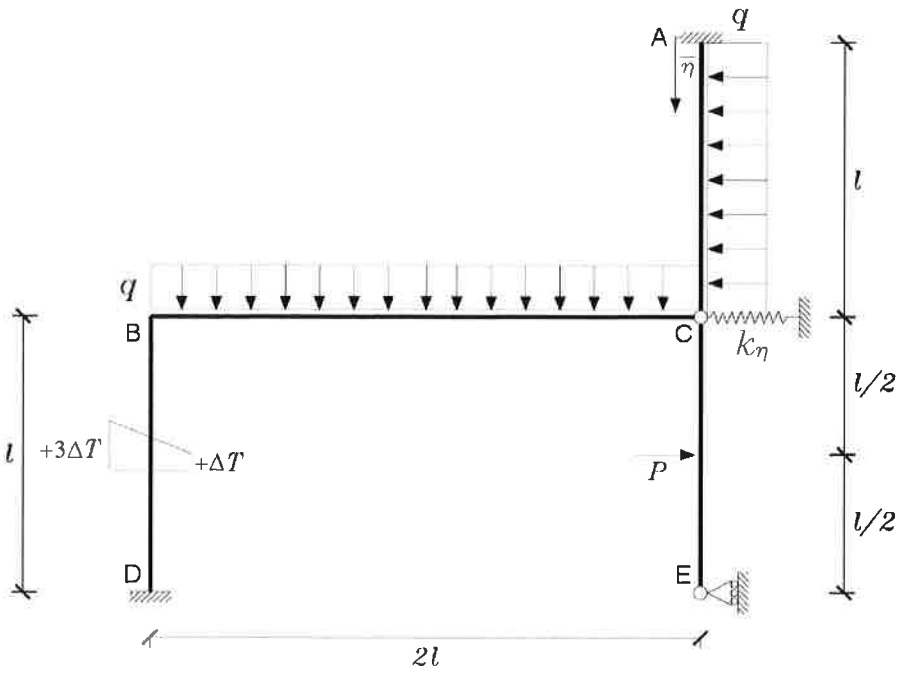
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 10 LUGLIO 2018

DOCENTI: **PROF. GIOVANNI PLIZZARI** ESERCITATORE: **ING. LUCA FACCONI**
PROF. FAUSTO MINELLI

DURATA: 2 ORE

Esercizio



$$K_\eta = \frac{7}{2}\frac{EJ}{l^3} \qquad \bar{\eta} = \frac{ql^4}{EJ} \qquad P = ql$$

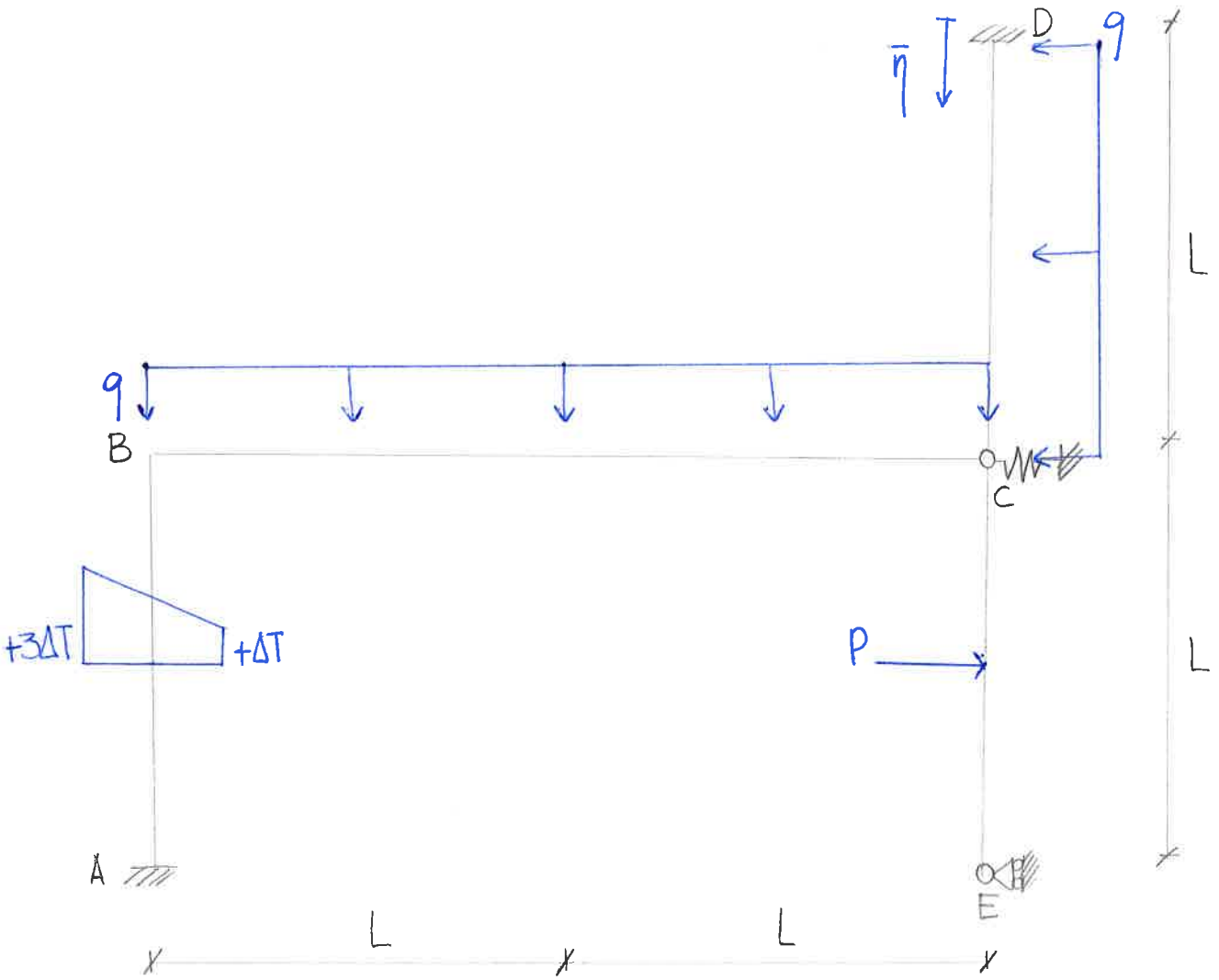
$$\alpha\Delta T = \frac{1}{4}\frac{ql^3}{EJ} \qquad \frac{\alpha\Delta T}{H} = \frac{1}{2}\frac{ql^2}{EJ}$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Quanto richiesto, ed i calcoli necessari per lo sviluppo del tema, vanno riportati nello stesso in maniera chiara e con tratto non cancellabile.

TEMA ESAME 10/07/2018



DATI

$k_\eta = \frac{7}{2}\frac{EJ}{L}$

$\bar{\eta} = \frac{qL^4}{EJ}$

$P = qL$

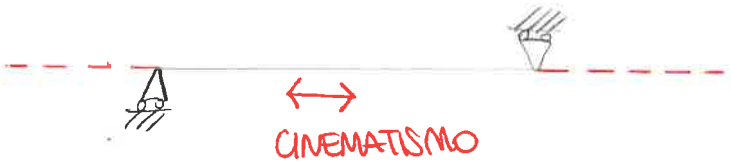
$\alpha\Delta T = \frac{1}{4}\frac{qL^3}{EJ}$

$\frac{\alpha\Delta T}{H} = \frac{1}{2}\frac{qL^2}{EJ}$

N SI VALUTA IL GRADO DI IPERSTATICITÀ DELLA STRUTTURA

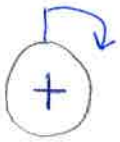
GdV = 12
GdL = 9
} 3 VOLTE IPERSTATICA

N SI OSSERVA CHE IL TELAIO È A NODI SPOSTABILI → BIELLA FITTIZIA IN C



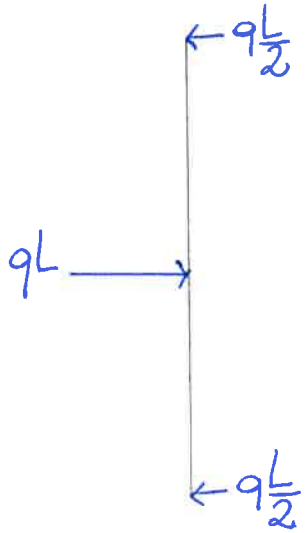
CONVENZIONE DI SEGNO

MOMENTI AL NODO



ROTAZIONI

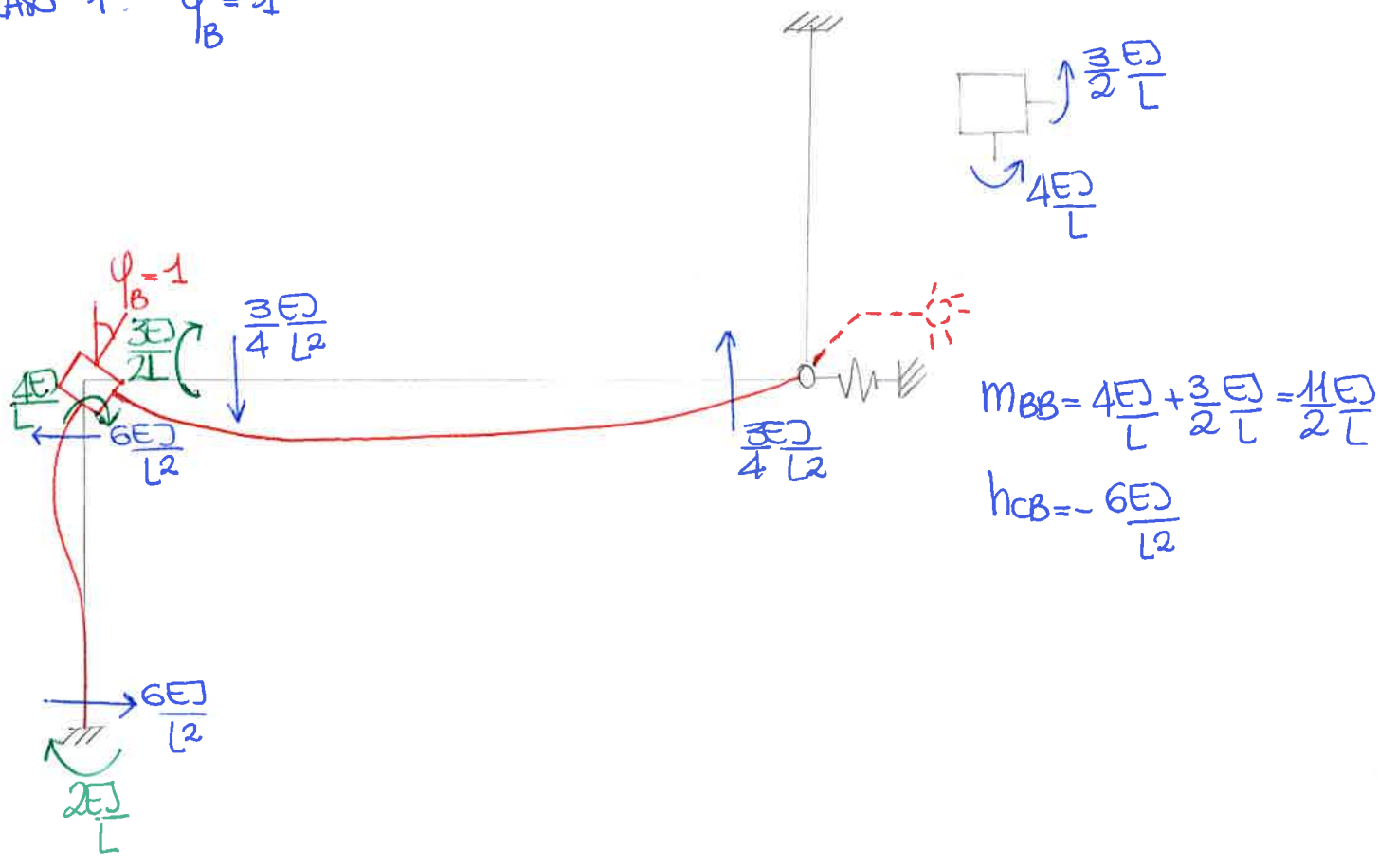
APPENDICE ISOSTATICA CE



SISTEMA RISOLVENTE

$$\begin{cases} m_{BB}\,\varphi_B + m_{BC}\,\eta_C + m_{B0} = 0 \\ h_{CB}\,\varphi_B + h_{CC}\,\eta_C + h_{C0} = 0 \end{cases}$$

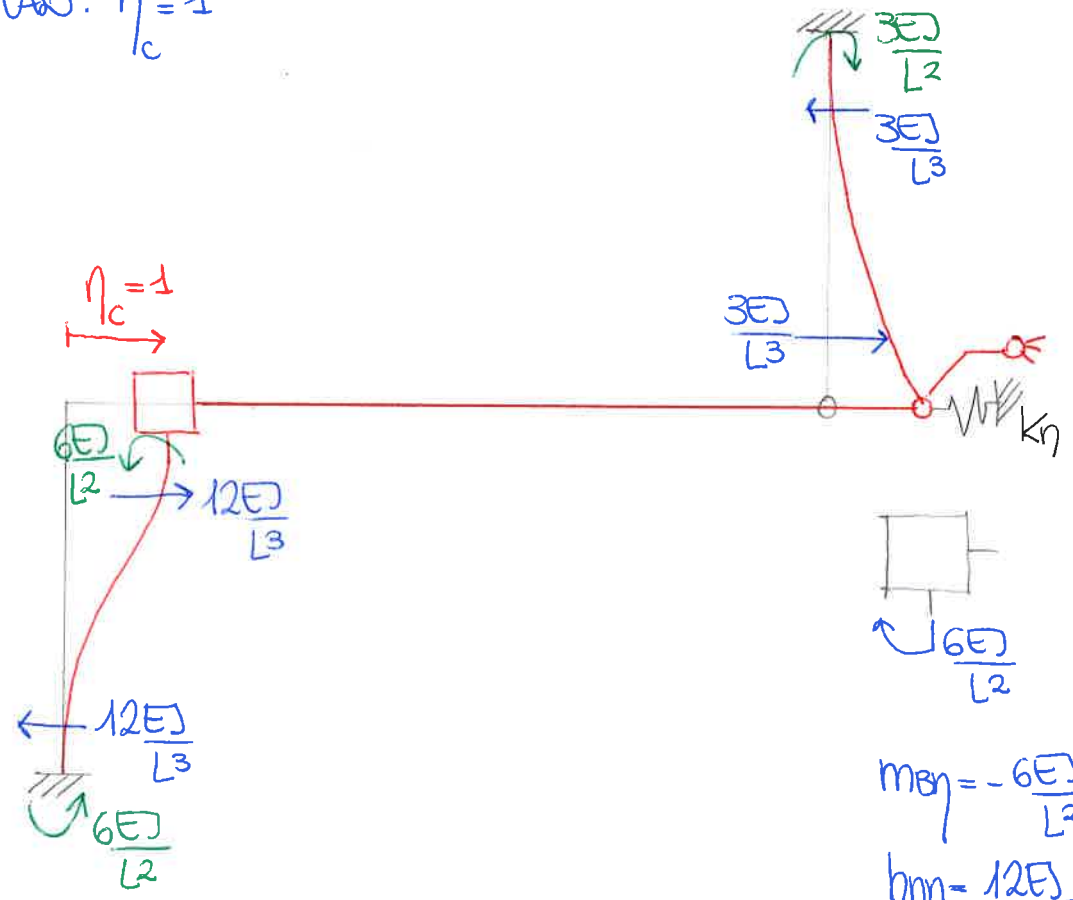
* CASO 1: $\varphi_B = 1$



$$m_{BB} = \frac{4EJ}{L} + \frac{3}{2}\frac{EJ}{L} = \frac{11}{2}\frac{EJ}{L}$$

$$h_{CB} = -\frac{6EJ}{L^2}$$

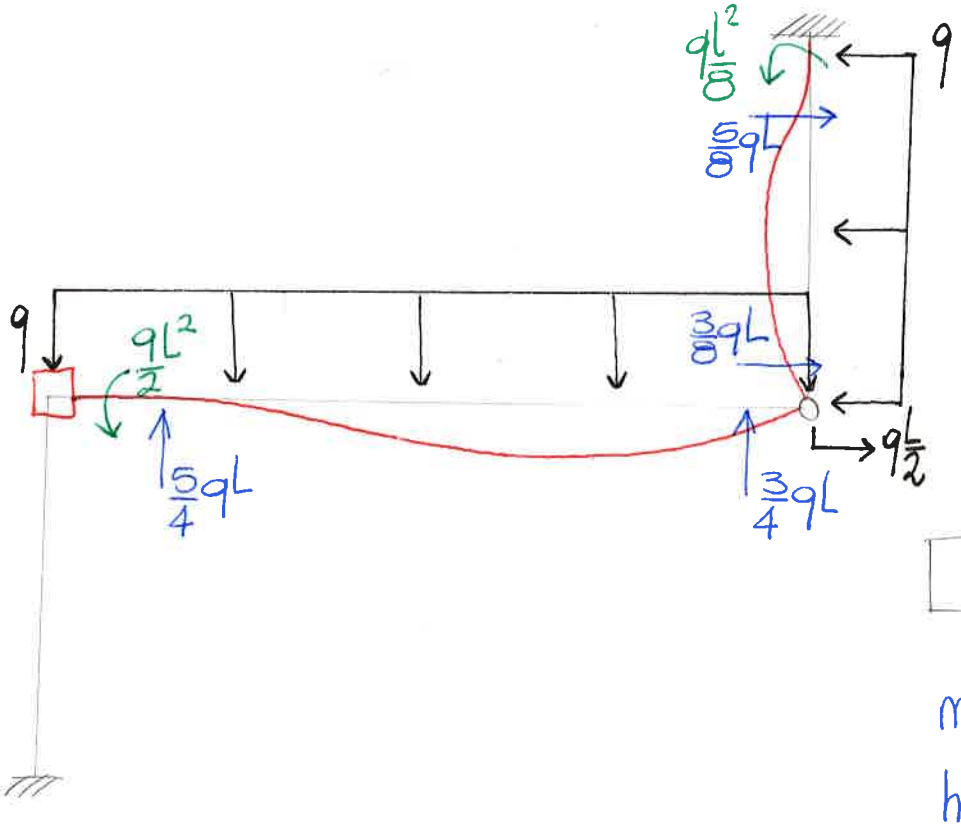
* 2 CASO: $\eta_c = 1$



$$m_{B\eta} = -\frac{6EJ}{L^2}$$

$$h_{\eta\eta} = \frac{12EJ}{L^3} + \frac{3EJ}{L^3} + \frac{7}{2}\frac{EJ}{L^3} = \frac{37}{2}\frac{EJ}{L^3}$$

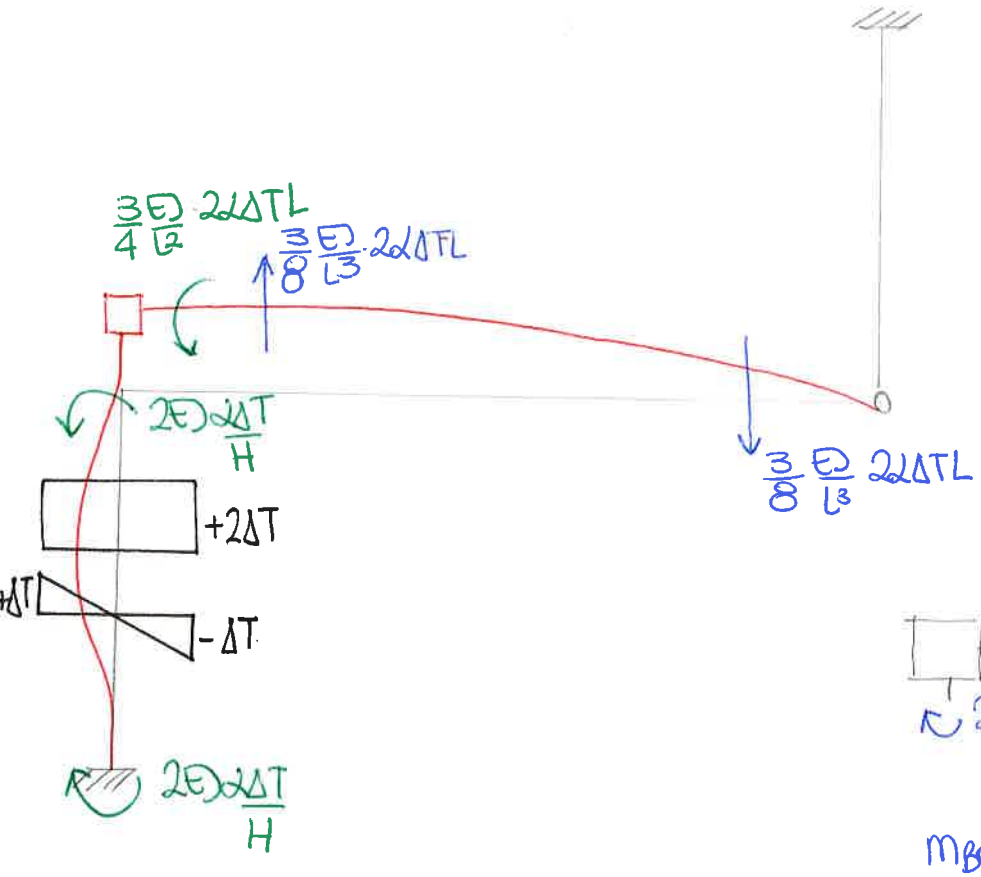
* CASO 3: $q \neq 0$



$$m_{B0,1} = -\frac{ql^2}{2}$$

$$h_{q0,1} = -\frac{ql}{2} + \frac{3}{8}ql - \frac{1}{8}ql$$

*CASO 4 $\Delta T \neq 0$



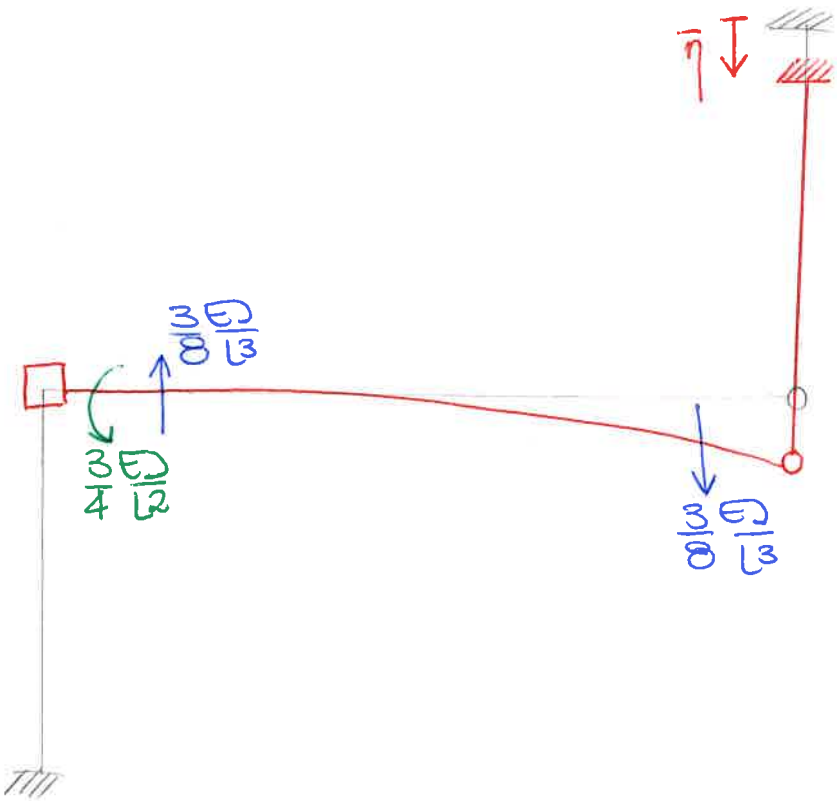
$\frac{3}{4}\frac{EJ}{L^2}2\alpha\Delta T = \frac{3}{8}qL^2$

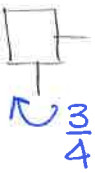
$2EJ\frac{\alpha\Delta T}{H} = qL^2$

$m_{B0,2} = -qL^2 - \frac{3}{8}qL^2 = -\frac{11}{8}qL^2$

$h_{\eta 0,2} = 0$

*CASO 5 $\eta \neq 0$





$\frac{3}{4}\frac{EJ}{L^2}\cdot\bar{\eta}$

$m_{B0,3} = -\frac{3}{4}qL^2$

$h_{\eta 0,3} = 0$

$$M_{BD} = -\frac{qL^2}{2} - \frac{11}{8}qL^2 - \frac{3}{4}qL^2 = -\frac{21}{8}qL^2$$

$$h_{PD} = -\frac{1}{8}qL$$

N SISTEMA RISOLVENTE

$$\begin{cases} \frac{11}{2}\frac{EJ}{L}\varphi_B - \frac{6EJ}{L^2}\eta_C - \frac{21}{8}qL^2 = 0 & (1) \\ -\frac{6EJ}{L^2}\varphi_B + \frac{37}{2}\frac{EJ}{L^3}\eta_C - \frac{1}{8}qL = 0 & (2) \end{cases}$$

MOLTIPLICO (2) PER $\frac{11}{12}L$ E LA SOMMO A (1)

$$-\frac{11}{2}\frac{EJ}{L}\varphi_B + \frac{407}{24}\frac{EJ}{L^2}\eta_C - \frac{11}{96}qL^2 = 0$$

(+)

$$\frac{11}{2}\frac{EJ}{L}\varphi_B - \frac{6EJ}{L^2}\eta_C - \frac{21}{8}qL^2 = 0$$

$$\left[-\frac{11}{2}\frac{EJ}{L} + \frac{11}{2}\frac{EJ}{L}\right]\varphi_B + \left[\frac{407}{24}\frac{EJ}{L^2} - \frac{6EJ}{L^2}\right]\eta_C + \left[-\frac{11}{96}qL^2 - \frac{21}{8}qL^2\right] = 0$$

$$\frac{263}{24}\frac{EJ}{L^2}\eta_C = \frac{263}{96}qL^2$$

$$\eta_C = \frac{263}{96}qL^2 \cdot \frac{24}{263}\frac{L^2}{EJ} \longrightarrow \eta_C = \frac{1}{4}q\frac{L^4}{EJ}$$

SOSTITUISCO η_C NELLA (2)

$$-\frac{6EJ}{L^2}\varphi_B + \frac{37}{2}\frac{EJ}{L^3} \cdot \frac{1}{4}q\frac{L^4}{EJ} - \frac{1}{8}qL = 0$$

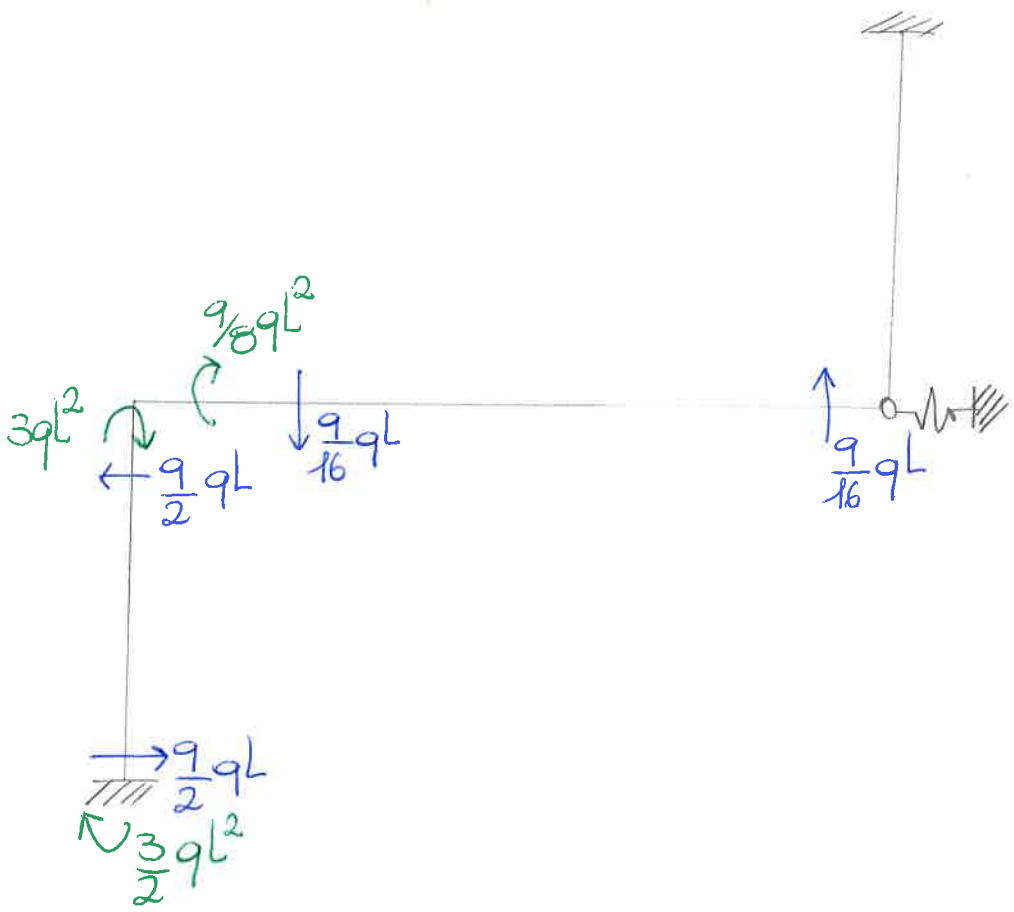
$$\frac{6EJ}{L^2}\varphi_B = \frac{36}{8}qL$$

$$\varphi_B = \frac{36}{8}qL \cdot \frac{L^2}{6EJ} \longrightarrow \varphi_B = \frac{3}{4}\frac{qL^3}{EJ}$$

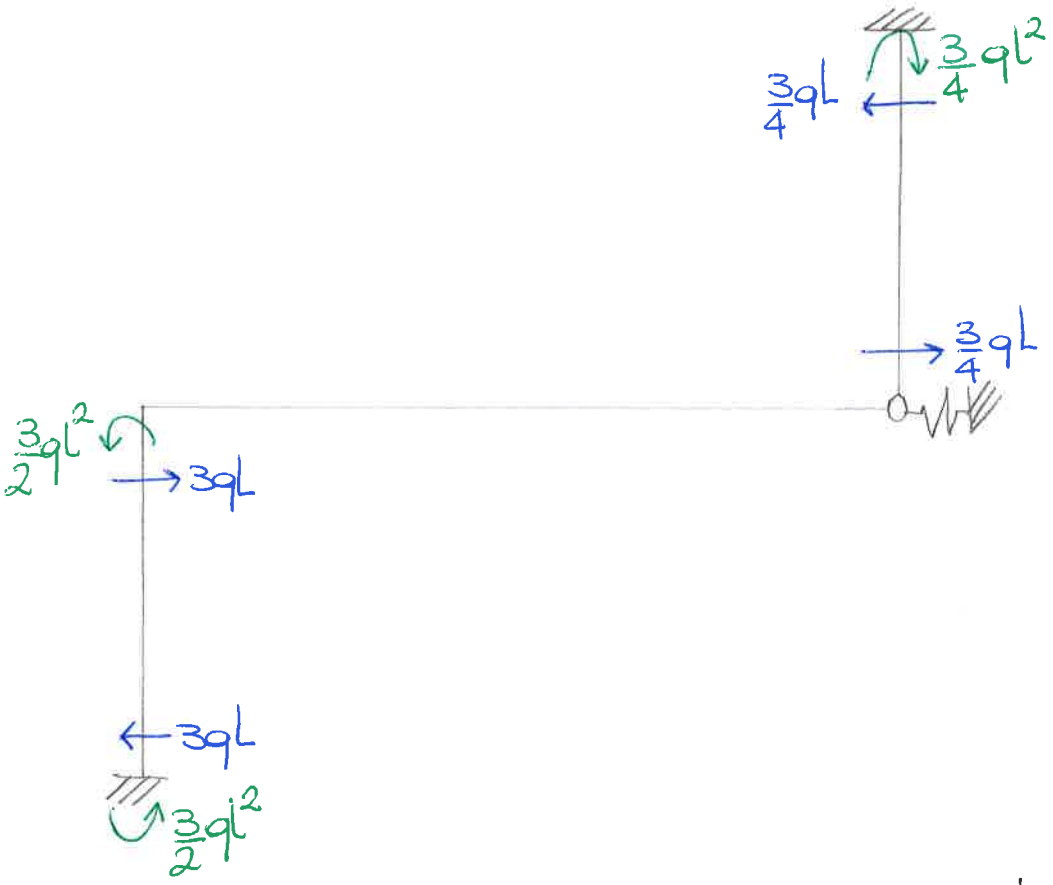
IN CONCLUSIONE

$$\begin{cases} \varphi_B = \frac{3}{4}\frac{qL^3}{EJ} \\ \eta_C = \frac{1}{4}\frac{qL^4}{EJ} \end{cases}$$

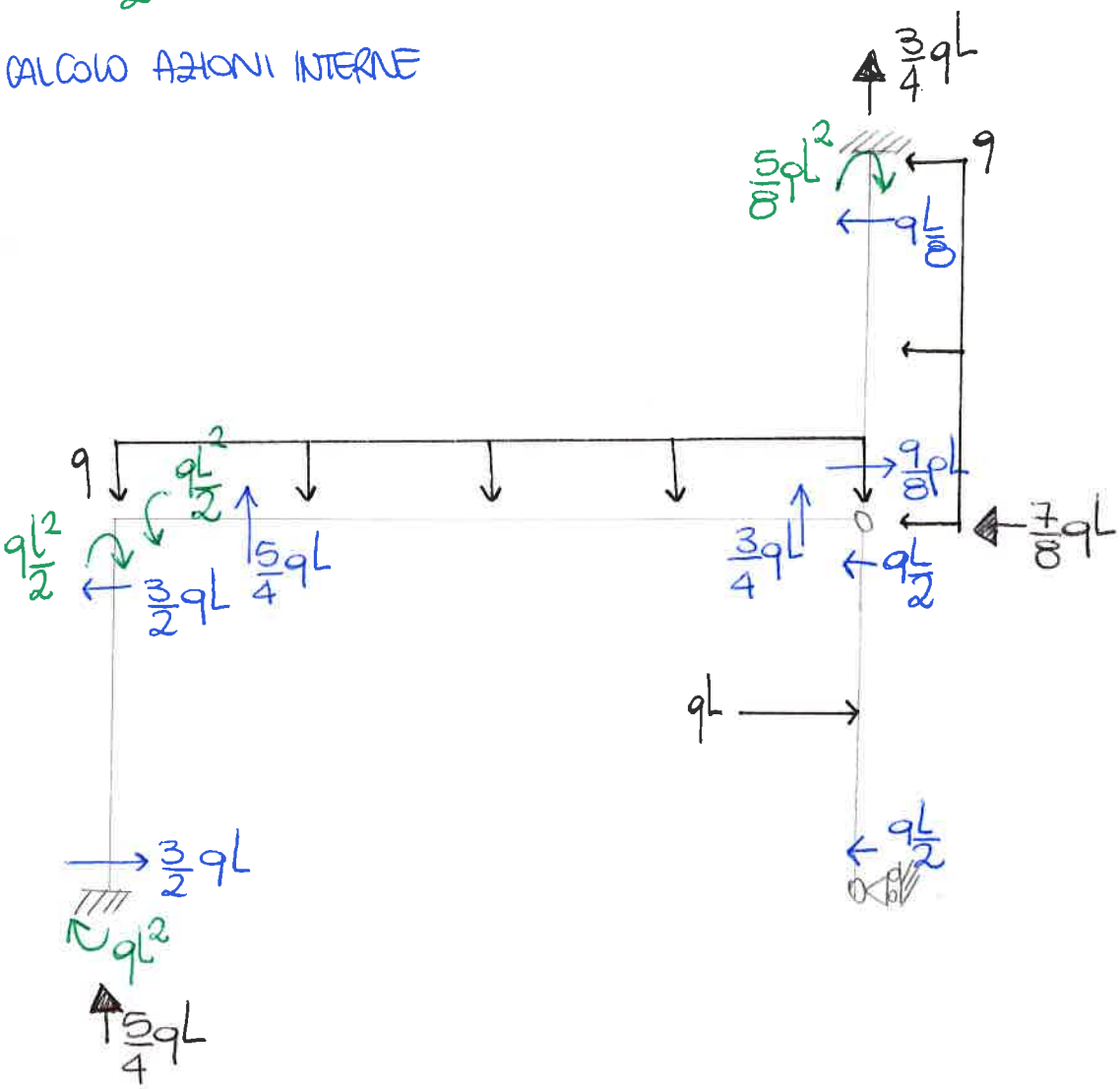
N $\varphi_B = \frac{3}{4}\frac{qL^3}{EJ}$



$\eta|_C = \frac{1}{4}\frac{ql^4}{EJ}$



NCALCOLO AZIONI INTERNE



EQ: $\uparrow_+ = 0$

$\frac{5}{4}qL - 2qL + \frac{3}{4}qL = 0$ ok!

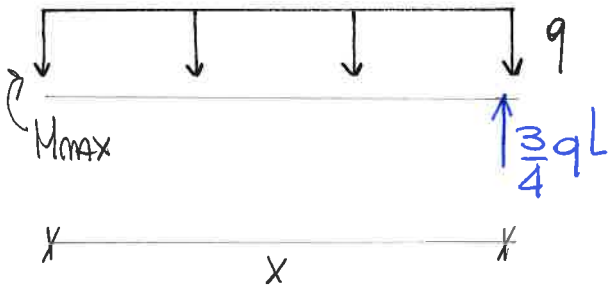
EQ: $\underset{+}{\rightarrow} = 0$

$\frac{3}{2}qL + qL - q\frac{L}{2} - qL - \frac{7}{8}qL - q\frac{L}{8} = 0$ ok!

EQ: $+\circlearrowleft_B = 0$

$qL^2 - \frac{3}{2}qLL + q2L \cdot L - \frac{3}{4}qL \cdot 2L - qL\frac{L}{2} - q\frac{L}{8} \cdot L + \frac{5}{8}qL = 0$ ok!

N MOMENTO MASSIMO

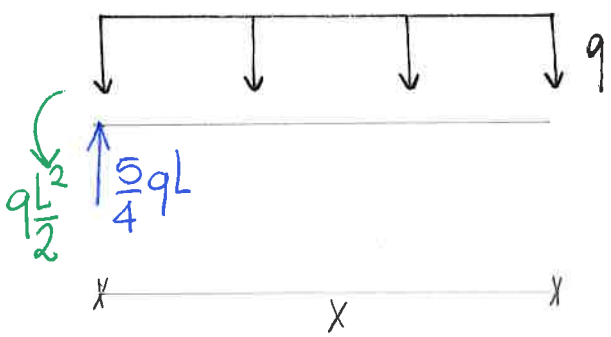


$+\frac{3}{4}qL \cdot x - q \cdot x \cdot \frac{x}{2} = M_{MAX}$

$\frac{dM_{MAX}}{dx} = +\frac{3}{4}qL - qx = 0 \longrightarrow \quad x = \frac{3}{4}L$

$M_{MAX} = +\frac{3}{4}qL \cdot \frac{3}{4}L - q \cdot \frac{9}{16}\frac{L^2}{2} = \frac{9}{32}qL^2$

N FLESSO 1

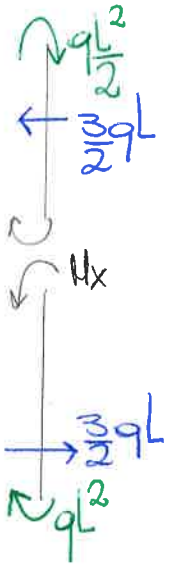


$$M = \frac{5}{4}qLx - q\frac{L^2}{2} - q\frac{x^2}{2} = 0$$

$$\frac{x^2}{2} - \frac{5}{4}xL + \frac{L^2}{2} = 0$$

$$x_{I,II} = \frac{5L \pm \sqrt{25L^2 - 16L^2}}{4} = \frac{5L \pm 3L}{4} \begin{cases} 4L \\ \frac{L}{2} \end{cases} \rightarrow x_1 = \frac{L}{2}$$

N FLESSO 2



$$y'' = -\frac{M(x)}{EJ} + 2\cdot\frac{\alpha\Delta T}{H}$$

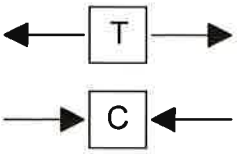
$$M(x) = -\frac{3}{2}qLx + qL^2$$

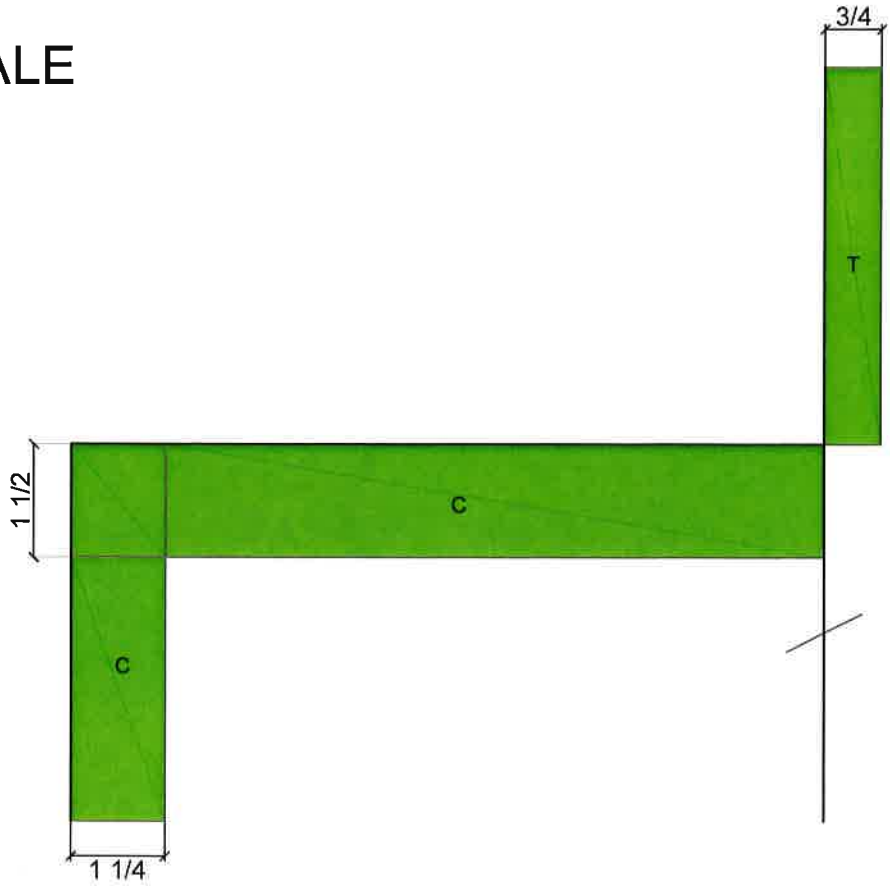
$$y'' = \frac{\frac{3}{2}qLx - qL^2}{EJ} + 2\cdot\frac{1}{2}\frac{qL^2}{EJ}$$

$$\frac{3}{2}qLx - qL^2 + qL^2 > 0 \rightarrow x > 0 \quad \forall x$$

AZIONE ASSIALE

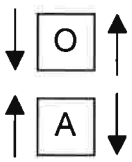
U.D.M. ql

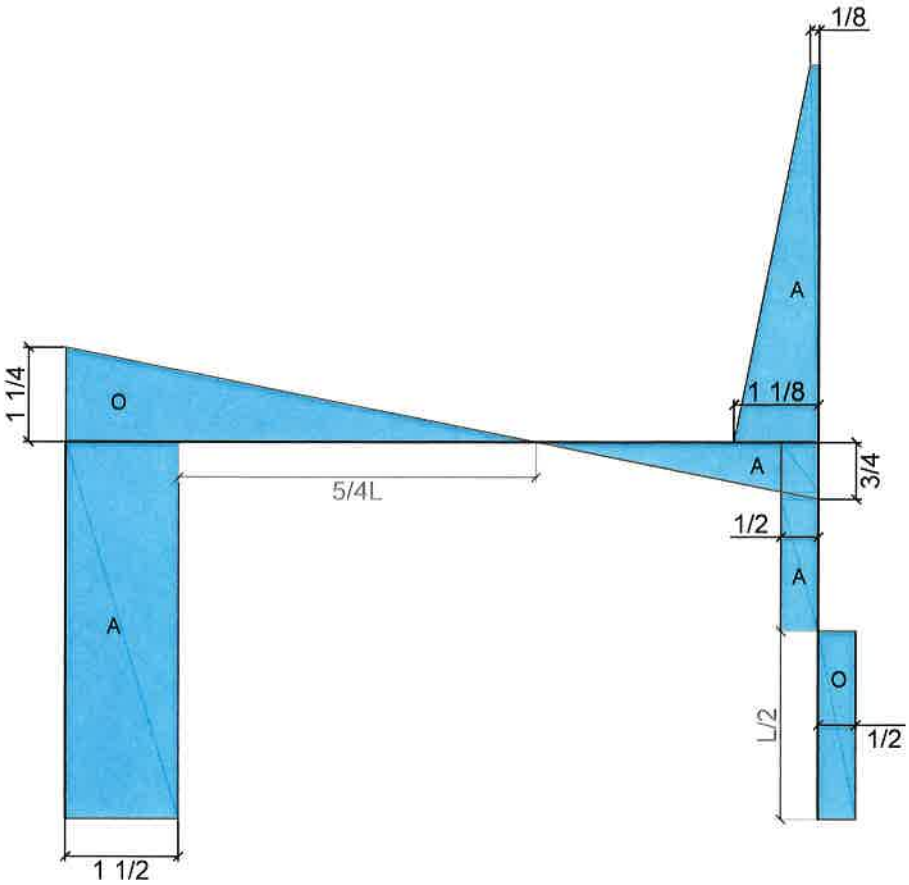




TAGLIO

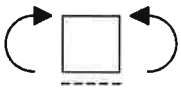
U.D.M. ql

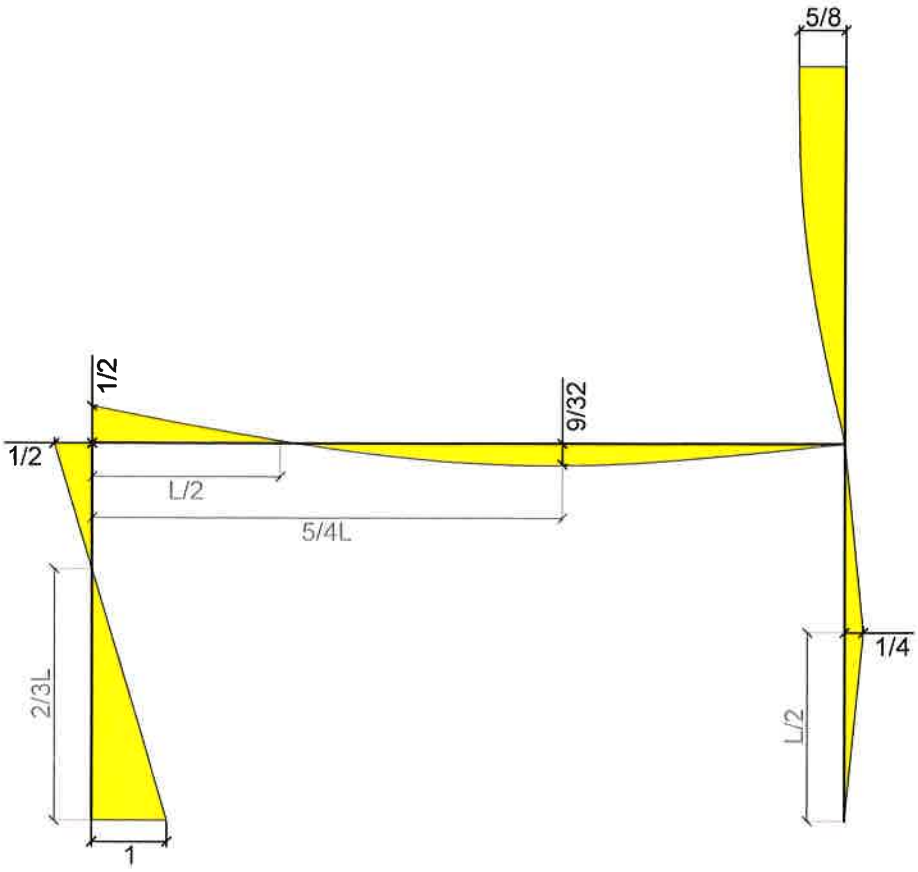




MOMENTO FLETTENTE

U.D.M. ql^2





DEFORMATA QUALITATIVA

U.D.M. Spostamenti ql^4/EJ

U.D.M. Rotazioni ql^3/EJ

U.D.M. Posizione flessi l