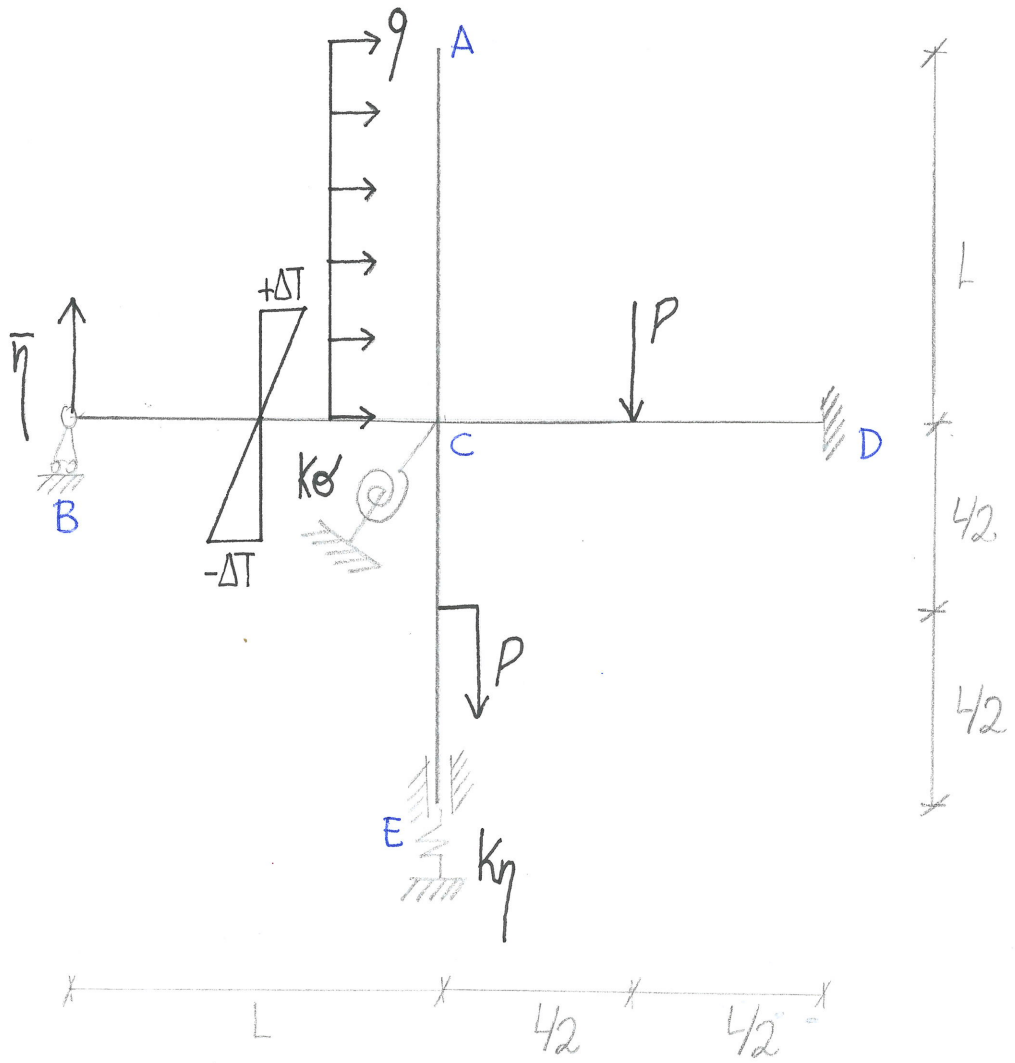
$$K_\eta = 24\frac{EJ}{l^3} \qquad k_\theta = 15\frac{EJ}{l}$$

$$\bar{\eta} = \frac{3}{2}\frac{ql^4}{EJ} \qquad \frac{\alpha\Delta T}{H} = \frac{1}{3}\frac{ql^2}{EJ} \qquad P = 2ql$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

TEMA ESAME 26/03/2018



DATI

$\bar{\eta} = \frac{3}{2}\frac{qL^4}{EJ}$

$\frac{\alpha\Delta T}{H} = \frac{qL^2}{3EJ}$

$K\vartheta = 15\frac{EJ}{L}$

$K\eta = 24\frac{EJ}{L}$

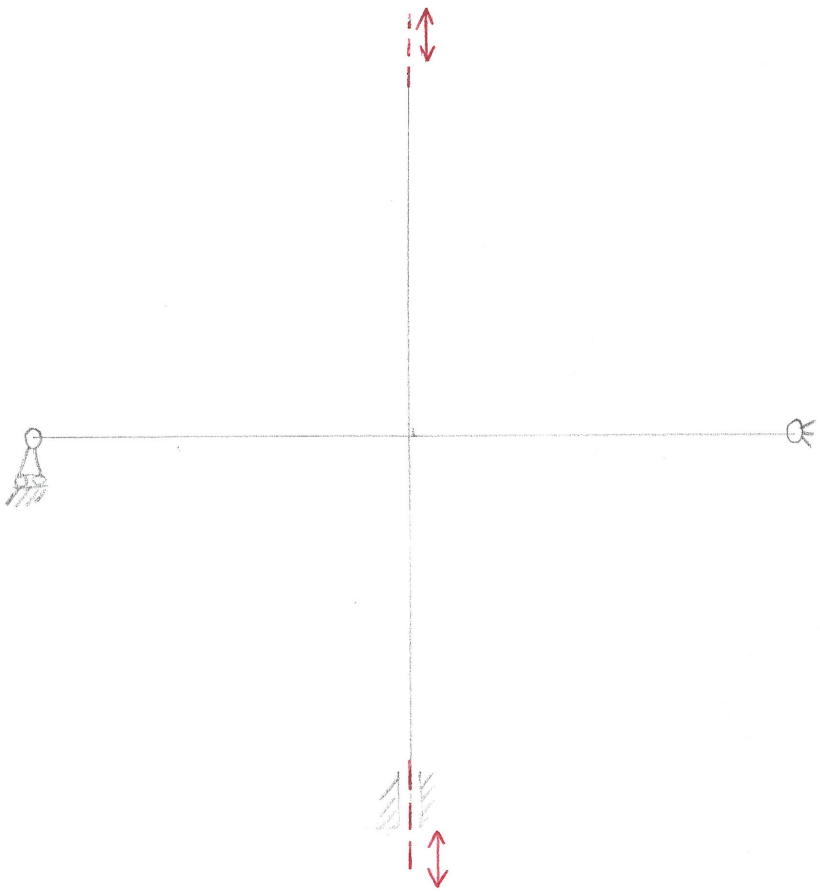
$P = 2qL$

N SI VALUTA IL GRADO DI IPERSTATICITÀ DELLA STRUTTURA

GdV = 8

GdL = 3

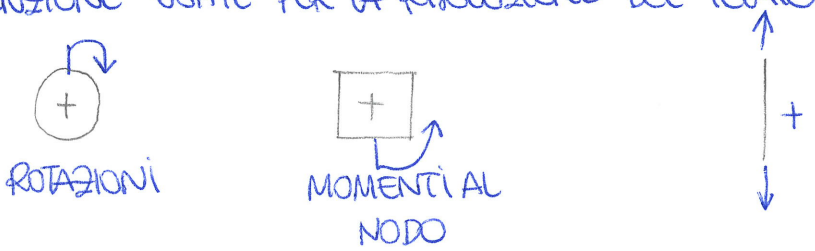
N SI VALUTA SE IL TELAIO È A NODI FISSI O SPOSTABILI



SI OSSERVA CHE È POSSIBILE UN CINEMATISMO IN DIREZIONE VERTICALE

IL TELAIO È A NODI SPOSTABILI → BIELLA FITTIZIA IN A

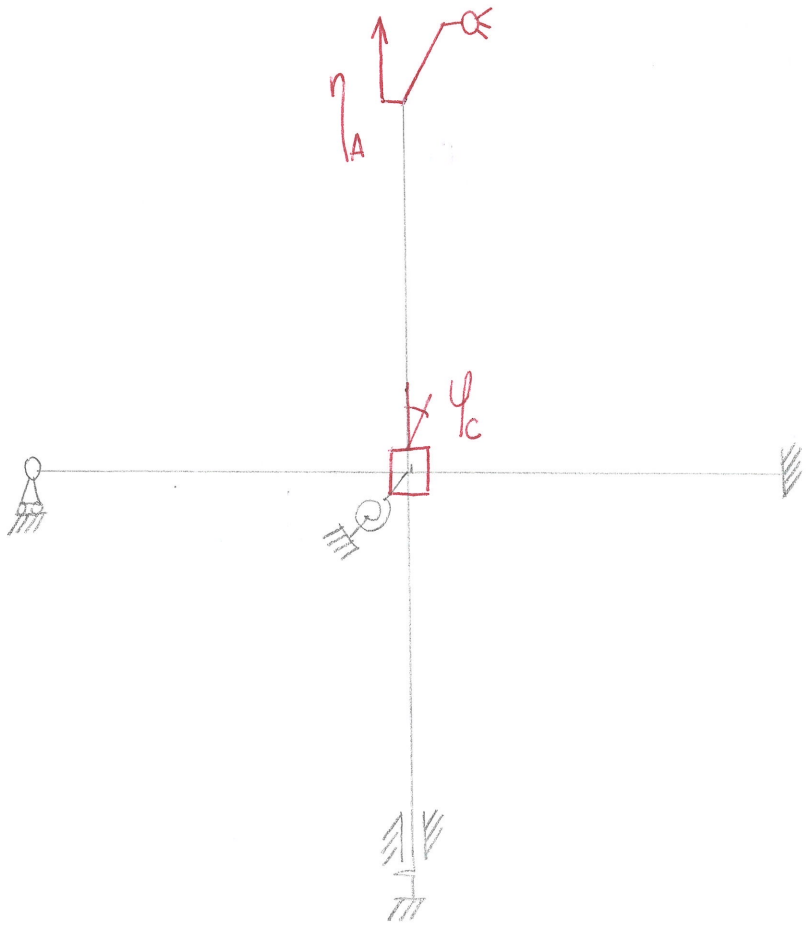
N CONVENZIONE USATE PER LA RISOLUZIONE DEL TELAIO



N RISOLVO CON IL METODO DEGLI SPOSTAMENTI

LE INCOGNITE SONO:

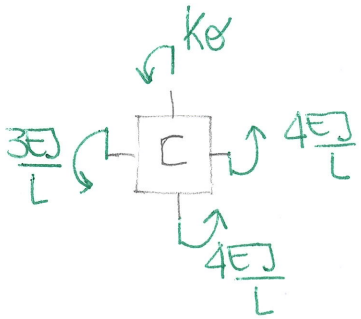
$$\begin{cases} \varphi_C \\ \eta_A \end{cases}$$

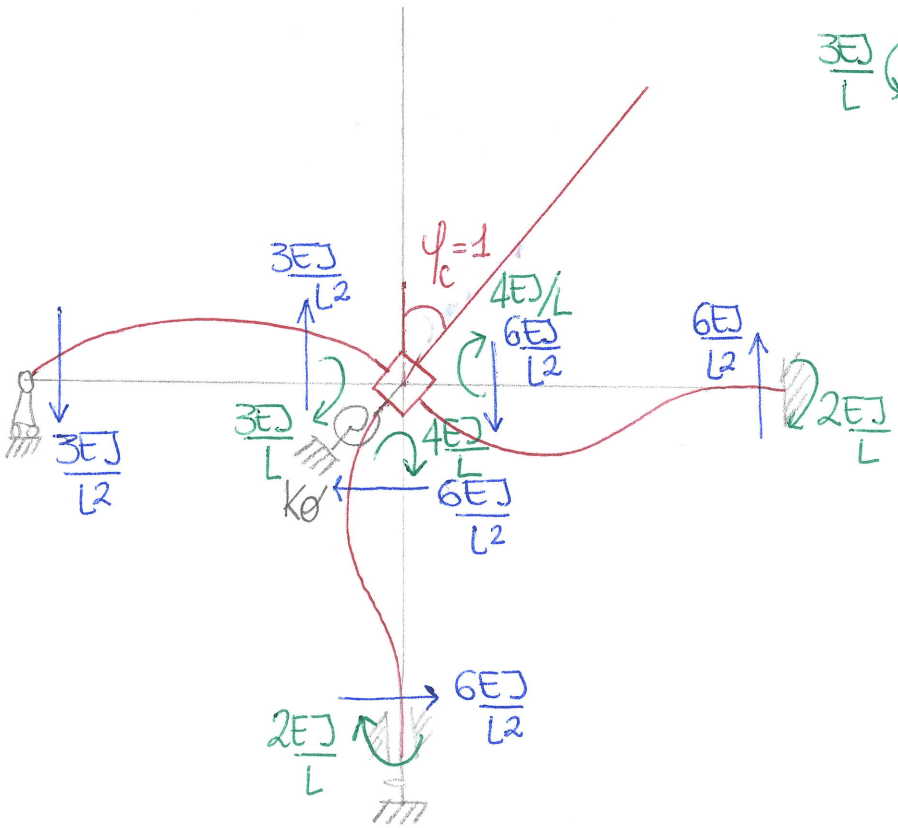


N IL SISTEMA RISOLVENTE RISULTA ESSERE:

$$\begin{cases} \Sigma M_C = 0 \\ \Sigma R_A = 0 \end{cases} \qquad \begin{cases} m_{CC}\,\varphi_C + m_{CA}\cdot\eta_A + m_{C0} = 0 \\ h_{AC}\,\varphi_C + h_{AA}\cdot\eta_A + h_{A0} = 0 \end{cases}$$

*CASO 1: $\varphi_C = 1$; $\eta_A = 0$; $\forall P_{EX} = 0$



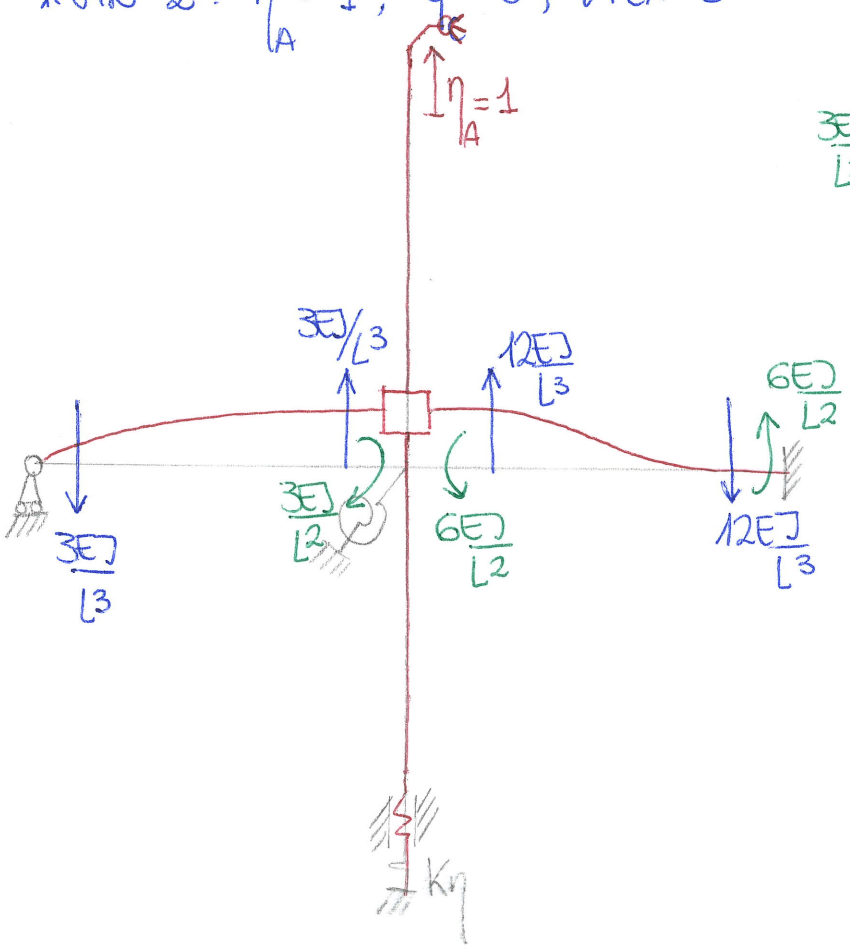


$$m_{CC} = \frac{3EJ}{L} + \frac{4EJ}{L} + \frac{4EJ}{L} + \frac{15EJ}{L} =$$

$$= 26\,\frac{EJ}{L}$$

$$h_{AC} = \frac{3EJ}{L^2} - \frac{6EJ}{L^2} = -\frac{3EJ}{L^2}$$

*CASO 2: $\eta_A = 1$; $\varphi = 0$; $\forall P_{EX} = 0$

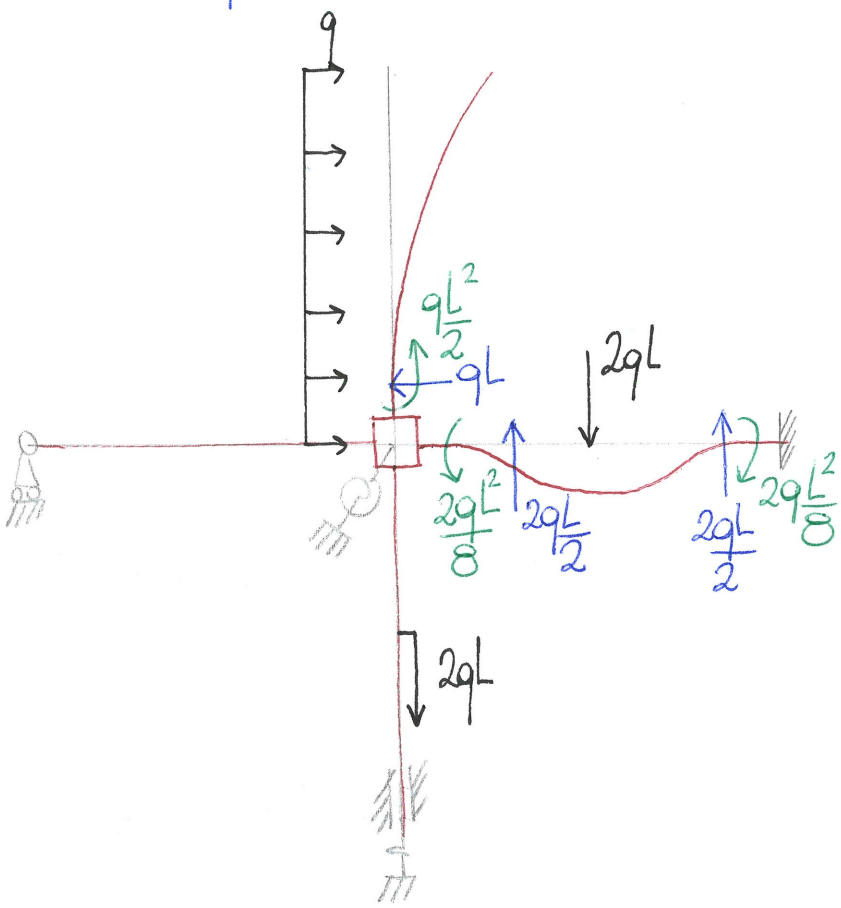


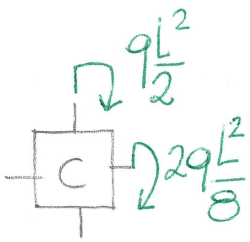
$$m_{CA} = \frac{3EJ}{L^2} - \frac{6EJ}{L^2} = -\frac{3EJ}{L^2}$$

$$h_{AA} = \frac{3EJ}{L^3} + \frac{12EJ}{L^3} + \frac{24EJ}{L^3} =$$

$$= 39\frac{EJ}{L^3}$$

*CASO 3: $q \neq 0$

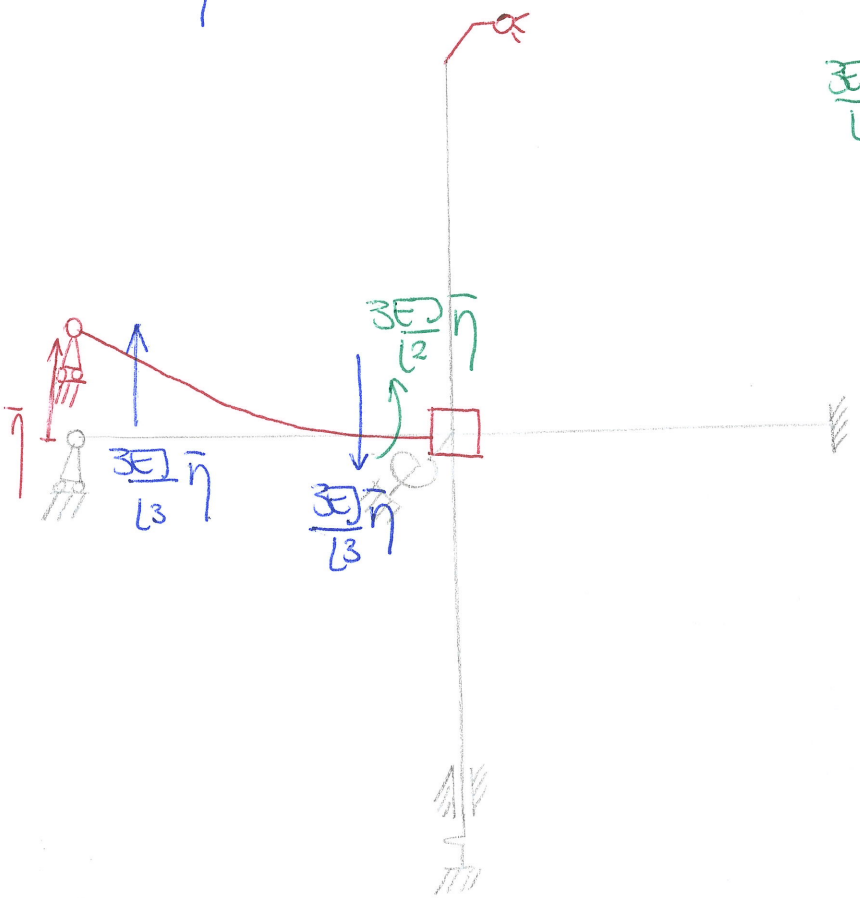


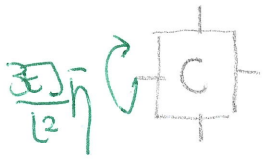


$$m_{CO,1} = -\frac{qL^2}{2} - \frac{2qL^2}{8} = -\frac{3}{4}qL^2$$

$$h_{AO,1} = 2qL + \frac{2qL}{2} = 3qL$$

* CASO 4 $\bar{\eta} \neq 0$

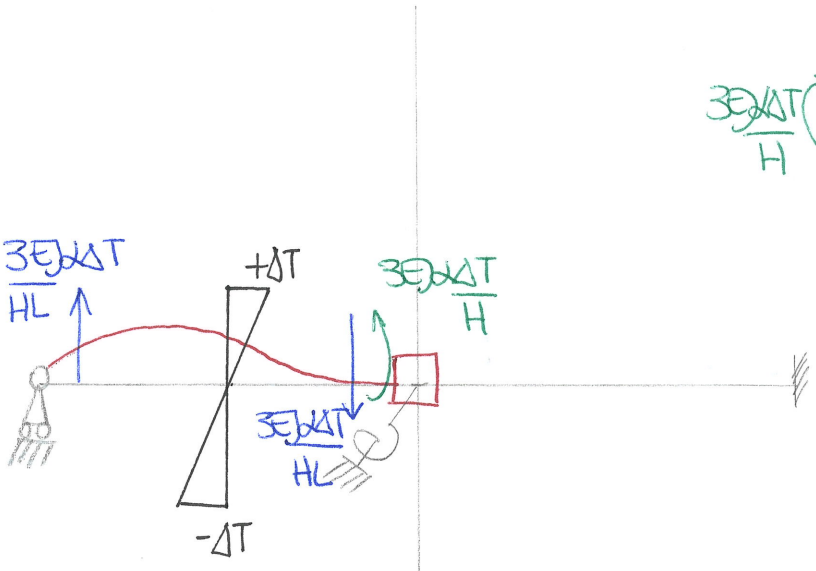


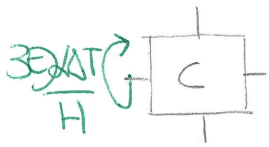


$$m_{C0,2} = -\frac{3EJ}{l^2}\cdot\bar{\eta} = -\frac{9}{2}ql^2$$

$$h_{A0,2} = -\frac{3EJ}{l^3}\bar{\eta} = -\frac{9}{2}ql^2$$

* CASO 5: $\Delta T \neq 0$





$$m_{C0,3} = -\frac{3EJ\alpha\Delta T}{H} = -ql^2$$

$$h_{A0,3} = -\frac{3EJ\alpha\Delta T}{HL} = -ql$$

$$m_{BO} = -\frac{3}{4}ql^2 - \frac{9}{2}ql^2 - ql^2 = -\frac{25}{4}ql^2$$

$$h_{AO} = 3ql - \frac{9}{2}ql^2 - ql = -\frac{5}{2}ql$$

N SISTEMA RISOLVENTE

$$\begin{cases} 26\frac{EJ}{L}\cdot\varphi_C - 3\frac{EJ}{L^2}\cdot\eta_A - \frac{25}{4}ql^2 = 0 & (1) \\ -3\frac{EJ}{L^2}\cdot\varphi_C + 39\frac{EJ}{L^3}\cdot\eta_A - \frac{5}{2}ql = 0 & (2) \end{cases}$$

MOLTIPLICO (2) PER $\frac{1}{13}$ E LA SOMMO A (1)

$$-\frac{3}{13}\frac{EJ}{L}\varphi_C + 3\frac{EJ}{L^2}\cdot\eta_A - \frac{5}{26}ql^2 = 0$$

(+)

$$26\frac{EJ}{L}\varphi_C - 3\frac{EJ}{L^2}\eta_A - \frac{25}{4}ql^2 = 0$$

$$\left[-\frac{3}{13}\frac{EJ}{L} + 26\frac{EJ}{L}\right]\varphi_C + \left[3\frac{EJ}{L^2} - 3\frac{EJ}{L^2}\right]\eta_A + \left[-\frac{5}{26}ql^2 - \frac{25}{4}ql^2\right] = 0$$

$$\frac{335}{13}\frac{EJ}{L}\varphi_C - \frac{335}{52}ql^2 = 0$$

$$\varphi_C = \frac{335}{52}ql^2\cdot\frac{13}{335}\frac{L}{EJ} \longrightarrow \varphi_C = \frac{1}{4}\frac{ql^3}{EJ}$$

SOSTITUISCO φ_C NELLA (2)

$$-3\frac{EJ}{L^2}\cdot\frac{1}{4}\frac{ql^3}{EJ} + 39\frac{EJ}{L^3}\eta_A - \frac{5}{2}ql = 0$$

$$39\frac{EJ}{L^3}\eta_A - \frac{3}{4}ql - \frac{5}{2}ql = 0$$

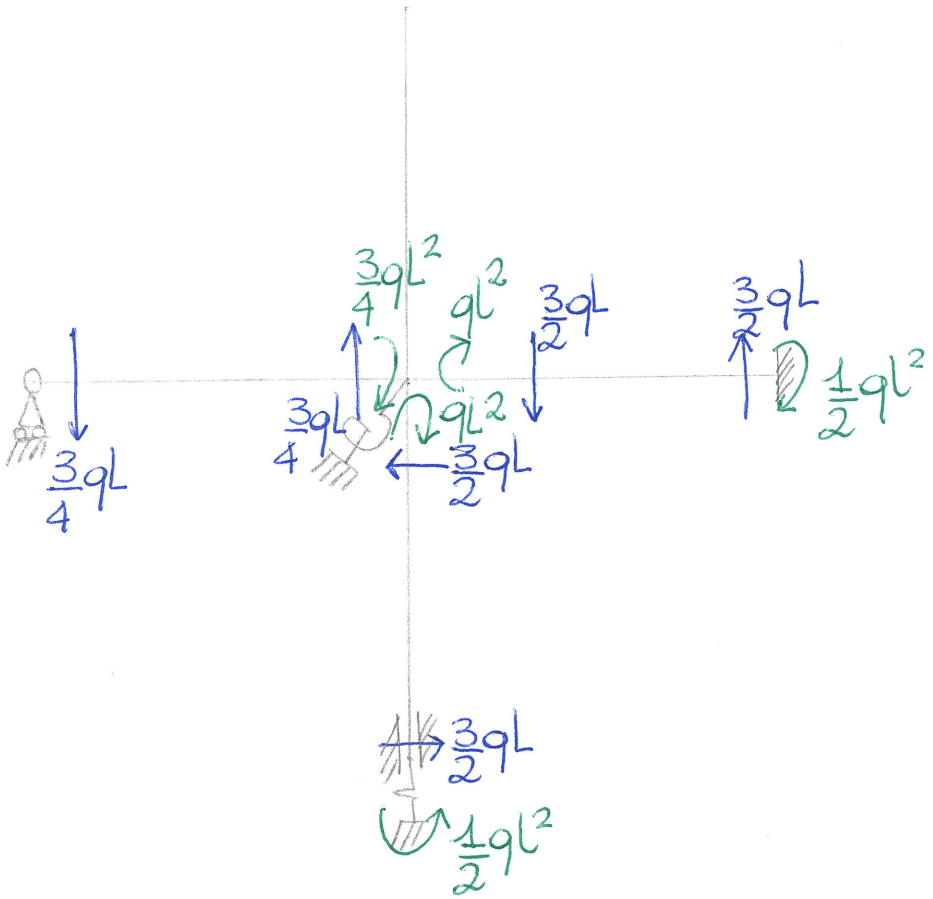
$$39\frac{EJ}{L^3}\eta_A = \frac{13}{4}ql$$

$$\eta_A = \frac{13}{4}ql\cdot\frac{1}{39}\frac{L^3}{EJ} \longrightarrow \eta_A = \frac{1}{12}\frac{ql^4}{EJ}$$

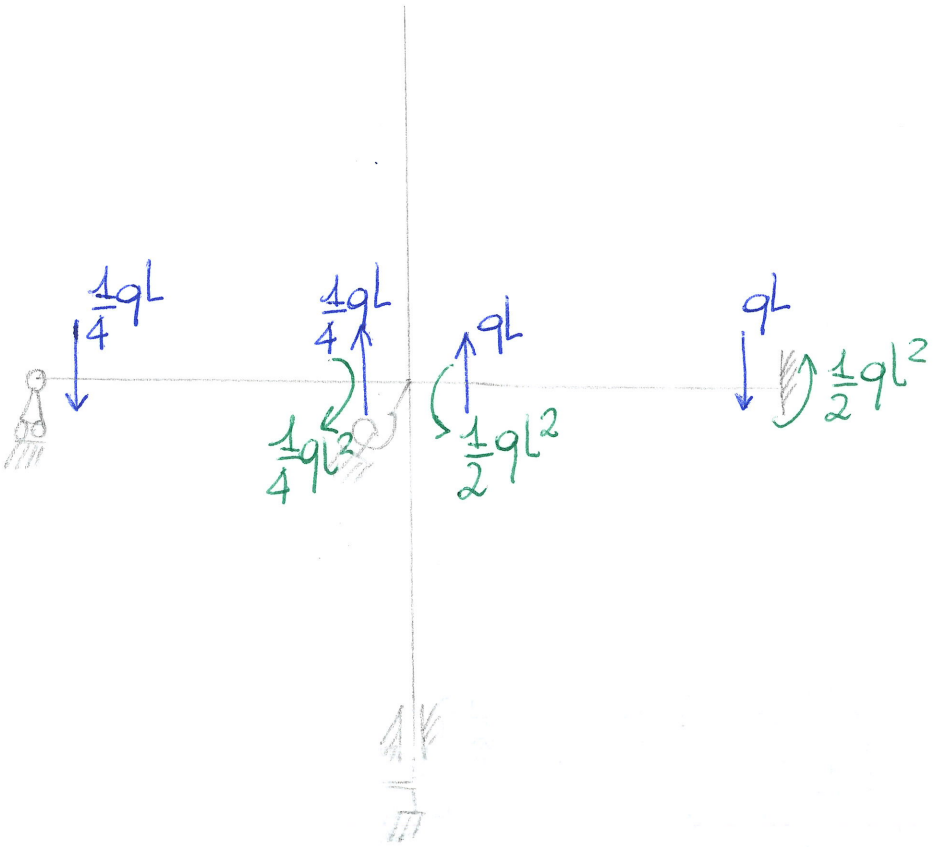
IN CONCLUSIONE

$$\begin{cases} \varphi_C = \frac{1}{4}\frac{qL^3}{EJ} \\ \eta_A = \frac{1}{12}\frac{qL^4}{EJ} \end{cases}$$

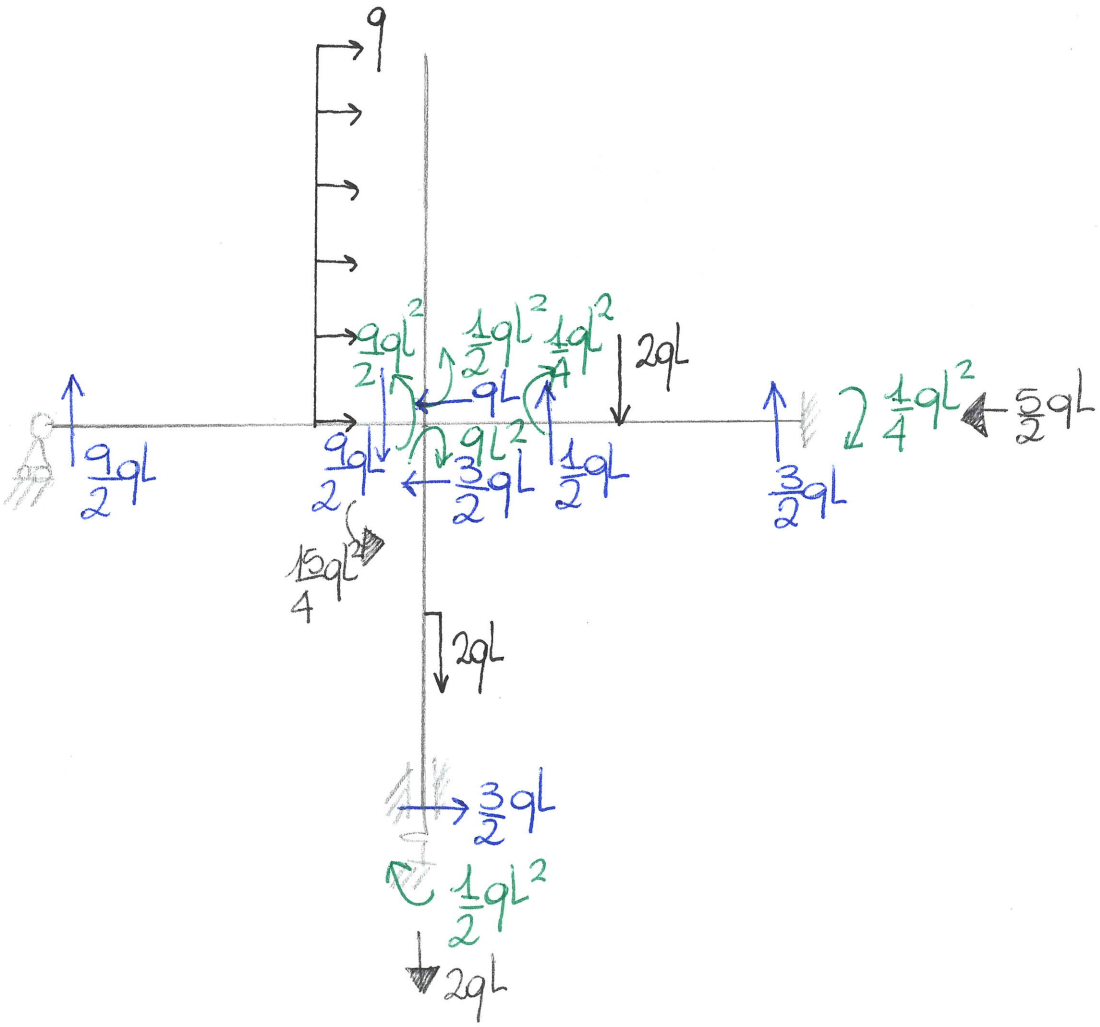
N $\varphi_C = \frac{1}{4}\frac{qL^3}{EJ}$



$\nu\eta_A = \frac{1}{12}\frac{qL^4}{EJ}$



νCALCOLO AZIONI INTERNE



EQ: $\uparrow + = 0$

$\frac{9}{2}qL + \frac{3}{2}qL - 2qL - 2qL - 2qL = 0$ ok!

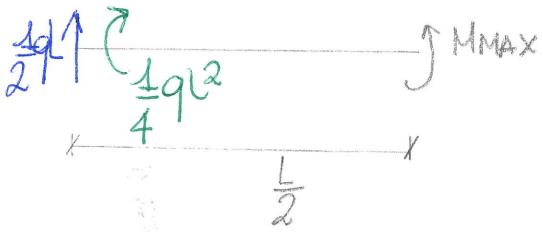
EQ: $\overset{+}{\rightarrow} = 0$

$\frac{3}{2}qL + qL - \frac{5}{2}qL = 0$ ok!

EQ: $\circlearrowleft_B = 0$

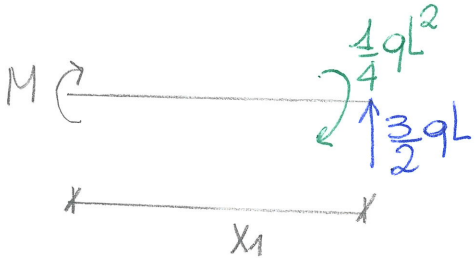
$-\frac{3}{2}qL \cdot L + \frac{1}{2}qL^2 + 2qLL + 2qLL - \frac{3}{2}qL \cdot 2L + \frac{1}{4}qL^2 + 2qL \cdot \frac{3}{2}L + qL \cdot \frac{L}{2} - \frac{15}{4}qL^2 = 0$ ok!

N MOMENTO MASSIMO



$M_{MAX} = \frac{1}{4}qL^2 + \frac{1}{2}qL \cdot \frac{L}{2} = \frac{1}{2}qL^2$

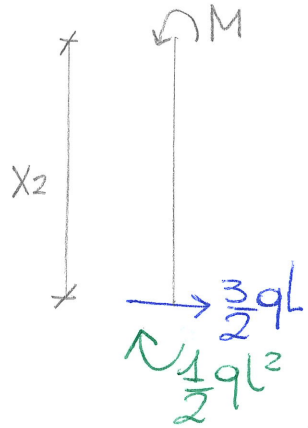
N FLESSO 1



$M = \frac{1}{4}qL^2 - \frac{3}{2}qL \cdot X_1 = 0$

$\frac{3}{2}X_1 = \frac{1}{4}L \rightarrow X_1 = \frac{2}{3} \cdot \frac{L}{4} = \frac{1}{6}L \simeq 0{,}17L$

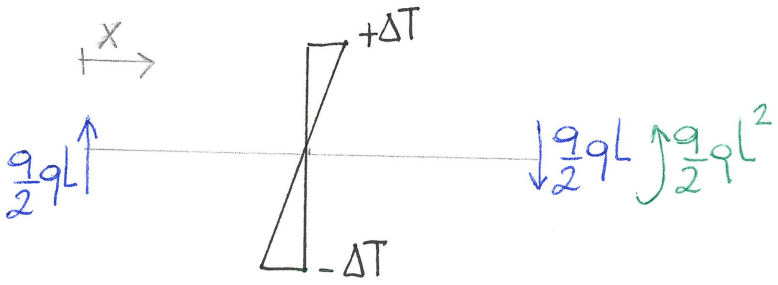
N FLESSO 2



$$M = -\frac{3}{2}qLX_2 + \frac{1}{2}qL^2 = 0$$

$$\frac{3}{2}X_2 = \frac{1}{2}L \rightarrow X_2 = \frac{2}{3}\cdot\frac{L}{2} = \frac{L}{3} \simeq 0{,}33L$$

N DEFORMATA TERMICA



$$y''(x) = -\frac{M(x)}{EJ} + \frac{2\alpha\Delta T}{EJ}$$

$$M(x) = \frac{9}{2}qL\cdot x$$

$$y''(x) = -\frac{9}{2}qL\frac{x}{EJ} + 2\cdot\frac{1}{3}\frac{qL^2}{EJ}$$

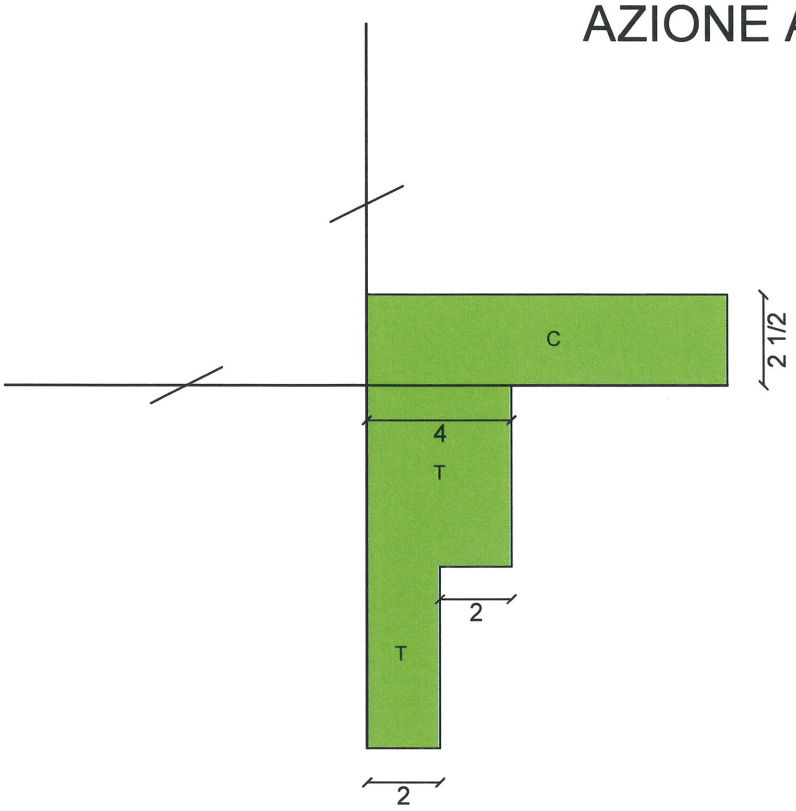
$$y''(x)\cdot EJ = -\frac{9}{2}qLx + \frac{2}{3}qL^2 \geq 0$$

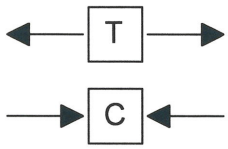
$$\frac{9}{2}x \leq \frac{2}{3}L \rightarrow x \leq \frac{2}{9}\cdot\frac{2}{3}L = \frac{4}{27}L \simeq 0{,}15L$$

$$x \leq 0{,}15L$$

AZIONE ASSIALE

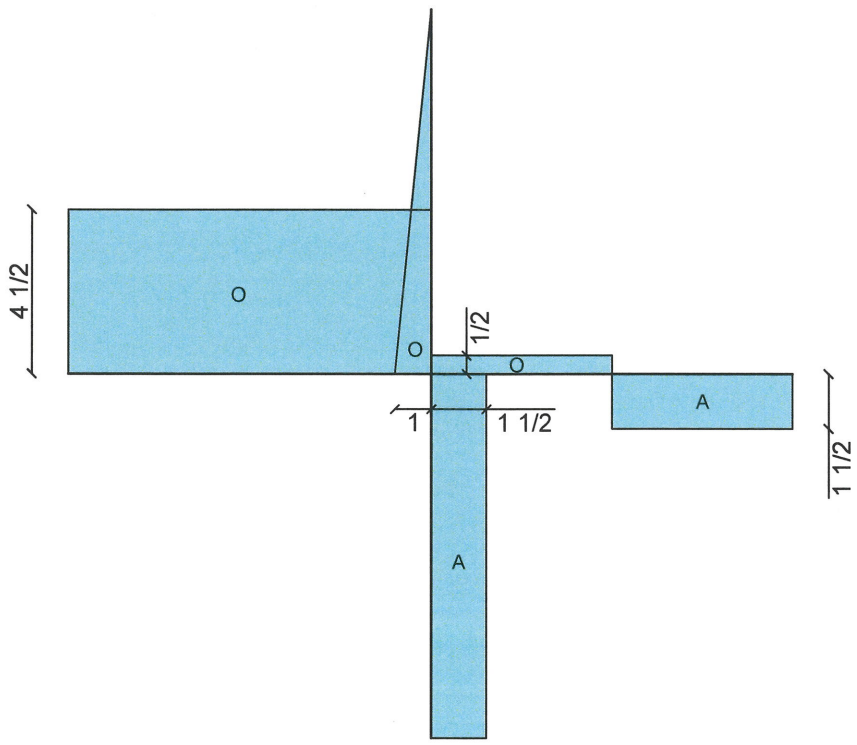
U.D.M. ql

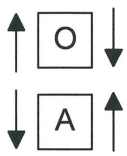




TAGLIO

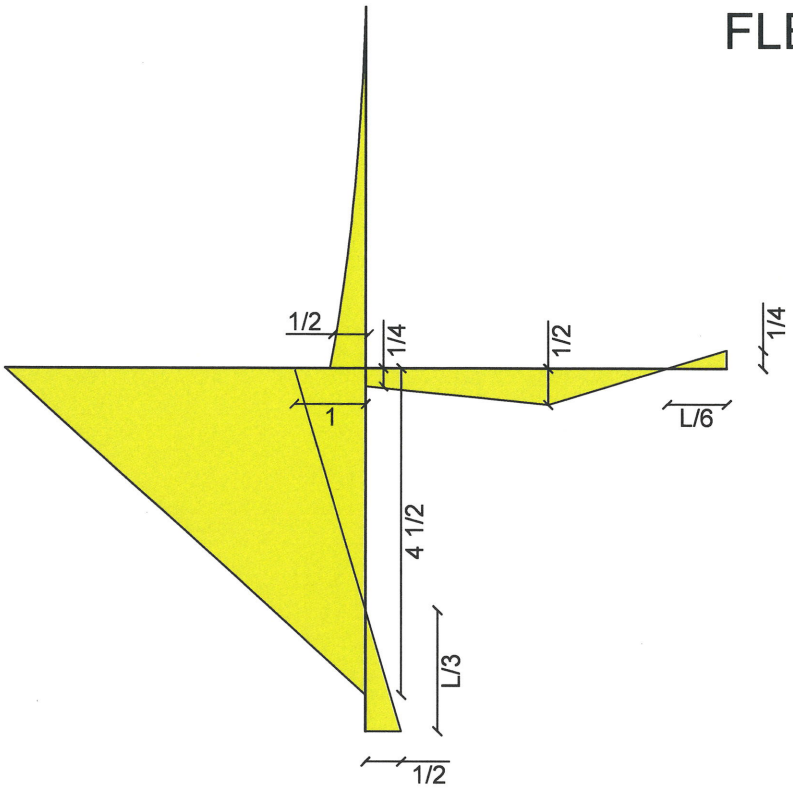
U.D.M. ql

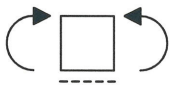




MOMENTO FLETTENTE

U.D.M. ql^2





DEFORMATA QUALITATIVA

U.D.M. Spostamenti ql^4/EJ

U.D.M. Rotazioni ql^3/EJ