

# TECNICA DELLE COSTRUZIONI

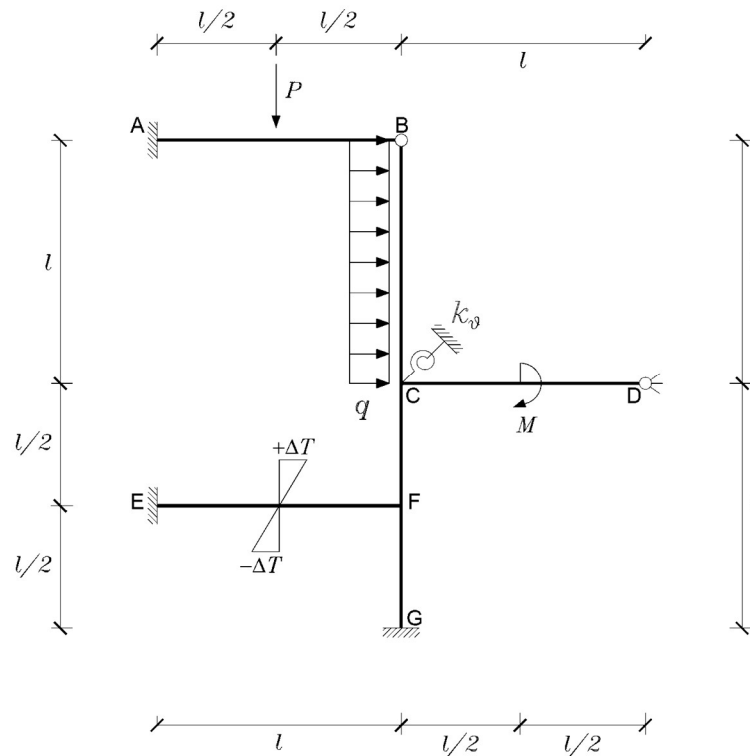
TEMA ESAME DEL 10 SETTEMBRE 2018

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PROF. FAUSTO MINELLI

ESERCITATORE: ING. LUCA FACCONI

DURATA: 2 ORE.

## Esercizio



$$\frac{\alpha \Delta T}{H} = \frac{7 q l^2}{6 E J}$$

$$k_\theta = 19 \frac{E J}{l}$$

$$M = 8 q l^2$$

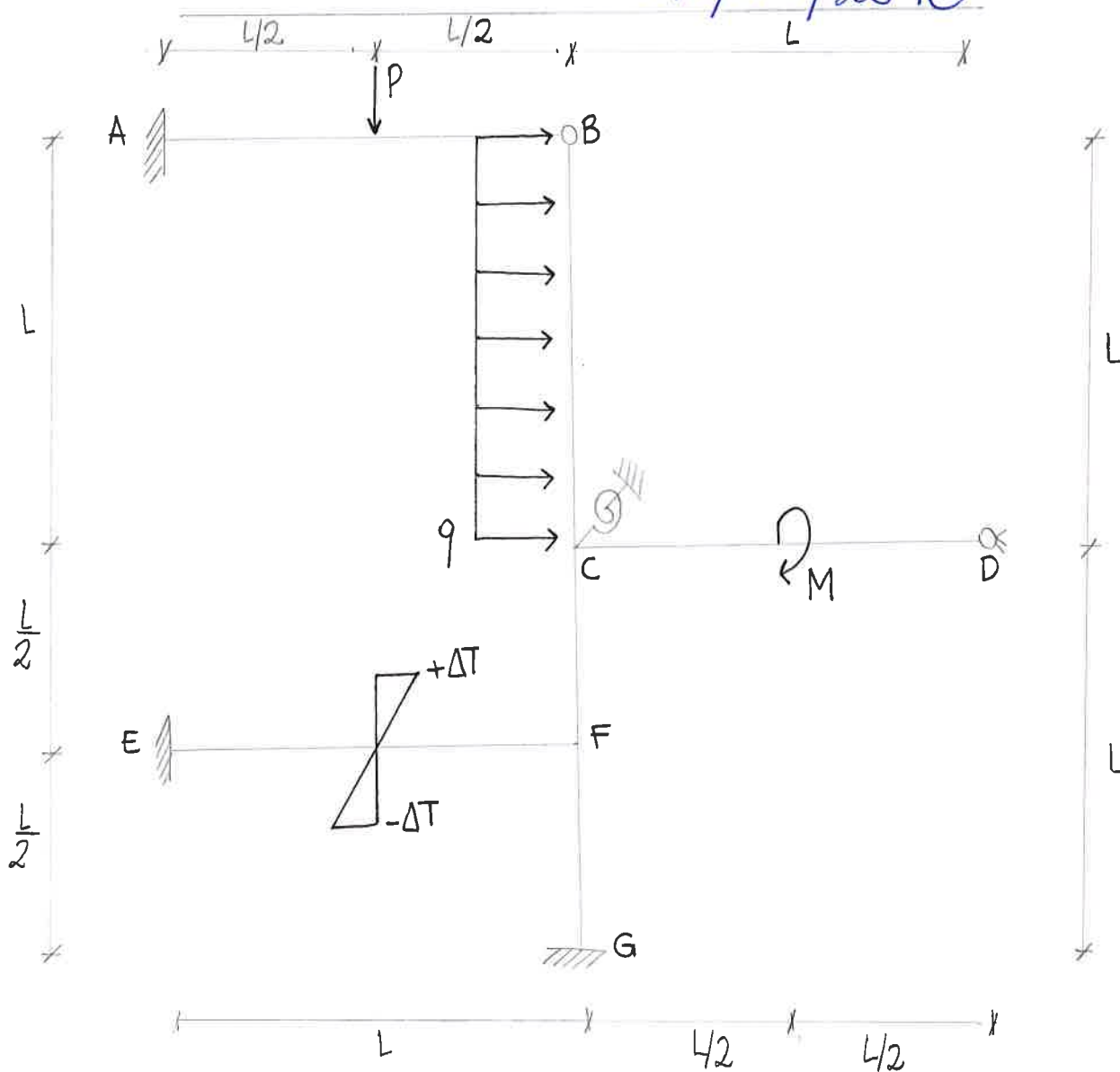
$$P = q l$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Quanto richiesto, ed i calcoli necessari per lo sviluppo del tema, vanno riportati nello stesso in maniera chiara e con tratto non cancellabile.

# TEMA ESAME 10/09/2018



$$\frac{\alpha \Delta T}{H} = \frac{7}{6} \frac{qL^2}{EJ}$$

$$k_{\theta} = 19 \frac{EJ}{L}$$

$$M = 8ql^2$$

$$P = qL$$

NSI VALUTA IL GRADO DI IPERSTATICITA' DELLA STRUTTURA

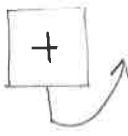
$$GdV = 14$$

$$GdL = 6$$

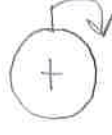
NSI OSSERVA CHE IL TELAILO È A NODI FISSI

NSI USA IL METODO DEGLI SPOSTAMENTI PER LA RIDUZIONE

N CONVENZIONE DI SEGNO



MOMENTI AL NODO

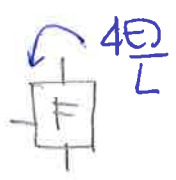
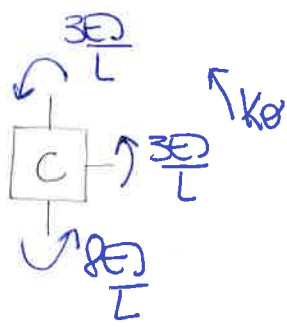
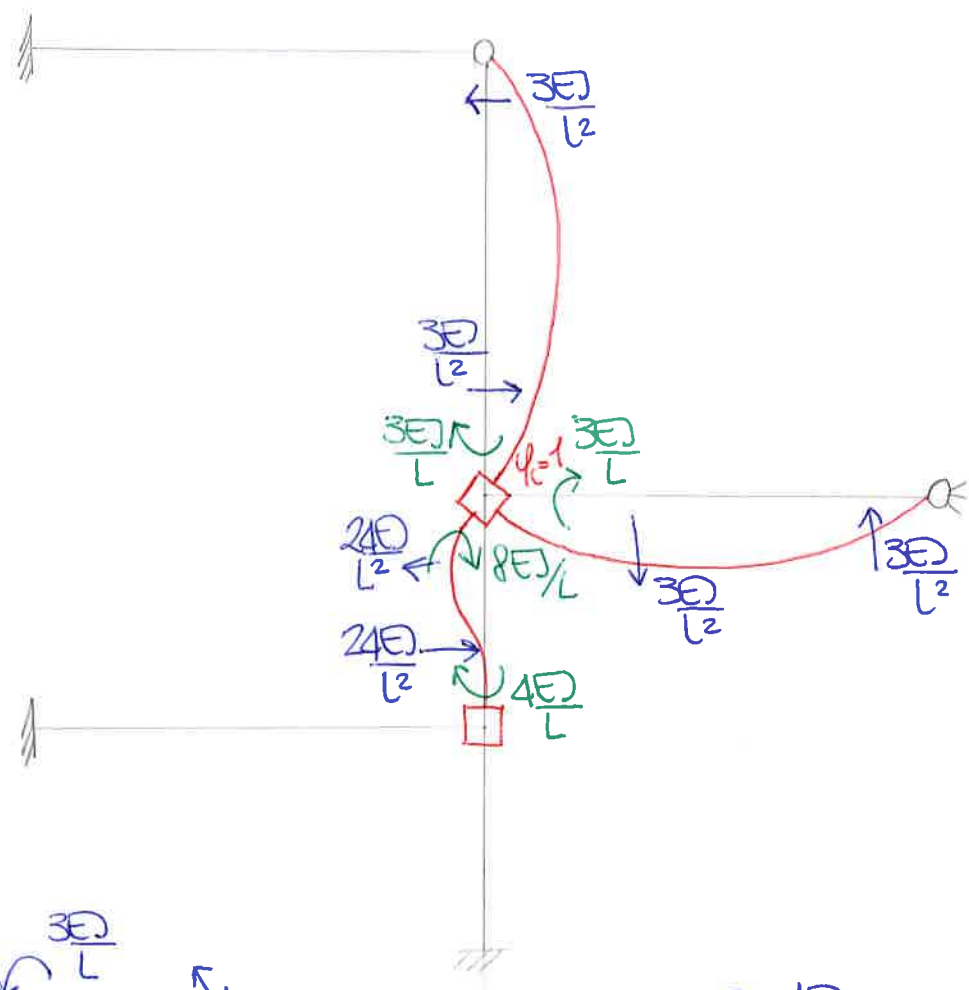


ROTAZIONI

N SISTEMA RISOLVENTE

$$\begin{cases} M_{cc} \varphi_c + M_{cf} \varphi_f + M_{co} = 0 \\ M_{fc} \varphi_c + M_{ff} \varphi_f + M_{fo} = 0 \end{cases}$$

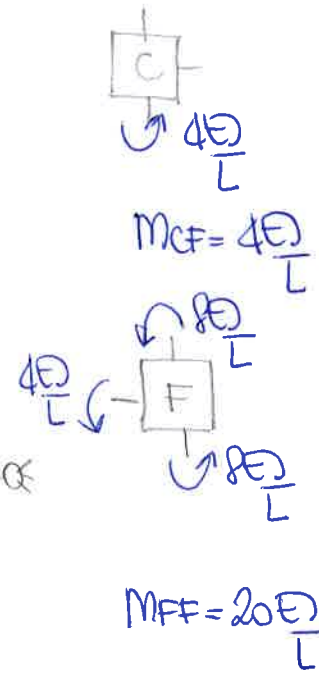
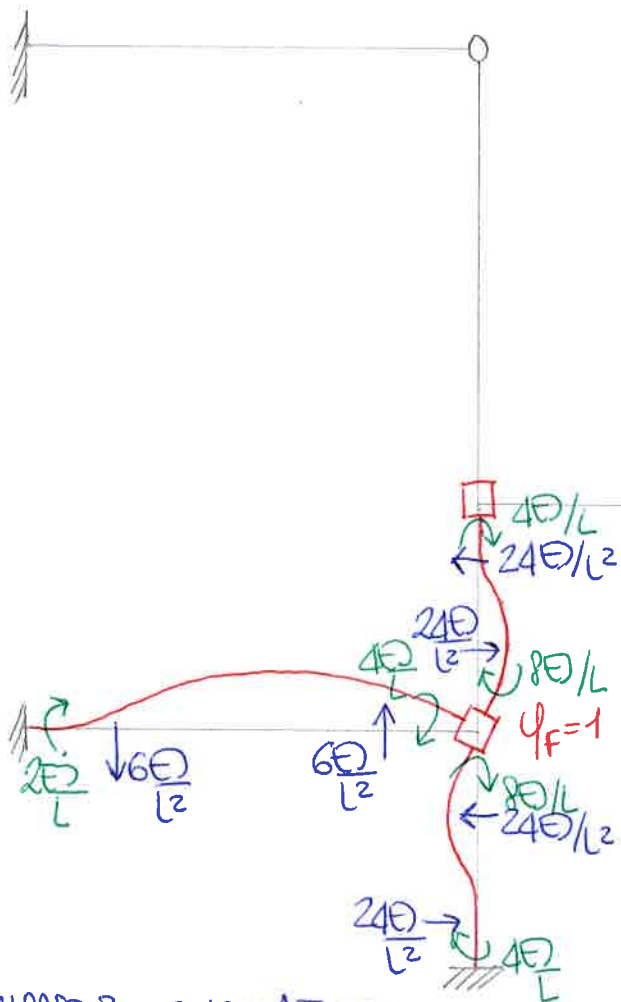
NGARO 1:  $\varphi_c = 1$



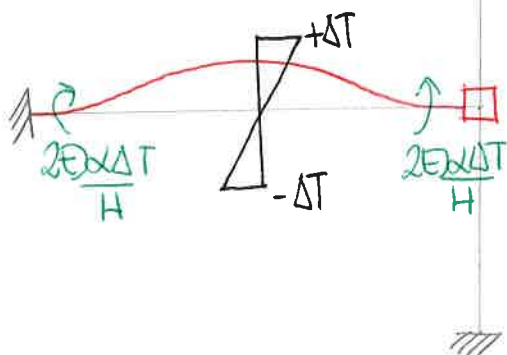
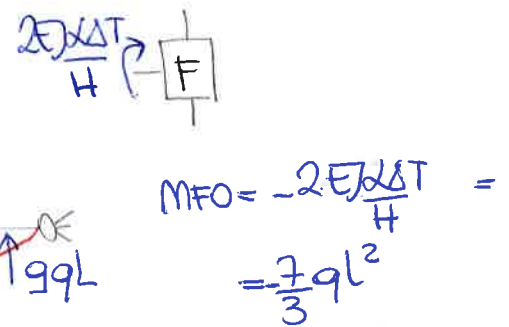
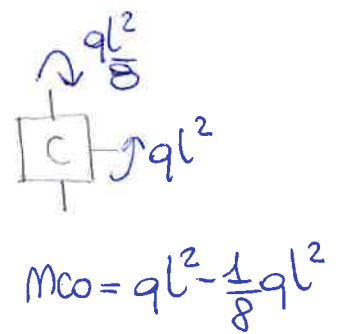
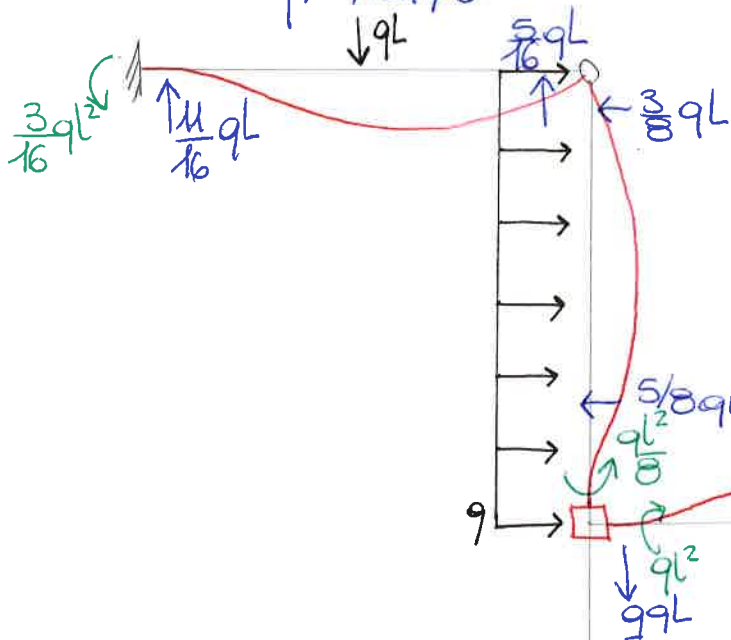
$$M_{cc} = 14 \frac{EJ}{L} + 19 \frac{EJ}{L} = 33 \frac{EJ}{L}$$

$$M_{fc} = 4 \frac{EJ}{L}$$

NCASO 2:  $\psi_F = 1$



NCASO 3:  $q \neq 0; \Delta T \neq 0$



IL SISTEMA RIDOLLENTE

$$\begin{cases} 33 \frac{E}{L} \varphi_c + 4 \frac{E}{L} \varphi_f + ql^2 - \frac{1}{8} ql^2 = 0 & (1) \end{cases}$$

$$\begin{cases} 4 \frac{E}{L} \varphi_c + 20 \frac{E}{L} \varphi_f - \frac{7}{3} ql^2 = 0 & (2) \end{cases}$$

MOLTIPLICO (2) PER  $-\frac{1}{5}$  E LA SOMMO A (1)

$$4 \frac{E}{L} \cdot \left(-\frac{1}{5}\right) \varphi_c + 20 \frac{E}{L} \cdot \left(-\frac{1}{5}\right) \varphi_f - \frac{7}{3} ql^2 \cdot \left(-\frac{1}{5}\right) = 0$$

$$-\frac{4}{5} \frac{E}{L} \varphi_c - 4 \frac{E}{L} \varphi_f + \frac{7}{15} ql^2 = 0$$

(+)

$$33 \frac{E}{L} \varphi_c + 4 \frac{E}{L} \varphi_f + ql^2 - \frac{1}{8} ql^2 = 0$$

$$\left(33 \frac{E}{L} - \frac{4}{5} \frac{E}{L}\right) \varphi_c + \left(4 \frac{E}{L} - 4 \frac{E}{L}\right) \varphi_f + \left(ql^2 - \frac{1}{8} ql^2 + \frac{7}{15} ql^2\right) = 0$$

$$\frac{161}{5} \frac{E}{L} \varphi_c = -\frac{161}{120} ql^2$$

$$\varphi_c = -\frac{1}{24} \frac{ql^3}{E}$$

SOSTITUISCO  $\varphi_c$  IN (2)

$$4 \frac{E}{L} \left(-\frac{1}{24} \frac{ql^3}{E}\right) + 20 \frac{E}{L} \varphi_f - \frac{7}{3} ql^2 = 0$$

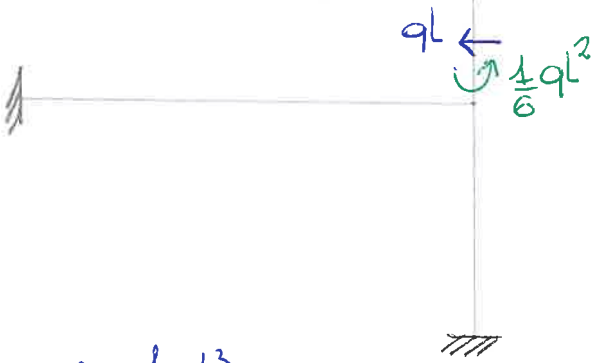
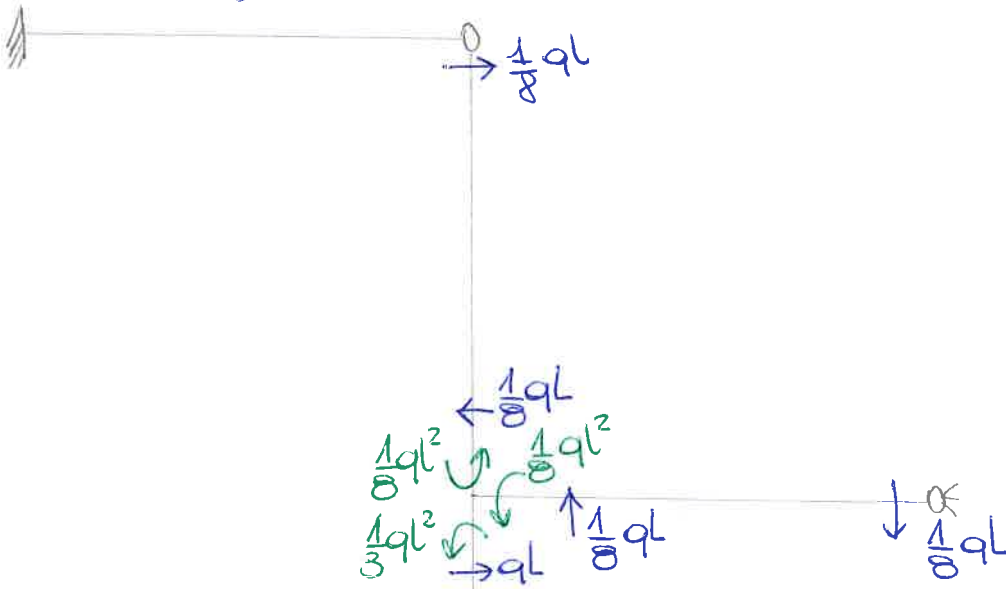
$$20 \frac{E}{L} \varphi_f = \frac{7}{3} ql^2 + \frac{1}{6} ql^2$$

$$20 \frac{E}{L} \varphi_f = \frac{15}{6} ql^2$$

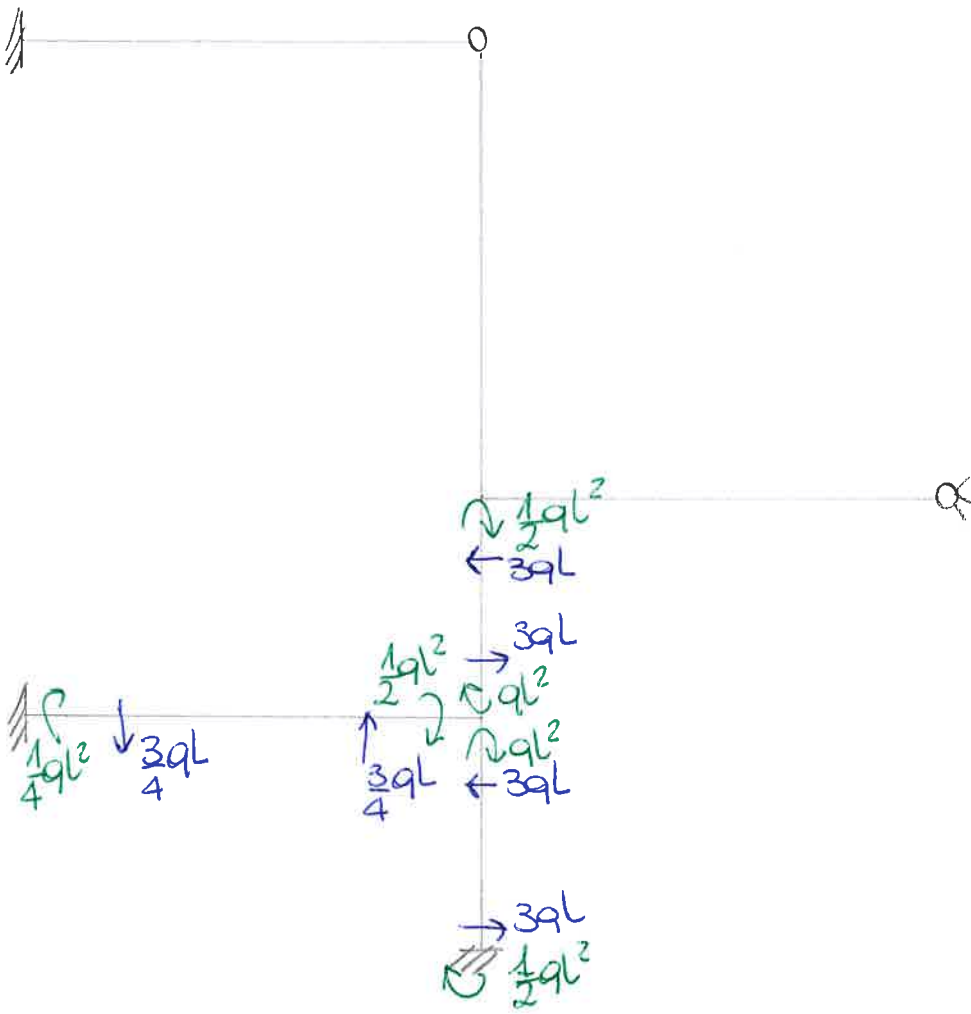
$$\varphi_f = \frac{1}{8} \frac{ql^3}{E}$$

$$\begin{cases} \varphi_c = -\frac{1}{24} \frac{ql^3}{E} \\ \varphi_f = \frac{1}{8} \frac{ql^3}{E} \end{cases}$$

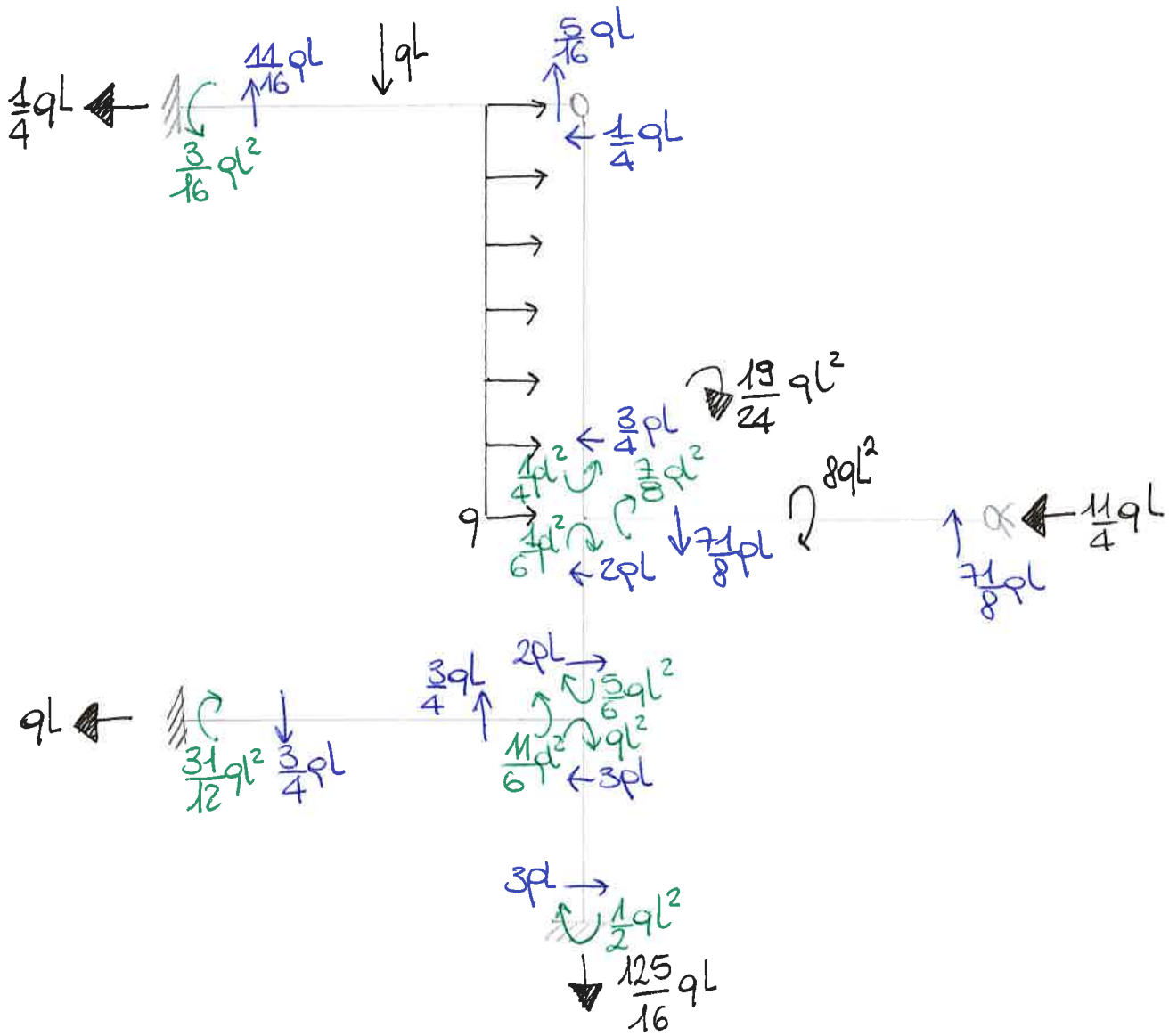
$$\psi_c = -\frac{1}{24} \frac{ql^3}{EI}$$



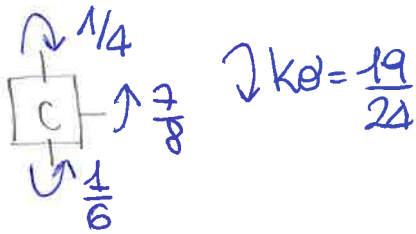
$$\psi_F = \frac{1}{8} \frac{ql^3}{EI}$$



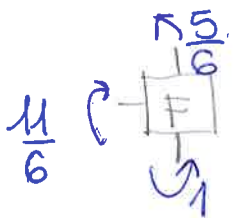
# NAZIONI INTERNE



EQUILIBRIO AL NODO



$$\frac{1}{6}qL^2 + \frac{7}{8}qL^2 - \frac{1}{4}pL^2 - \frac{19}{24}pL^2 = 0 \quad \text{ok!}$$



$$qL^2 + \frac{5}{6}qL^2 - \frac{11}{6}qL^2 = 0 \quad \text{ok!}$$

$$\text{EQ } \uparrow + = 0$$

$$\frac{11}{16} qL - qL - \frac{3}{4} qL + \frac{71}{8} qL - \frac{125}{16} qL = 0 \quad \text{ok!}$$

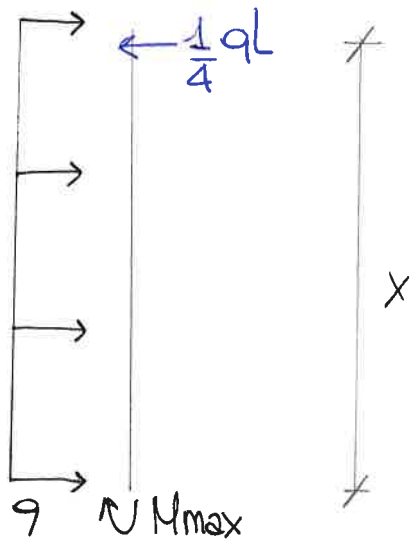
$$\text{EQ } \rightarrow = 0$$

$$-\frac{1}{4} qL + qL - \frac{11}{4} qL - qL + 3qL = 0 \quad \text{ok!}$$

$$\text{EQ } \uparrow_G = 0$$

$$\frac{1}{2} qL^2 - \frac{3}{4} qL \cdot L + \frac{31}{12} qL^2 - qL \cdot \frac{L}{2} + 8qL^2 - \frac{71}{8} qL \cdot L - \frac{11}{4} qL + qL \cdot \frac{3}{2} L - qL \cdot \frac{L}{2} - \frac{3}{16} qL + \frac{11}{16} qL - \frac{1}{4} qL \cdot 2L + \frac{19}{24} qL^2 = 0 \quad \text{ok!}$$

N MOMENTO MASSIMO



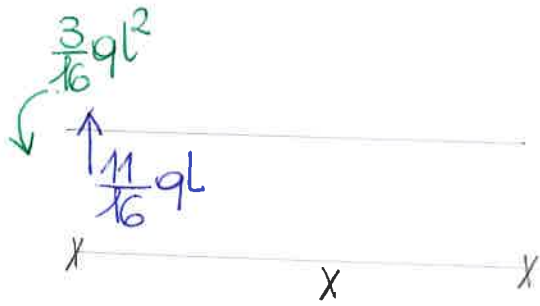
$$M_{\text{MAX}} = \frac{1}{4} qL \cdot x - qx \cdot \frac{x}{2}$$

$$\frac{dM_{\text{MAX}}}{dx} = \frac{1}{4} qL - qx = 0 \rightarrow x = \frac{1}{4} L$$

$$M_{\text{MAX}}(x = \frac{1}{4} L) = \frac{1}{4} qL \cdot \frac{1}{4} L - q \cdot \left(\frac{1}{4} L\right)^2 \cdot \frac{1}{2} = \frac{1}{32} qL^2$$



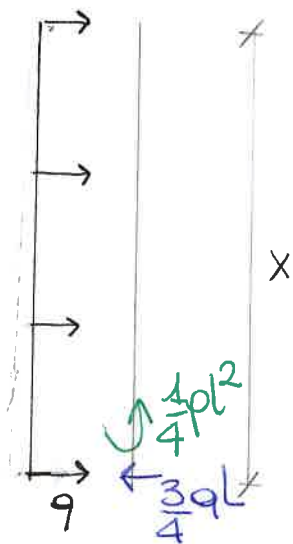
N FUESO 1. ASTA AB



$$M = -\frac{11}{16} qlx + \frac{3}{16} ql^2 = 0$$

$$x_1 = \frac{3}{11} \approx 0,27L$$

N FUESO 2. ASTA BC



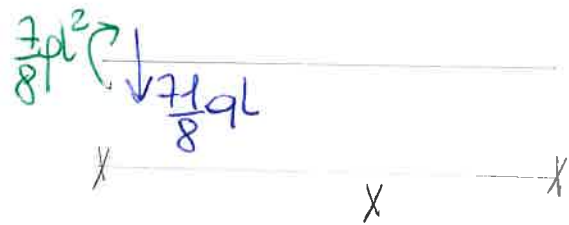
$$M = \frac{3}{4} qlx - \frac{1}{4} ql^2 - 9 \frac{x^2}{2} = 0$$

$$-2x^2 + 3x - 1l = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{-4} \begin{cases} \frac{1}{2}L = 0,5L \\ L \end{cases}$$

$$\rightarrow x_2 = 0,5L$$

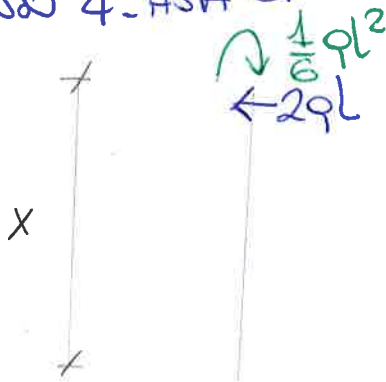
N FLESSO 3-ASTA CD



$$M = \frac{7}{8} ql^2 - \frac{7}{8} qlx = 0$$

$$x_3 = \frac{7}{71} L \approx 0,1L$$

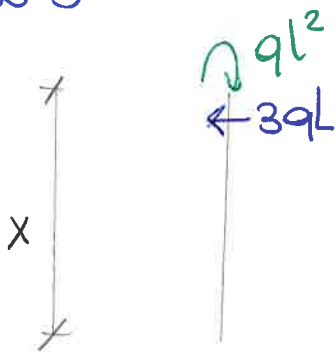
N FLESSO 4-ASTA CF



$$M = \frac{1}{6} ql^2 - \frac{1}{2} qlx = 0$$

$$x_4 = \frac{1}{12} L \approx 0,08L$$

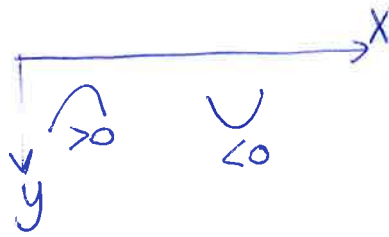
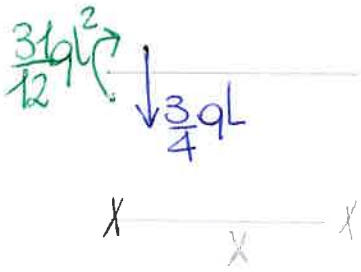
N FLESSO 5-ASTA FG



$$M = ql^2 - 3qlx = 0$$

$$x_5 = \frac{1}{3} L \approx 0,33L$$

# N DEFORMATA TERMICA



$$y'' = -\frac{M(x)}{EJ} + 2\frac{\alpha\Delta T}{H}$$

$$M(x) = \frac{31}{12}ql^2 - \frac{3}{4}qlx$$

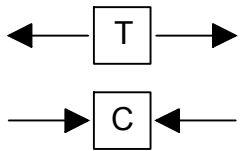
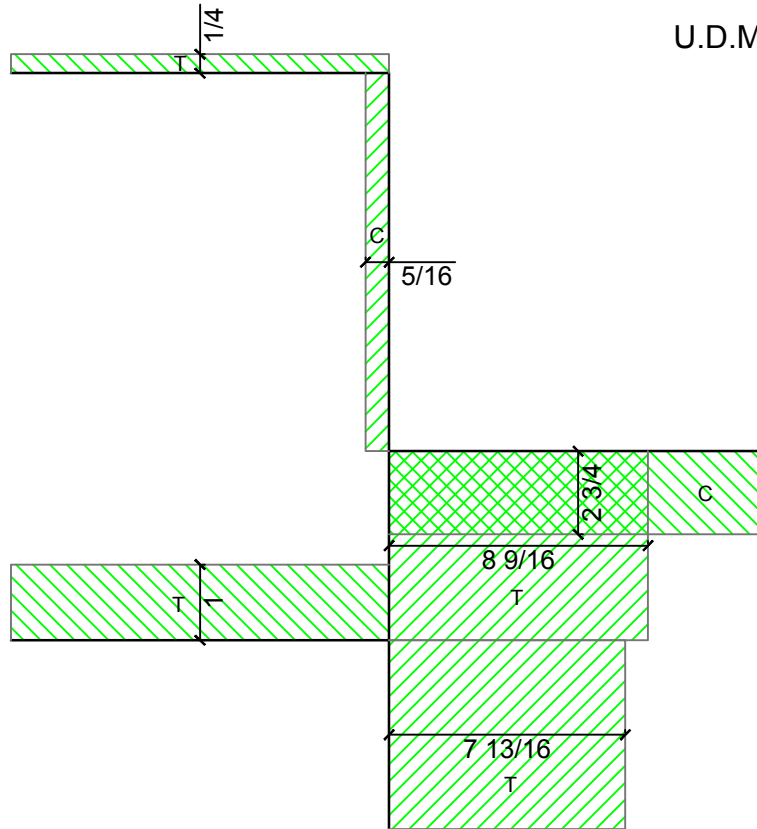
$$y' = \frac{\frac{3}{4}qlx - \frac{31}{12}ql^2}{EJ} + 2 \cdot \frac{7}{6} \frac{ql^2}{EJ} = 0$$

$$\frac{3}{4}x > \frac{31}{12} - \frac{7}{3}$$

$$x > \frac{1}{3}$$

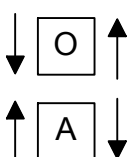
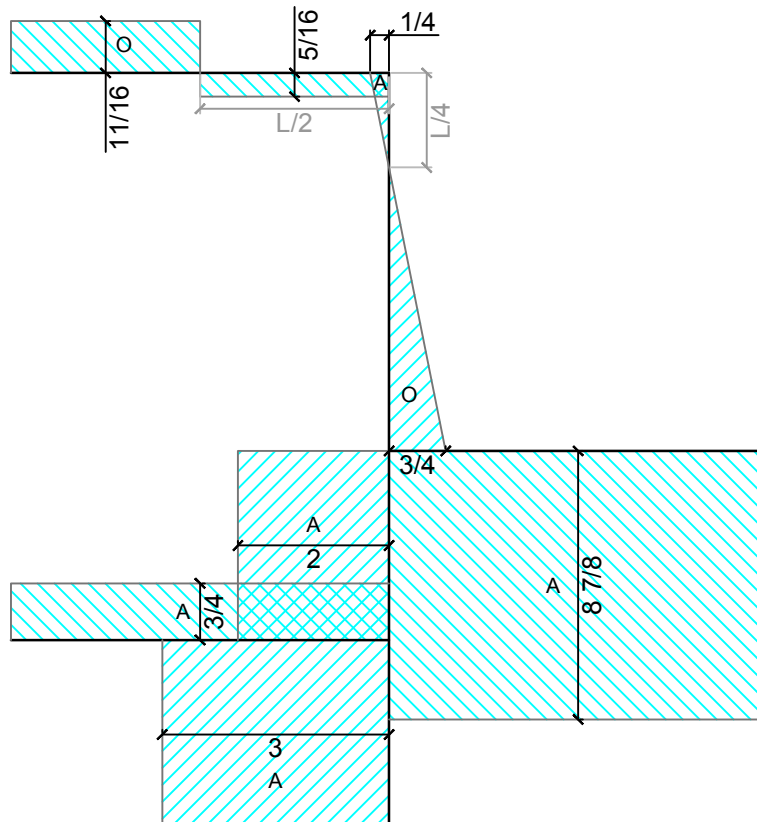
# AZIONE ASSIALE

U.D.M. ql (Non in scala)



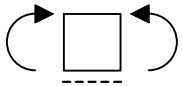
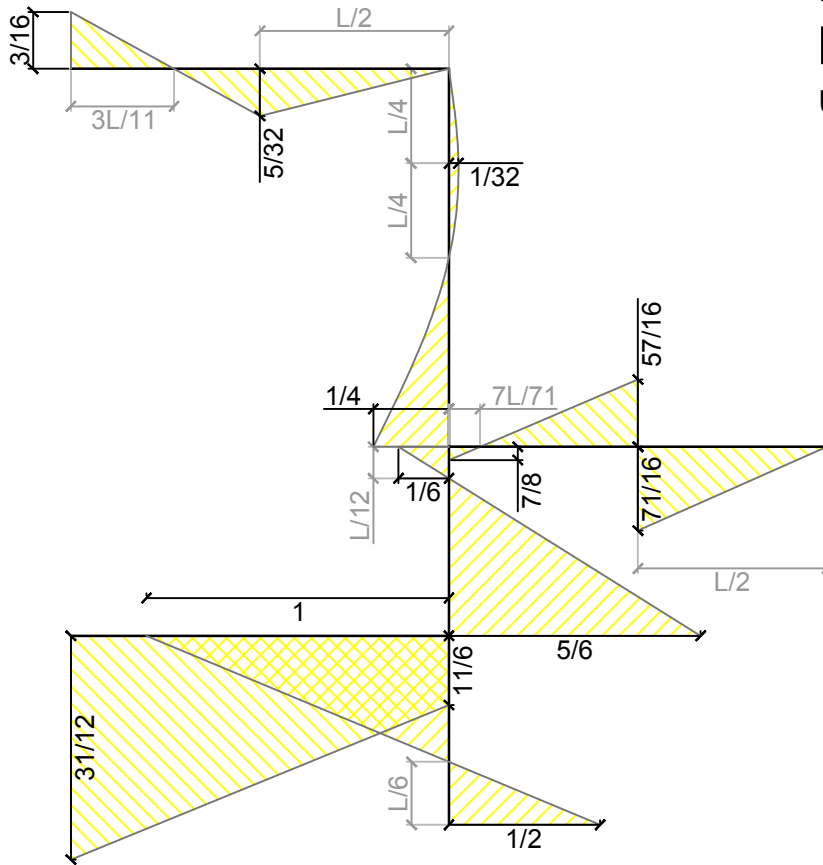
# TAGLIO

U.D.M. ql (Non in scala)



# MOMENTO FLETTENTE

U.D.M.  $ql^2$  (Non in scala)



# DEFORMATA QUALITATIVA

U.D.M. Spostamenti  $ql^4/EJ$   
 U.D.M. Rotazioni  $ql^3/EJ$   
 U.D.M. Posizione flessi I

