

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 11 FEBBRAIO 2019

DOCENTI:

PROF. GIOVANNI PLIZZARI

PROF. FAUSTO MINELLI

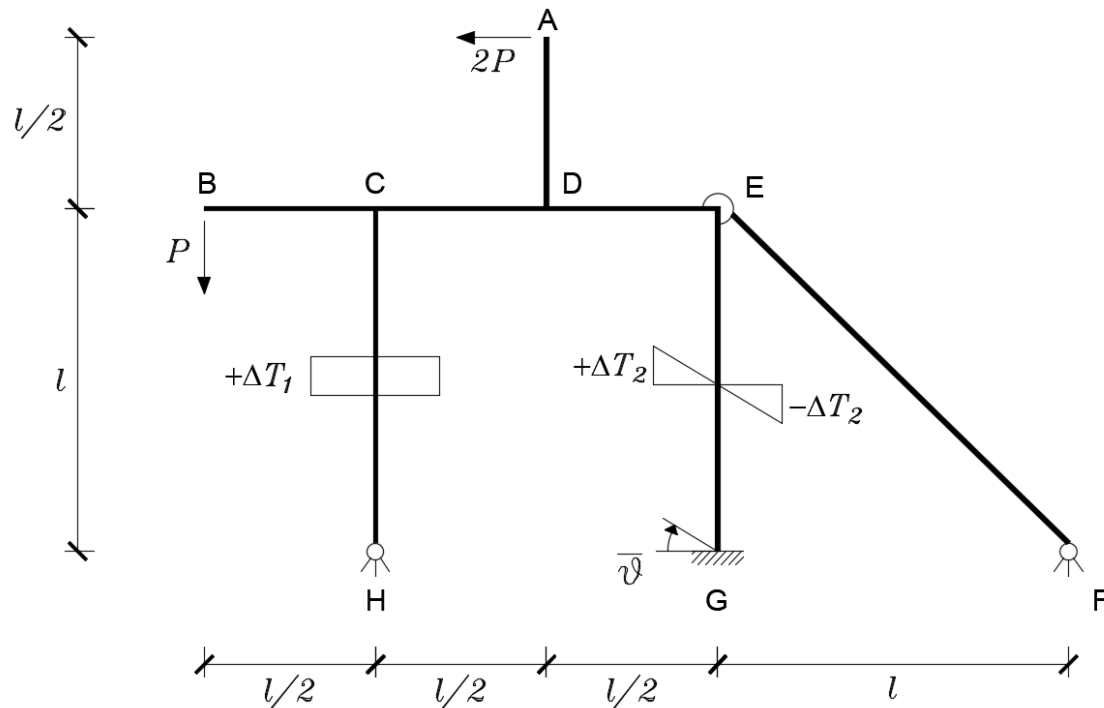
ESERCITATORI:

ING. LUCA FACCONI

ING. FRANCESCA FEROLDI

DURATA: 2 ORE.

Esercizio



$$\alpha \Delta T_1 = \frac{7 ql^3}{18 EJ}$$

$$\frac{\alpha \Delta T_2}{H} = \frac{1 ql^2}{2 EJ}$$

$$P = ql$$

$$\bar{\vartheta} = \frac{7 ql^3}{8 EJ}$$

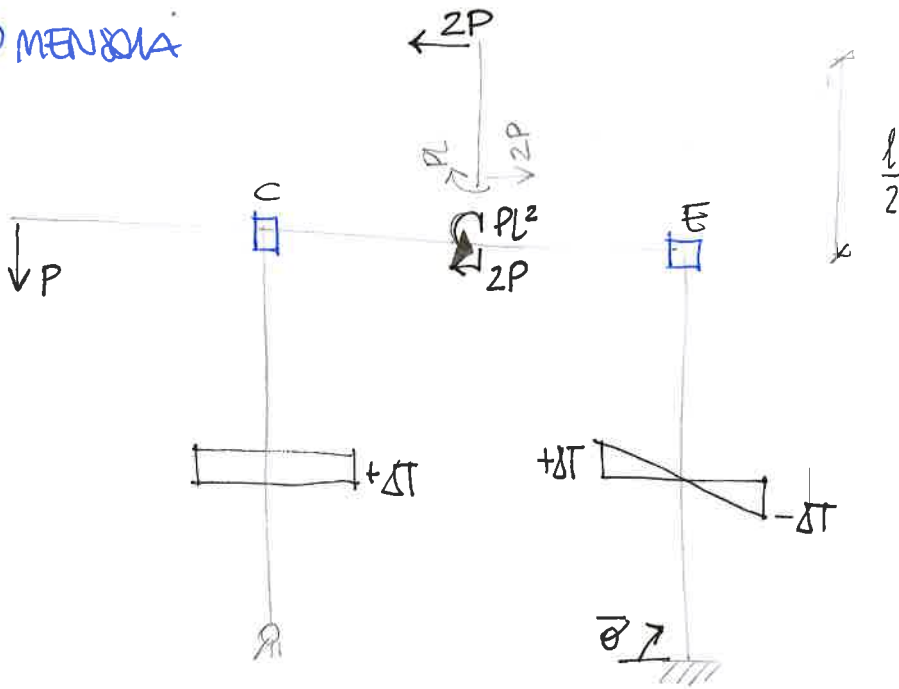
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

Nodi Fissi
BLOCCHE IN C ED E
 EF BIELLA

AD MENZOLA

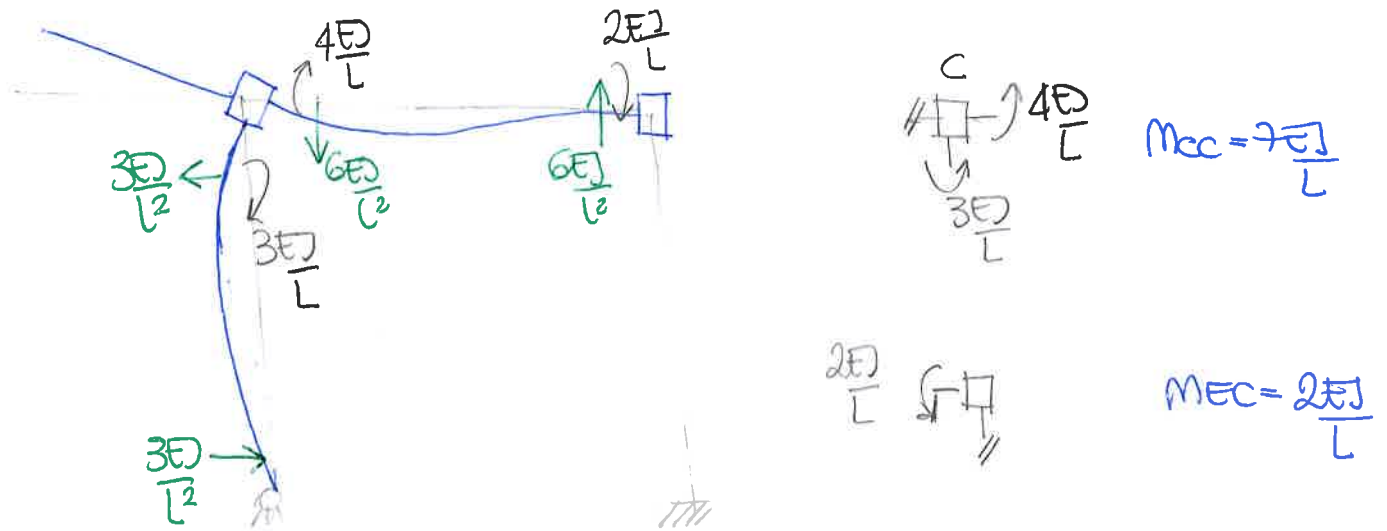


$l/2$

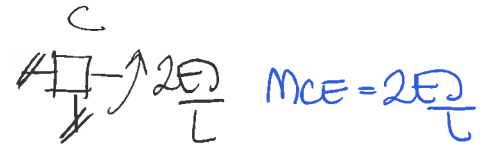
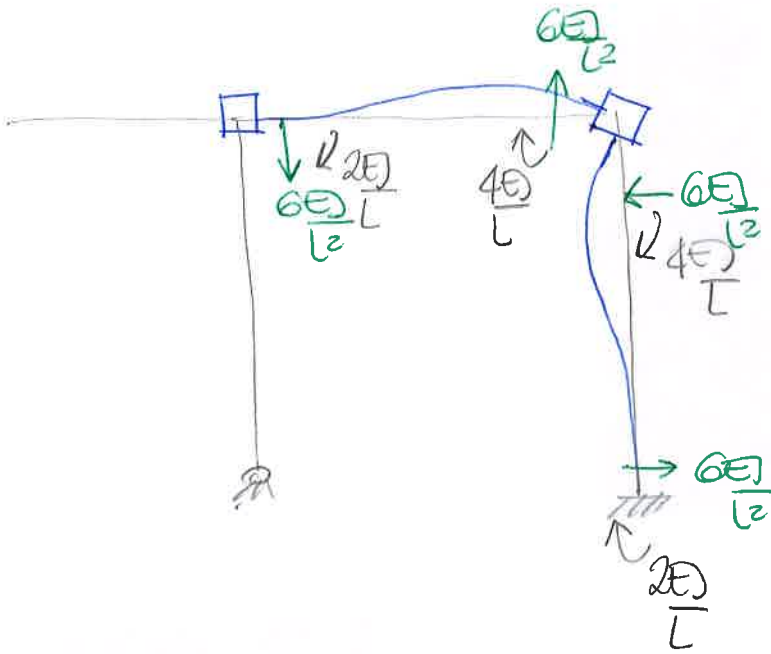
$$M_{CC} \psi_C + M_{CE} \psi_E + M_{CO} = 0$$

$$M_{EC} \psi_C + M_{EE} \psi_E + M_{EO} = 0$$

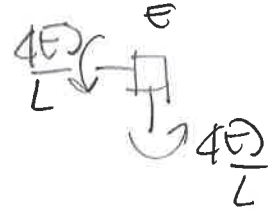
CASO 1: $\psi_C = 1$



CAD 2 $\varphi_E = 1$



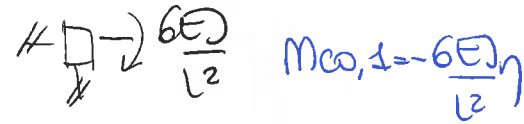
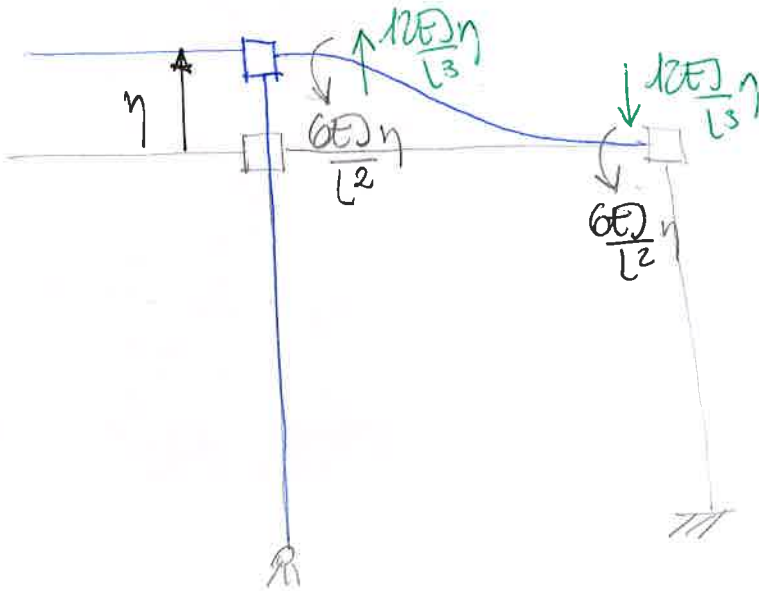
$$M_{CE} = 2EJ/L$$



$$M_{CE} = 8EJ/L$$

CAD 3 $\Delta T_1 \neq 0$

$$\Delta T_1 = \frac{7}{18} \frac{qL^3}{EJ}$$



$$M_{CO,1} = -\frac{6EJ}{L^2} \eta$$



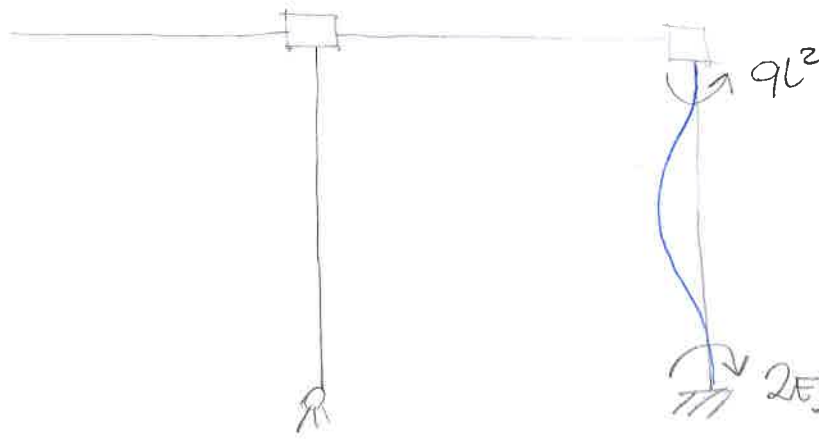
$$M_{CO,1} = -\frac{6EJ}{L^2} \eta$$

$$\eta = \alpha \Delta T_1 L = \frac{7}{18} \frac{qL^3}{EJ} \cdot L = \frac{7}{18} \frac{qL^4}{EJ}$$

$$M_{CO,1} = -\frac{6EJ}{L^2} \cdot \frac{7}{18} \frac{qL^4}{EJ} = -\frac{7}{3} qL^2$$

$$M_{EO,1} = -\frac{4}{3} qL^2$$

CASO 4 $\Delta T_2 \neq 0$

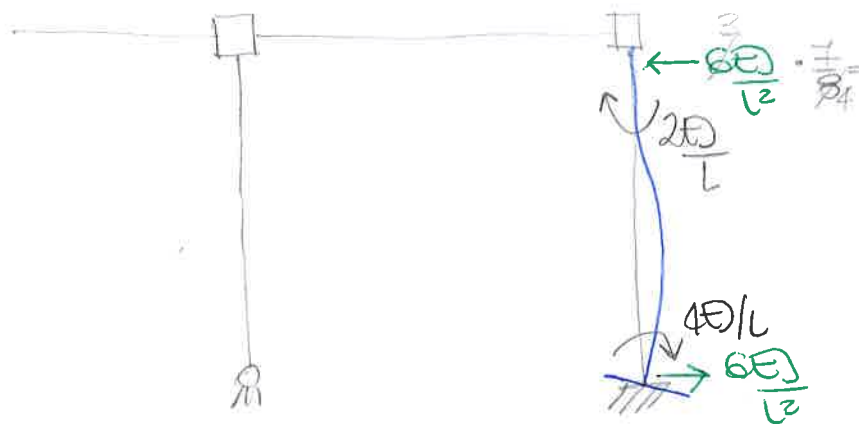


$M_{C0,2} = 0$

$M_{E0,2} = ql^2$

$\frac{2EI \Delta T}{H} = \frac{2EI}{H} \cdot \frac{1}{2} q l^2 = ql^2$

CASO 5: $\bar{\theta} \neq 0$

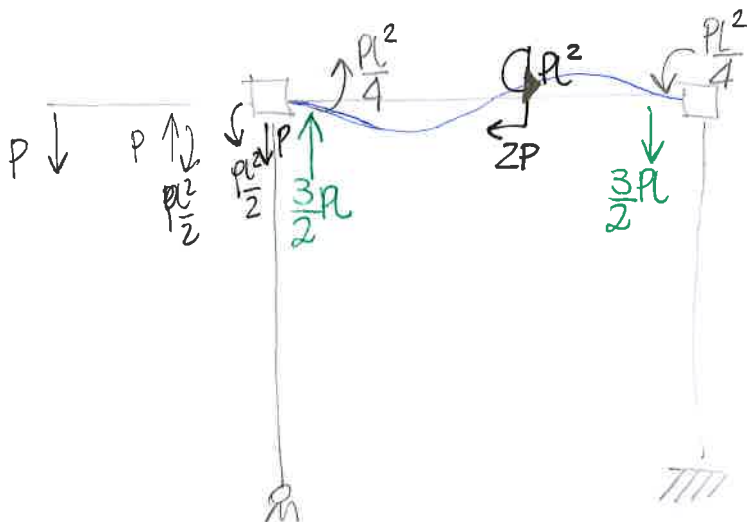


$M_{C0,3} = 0$

$2EI \bar{\theta}$

$M_{E0,3} = 2EI \bar{\theta} = \frac{7}{4} ql^2$

CASO 6 $P \neq 0$



$M_{C0,4} = -\frac{Pl^2}{4} + \frac{ql^2}{2} = -\frac{Pl^2}{4} + \frac{1}{4} ql^2$

$M_{E0,4} = -\frac{Pl^2}{4}$

$$\begin{cases} 7 \frac{EI}{L} \varphi_c + 2 \frac{EI}{L} \varphi_E - \frac{7}{3} pL^2 + \frac{1}{4} pL^2 = 0 \\ 2 \frac{EI}{L} \varphi_c + 8 \frac{EI}{L} \varphi_E - \frac{7}{3} qL^2 - pL^2 + \frac{7}{4} qL^2 - \frac{pL^2}{4} = 0 \end{cases}$$

$\underbrace{\quad}_{-25/12}$
 $\underbrace{\quad}_{-11/6}$

$$\begin{cases} 7 \frac{EI}{L} \varphi_c + 2 \frac{EI}{L} \varphi_E - \frac{25}{12} qL^2 = 0 \\ 2 \frac{EI}{L} \varphi_c + 8 \frac{EI}{L} \varphi_E - \frac{11}{6} qL^2 = 0 \end{cases}$$

moltiplica (2) per $\frac{1}{4}$

$$\frac{1}{2} \frac{EI}{L} \varphi_c + \frac{1}{2} \frac{EI}{L} \varphi_E - \frac{11}{24} qL^2 = 0$$

$$\frac{1}{2} \frac{EI}{L} \varphi_c + 2 \frac{EI}{L} \varphi_E - \frac{11}{24} qL^2 = 0$$

(1) - (2)

$$(7 - \frac{1}{2}) \frac{EI}{L} \varphi_c + (2 - 2) \frac{EI}{L} \varphi_E + (-\frac{25}{12} + \frac{11}{24}) pL^2 = 0$$

$$\frac{14-1}{2} \frac{EI}{L} \varphi_c + \frac{-50+11}{24} qL^2 = 0$$

$$\frac{13}{2} \frac{EI}{L} \varphi_c - \frac{39}{24} qL^2 = 0$$

$$\varphi_c = \frac{\frac{39}{24} qL^2 \cdot \frac{2}{13} L}{\frac{13}{2} EI} = \boxed{\frac{1}{4} \frac{qL^3}{EI}} \varphi_c$$

sostituisco φ_c in (2)

$$8 \frac{EI}{L} \cdot \frac{1}{4} \frac{ql^3}{EI} + 8 \frac{EI}{L} \varphi_E - \frac{11}{6} ql^2 = 0$$

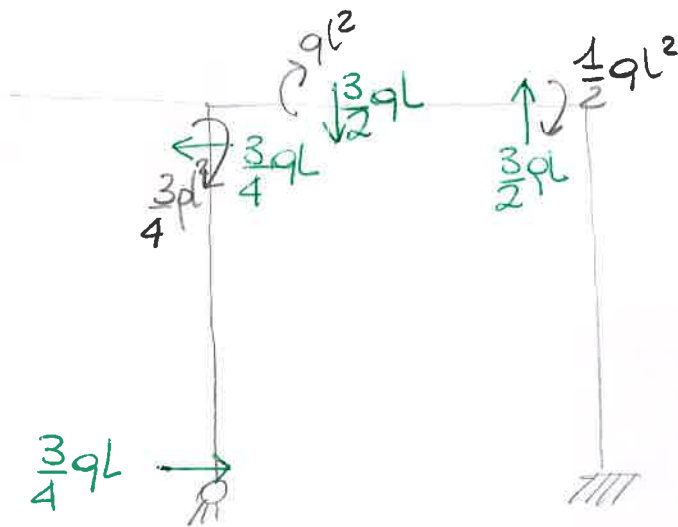
$$\frac{1}{2} ql^2 + 8 \frac{EI}{L} \varphi_E - \frac{11}{6} ql^2 = 0$$

$$\frac{8EI}{L} \varphi_E = \frac{11}{6} - \frac{1}{2} ql^2$$

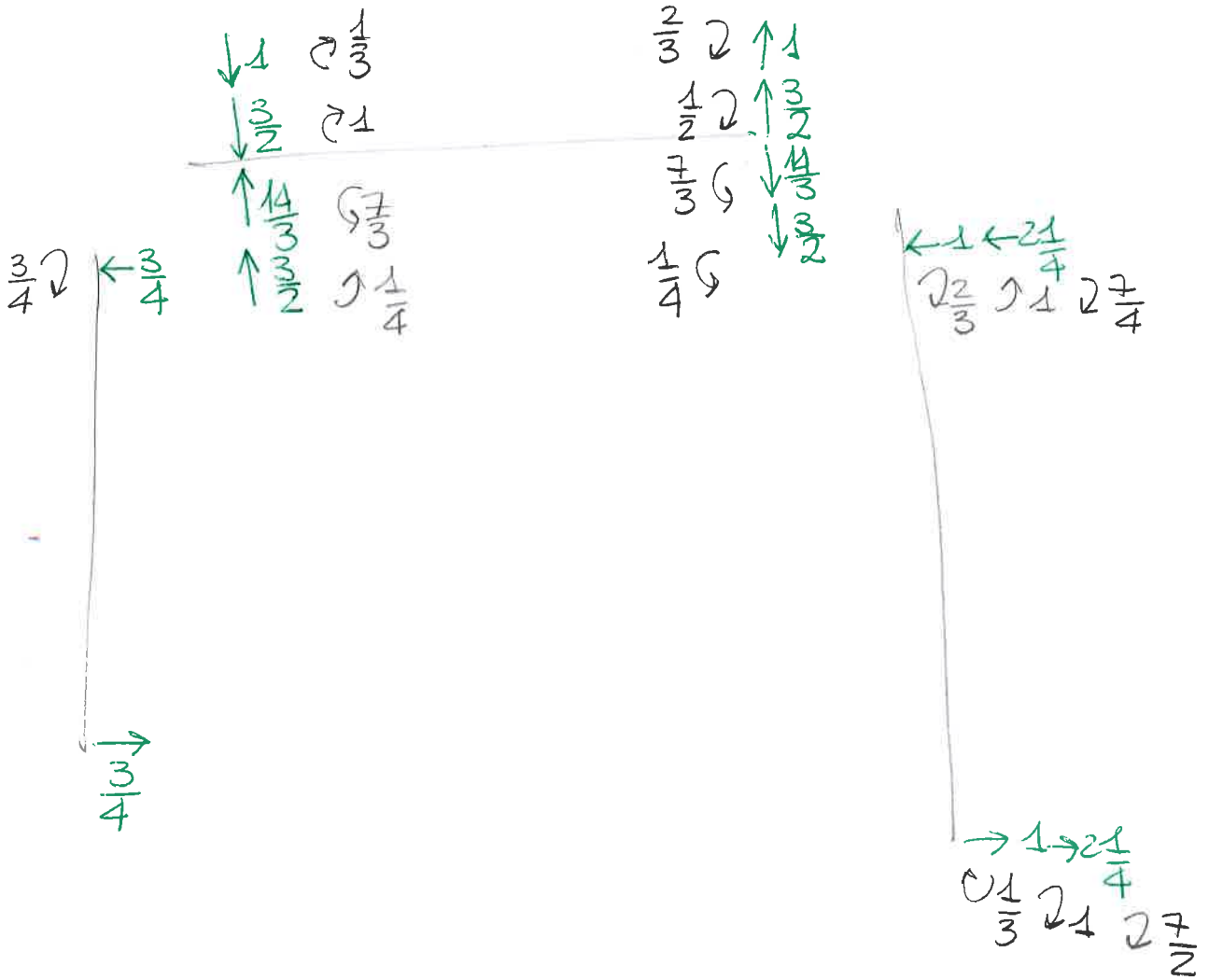
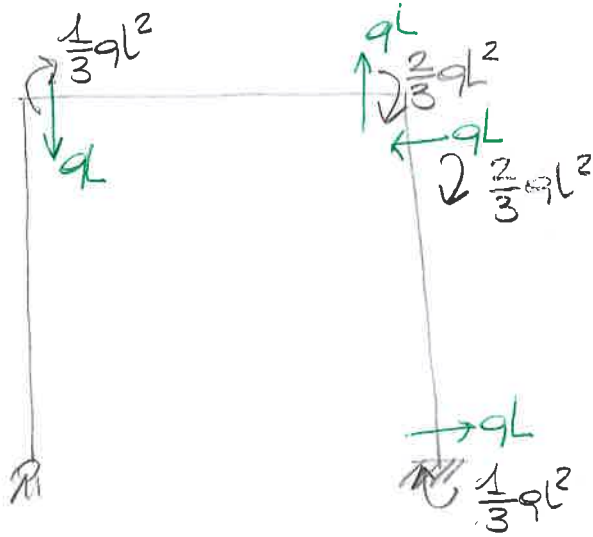
$$\frac{8EI}{L} \varphi_E = \frac{11-3}{6} ql^2$$

$$\varphi_E = \frac{8}{6} ql^2 \cdot \frac{L}{8EI} = \boxed{\frac{1}{6} \frac{ql^3}{EI}}$$

$$\varphi_c = \frac{1}{4} \frac{ql^3}{EI}$$

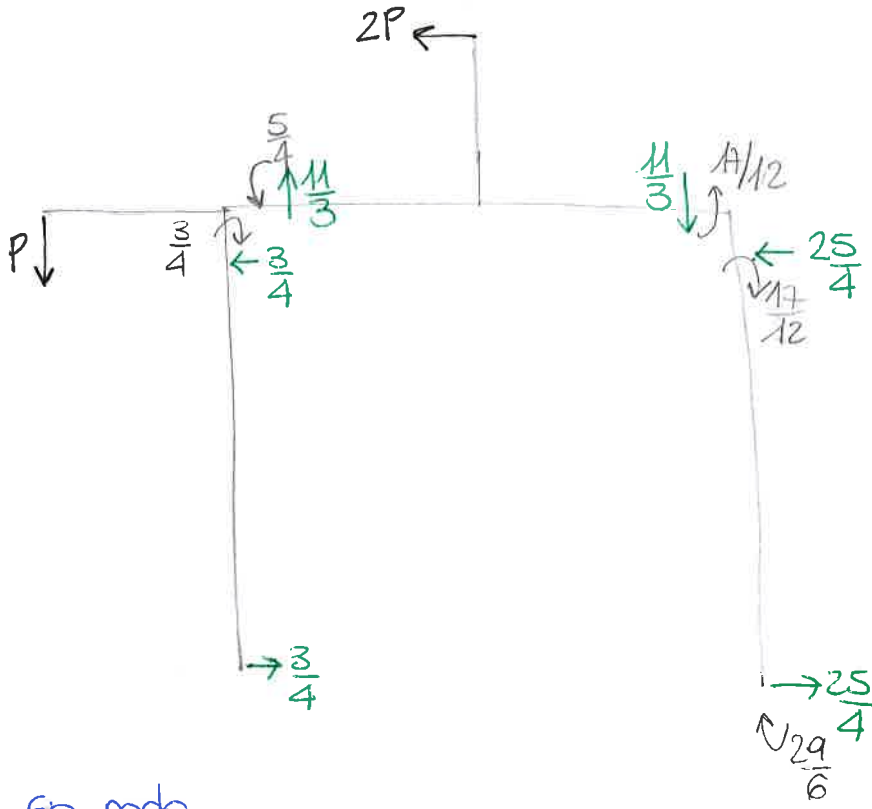


$$q_E = \frac{1}{6} q l^3 / E$$

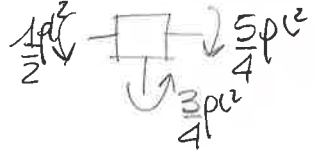


$$\frac{4}{6} - 1 + \frac{14}{8} = \frac{16 - 24 + 42}{24} = \frac{34}{24} = \frac{17}{12}$$

$$\frac{2}{4} + \frac{4}{6} - \frac{7}{3} - \frac{1}{4} = \frac{12 + 16 - 56 - 6}{24} = \frac{-34}{24} = -\frac{17}{12}$$

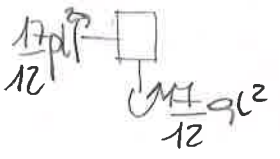


Eq node C



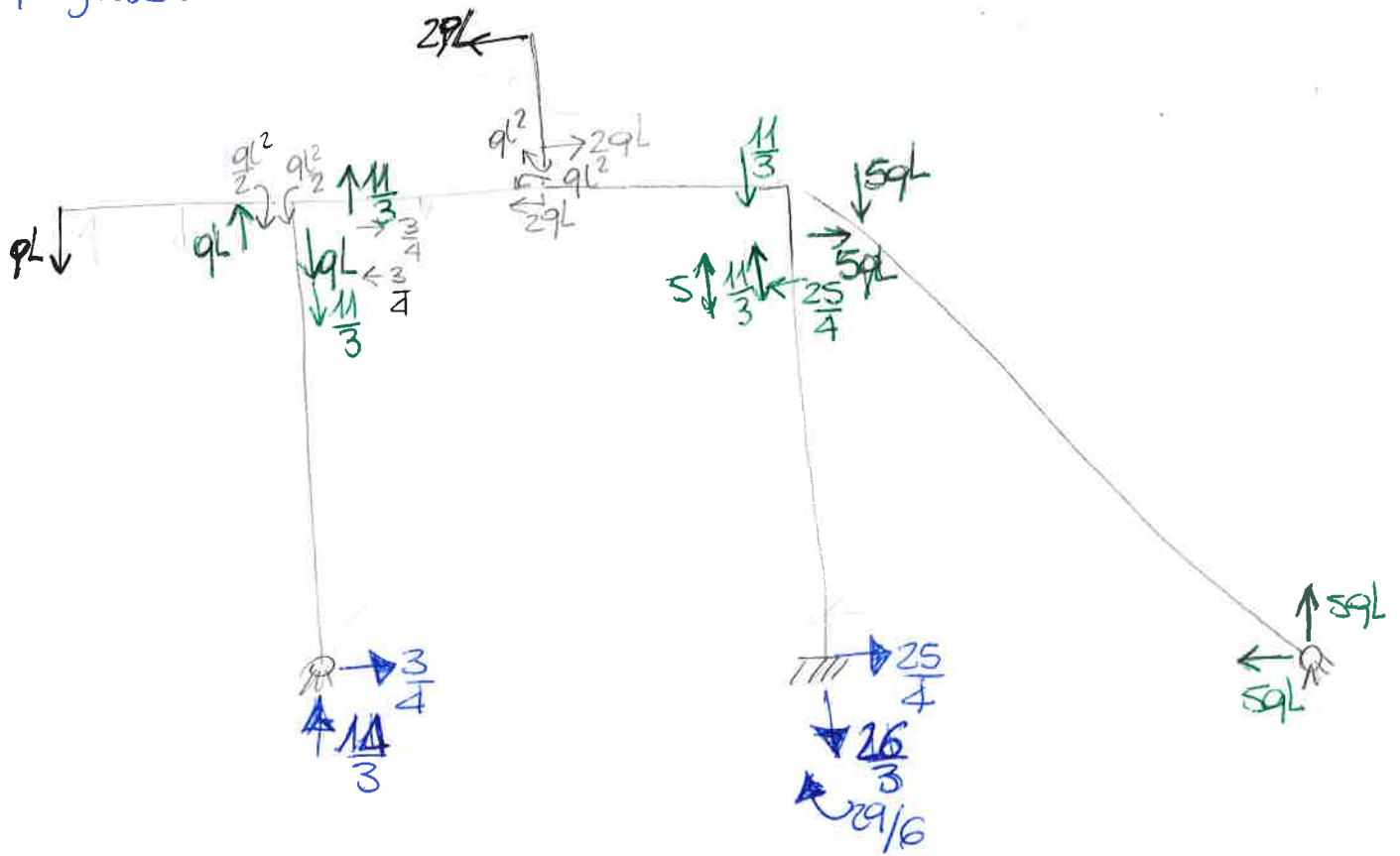
$$\frac{1}{2} q l^2 + \frac{3}{4} q l^2 - \frac{5}{4} q l^2 = \frac{2 + 3 - 5}{4} = 0 \text{ ok!}$$

E



$$= 0 \text{ ok!}$$

Eq globali



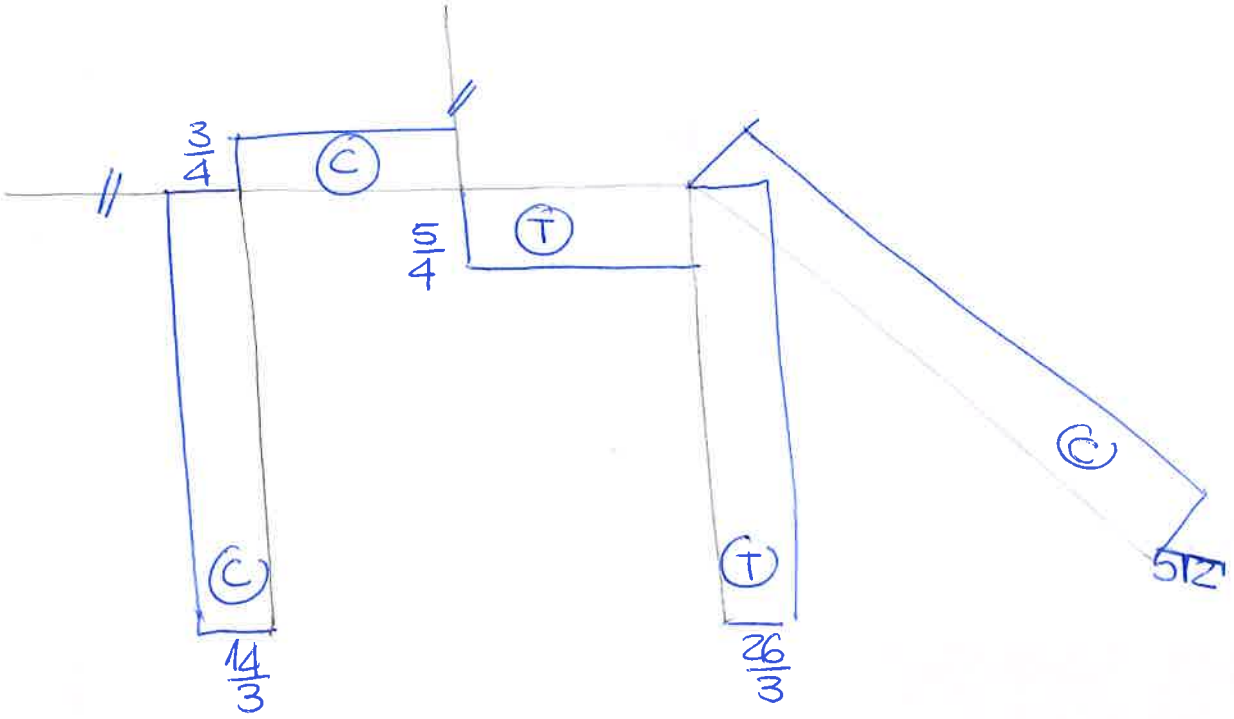
$$\uparrow = 0 \quad \frac{14}{3}qL - qL - \frac{26}{3} + 5qL = \frac{14 - 3 - 26 + 15}{3} = 0 \text{ ok!}$$

$$\rightarrow = 0 \quad \frac{3}{4}qL - 29L + \frac{25}{4}qL - 5qL = \frac{3 - 8 + 25 - 20}{4} = 0 \text{ ok!}$$

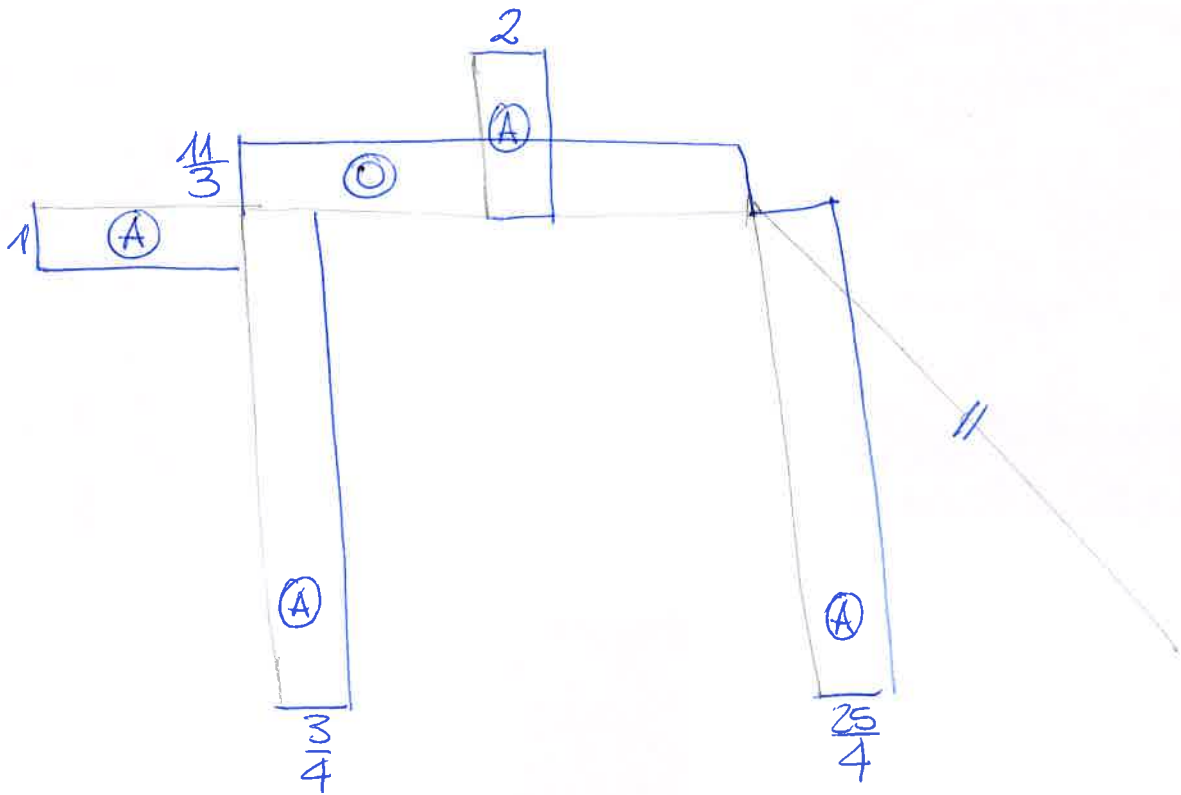
$$+\circlearrowleft = 0 \quad -qL \cdot \frac{L}{2} - 29L \cdot \frac{3}{2}L + \frac{26}{3}pL \cdot L - 5qL \cdot 2L + \frac{29}{6}pL^2$$

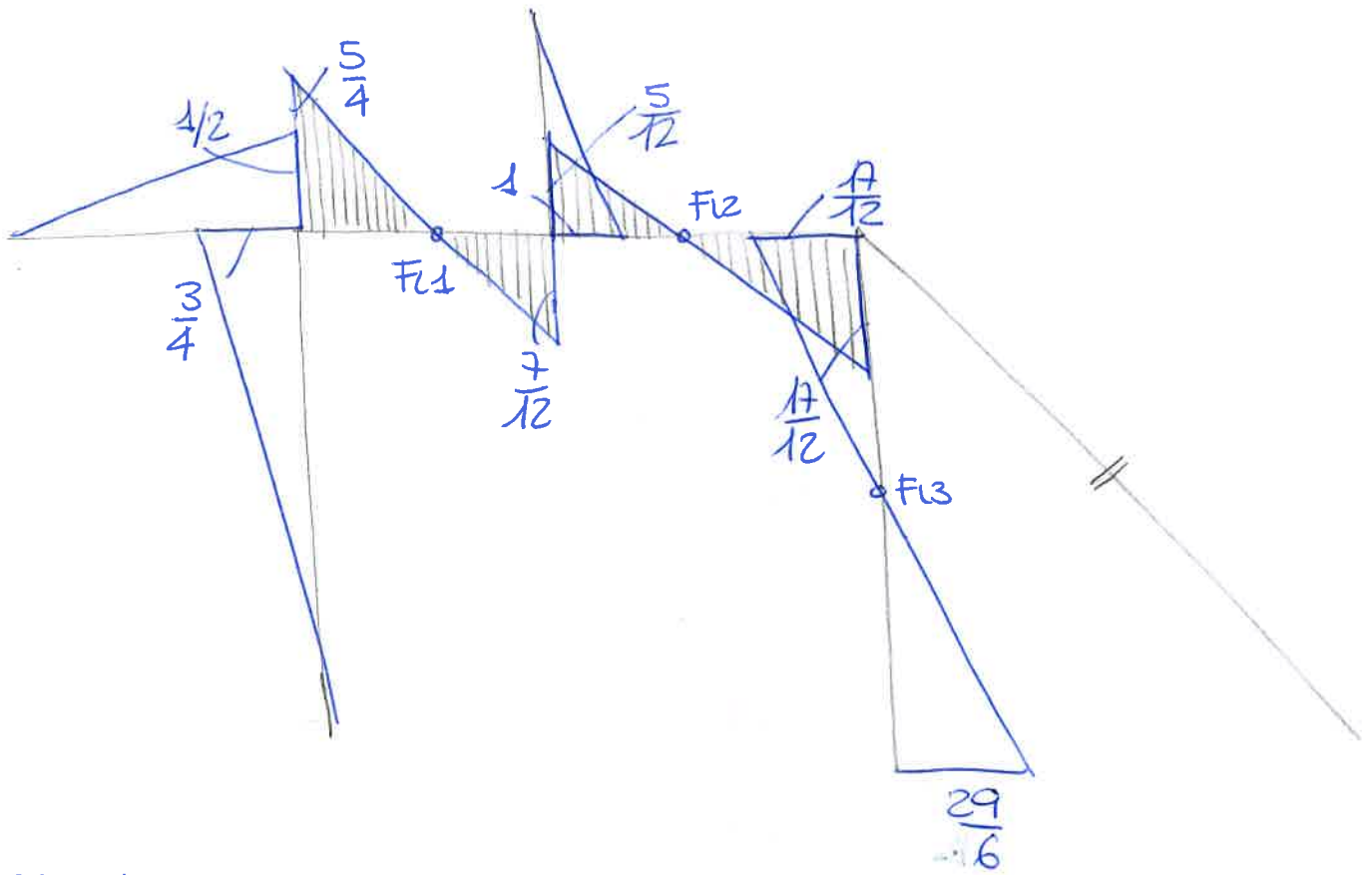
$$-\frac{1}{2}qL^2 - 39L^2 + \frac{26}{3}pL^2 - 10pL^2 + \frac{29}{6}pL^2 = \frac{-3 \cdot 18 + 52 - 60 + 29}{6} = 0 \text{ ok!}$$

(N)



(T)





FUSSO 1



$$\frac{11}{3} q l \cdot x - \frac{5}{4} q l^2 = 0$$

$$\frac{11}{3} x = \frac{5}{4} l \rightarrow x = \frac{5 \cdot 3}{4 \cdot 11} l = \frac{15}{44} l = 0,34l$$

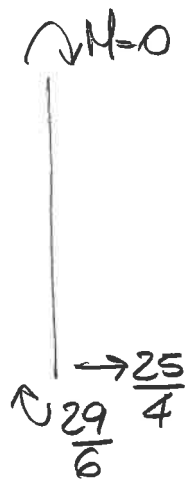
FUSSO 2



$$\frac{11}{3} q l x - \frac{17}{12} q l^2 = 0$$

$$\frac{11}{3} x = \frac{17}{12} l \rightarrow x = \frac{17 \cdot 3}{12 \cdot 11} l = \frac{51}{132} l = \frac{17}{44} l = 0,39l$$

Esercizio 3 (termico)



$$y'' = -\frac{M(x)}{EI} + \frac{2\alpha\Delta T}{H}$$

$$-\left(\frac{29}{6}ql^2 - \frac{25}{4}ql \cdot x\right) + 2 \cdot \frac{1}{2}ql^2 =$$

$$= -\frac{29}{6}ql^2 + \frac{25}{4}qlx + ql^2 =$$

$$= \frac{25}{4}lx - \frac{29}{6}L + L > 0$$

$$\frac{25}{4}x > \frac{29-6}{6}L$$

$$\frac{25}{4}x > \frac{23}{6}L$$

$$x > \frac{23}{6} \cdot \frac{4}{25}L = \frac{46}{75}$$

$$x > \frac{46}{75}L \quad (0,61L)$$

Deformata

