

TECNICA DELLE COSTRUZIONI

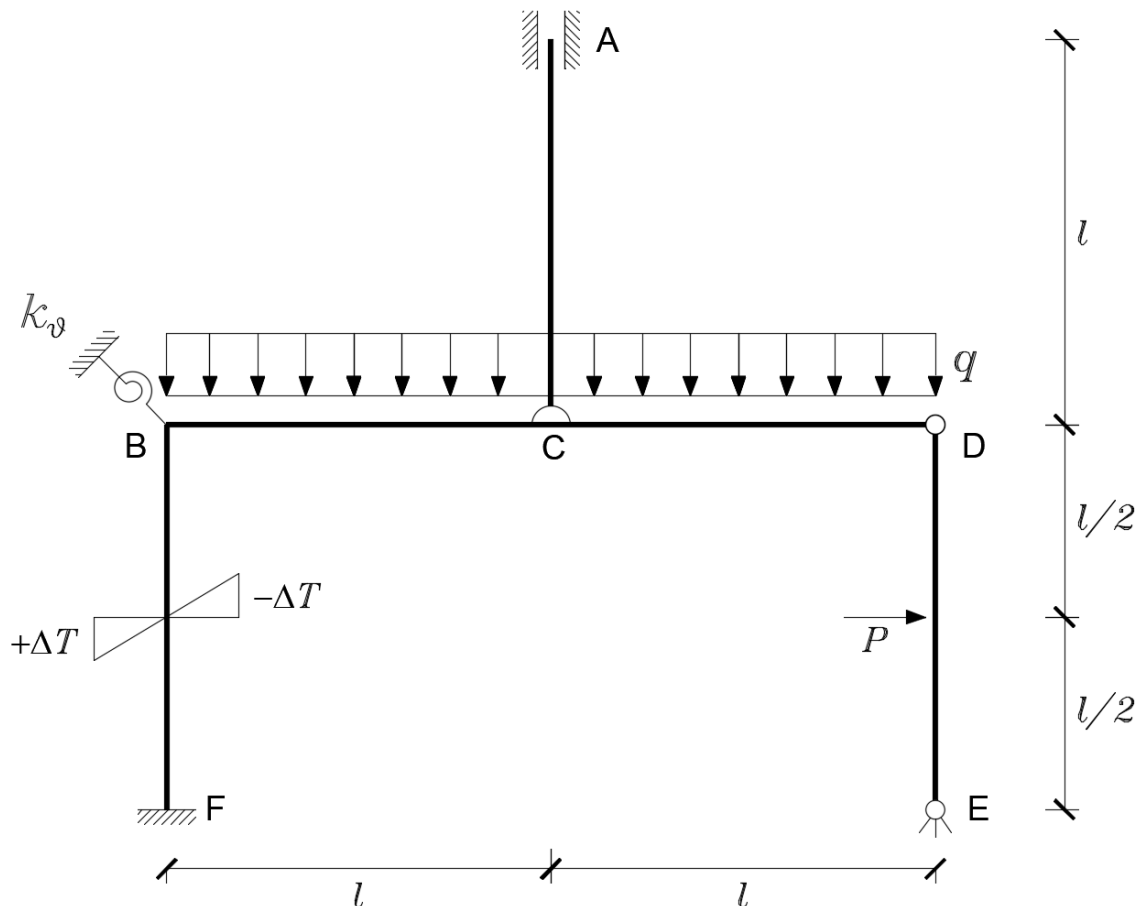
TEMA ESAME DEL 15 APRILE 2019

DOCENTI:
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ESERCITATORI:
ING. FRANCESCA FEROLDI

DURATA: 2 ORE.

Esercizio



$$k_{\theta} = \frac{1}{2} \frac{EJ}{l}$$

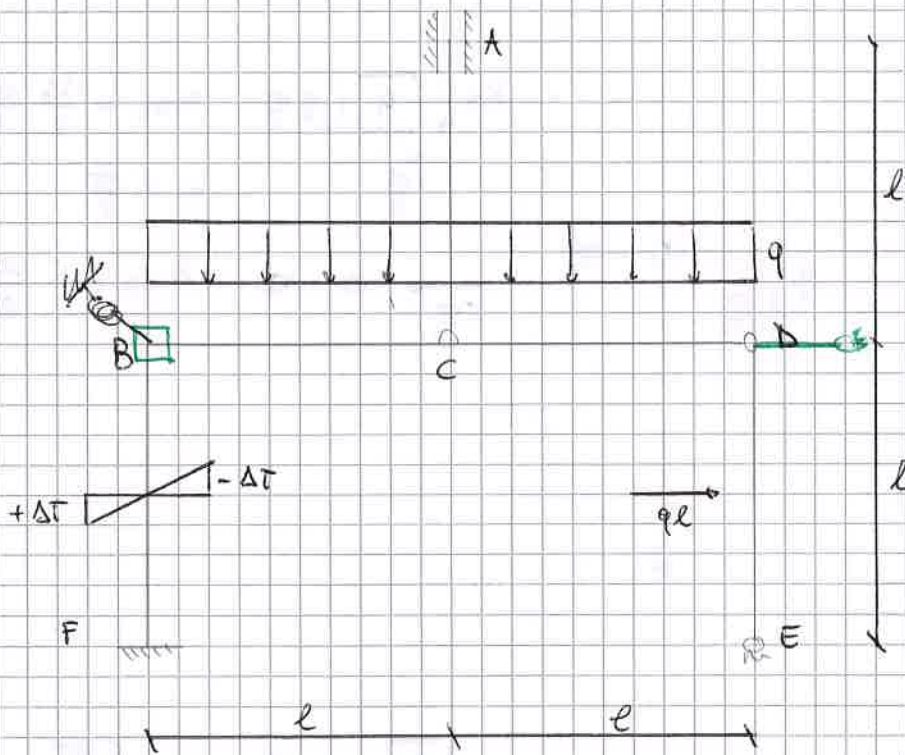
$$\frac{\alpha \Delta T}{H} = \frac{1}{4} \frac{ql^2}{EJ}$$

$$P = ql$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.



$$k_{\theta} = \frac{1}{2} \frac{EI}{l}$$

$$\frac{\Delta \Delta T}{k} = \frac{1}{3} \frac{ql^2}{EI}$$

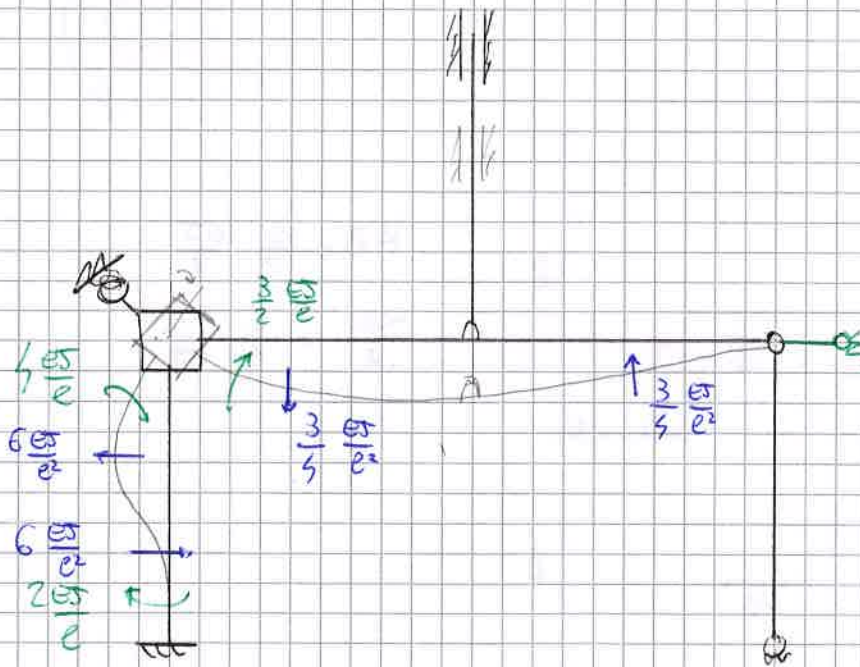
Convenzioni di segno



telaio a nodi spostabili - bicella fittizia in D e nodo in B.
Sistema risolvibile

$$\begin{cases} m_{BB} \varphi_B + m_{BD} \varphi_D + m_{BD} = 0 \\ k_{DB} \varphi_B + k_{DD} \varphi_D + k_{DD} = 0 \end{cases}$$

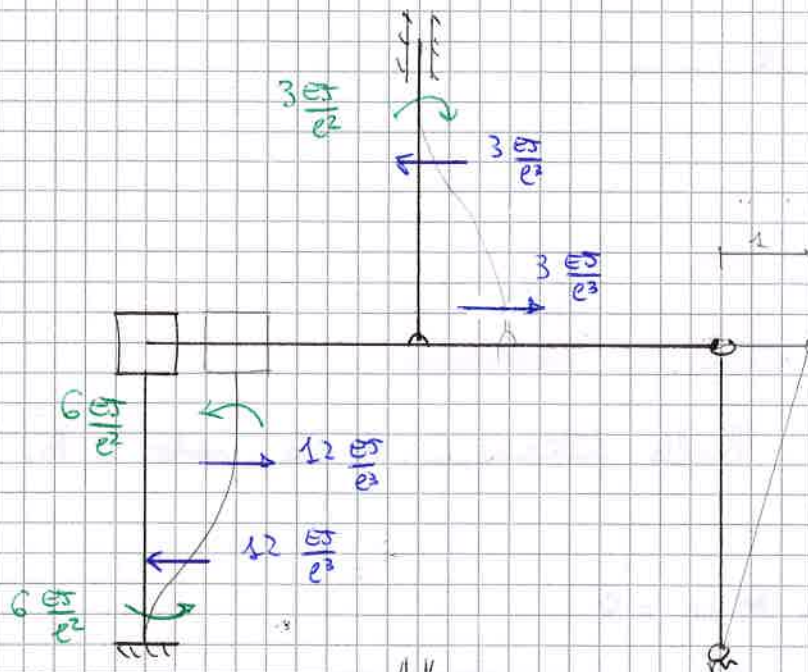
• rotazione $\varphi_B = 1$, resto = 0



$$K_B \downarrow \begin{matrix} \square B \\ \downarrow \frac{1}{2} \frac{EI}{e} \end{matrix} \quad m_{BB} = \frac{1}{2} \frac{EI}{e} + K_B = 6 \frac{EI}{e}$$

$$k_{DB} = 6 \frac{EI}{e^2}$$

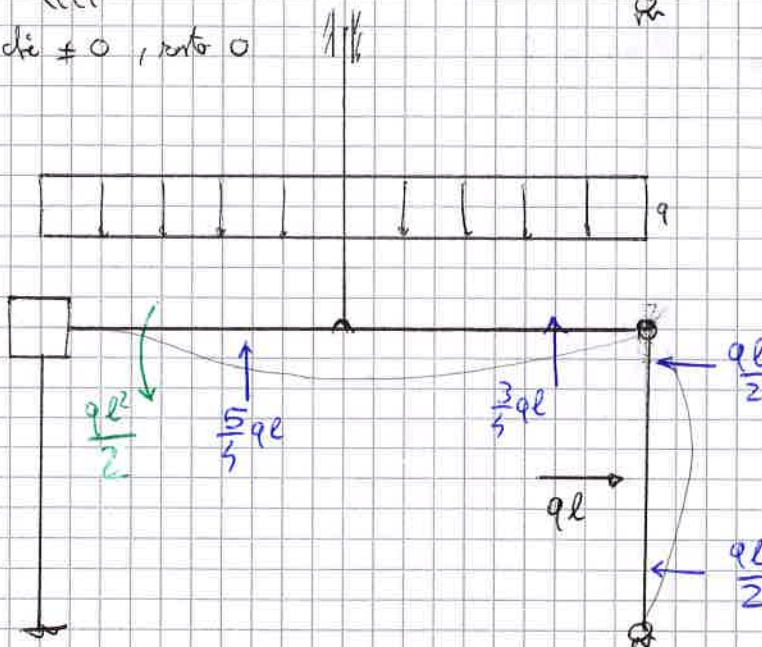
• $M_B = 1$, resto = 0



$$\begin{matrix} \square B \\ \downarrow \frac{1}{2} \frac{EI}{e^2} \end{matrix} \quad m_{BB} = -6 \frac{EI}{e^2}$$

$$k_{DB} = -12 \frac{EI}{e^3}$$

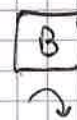
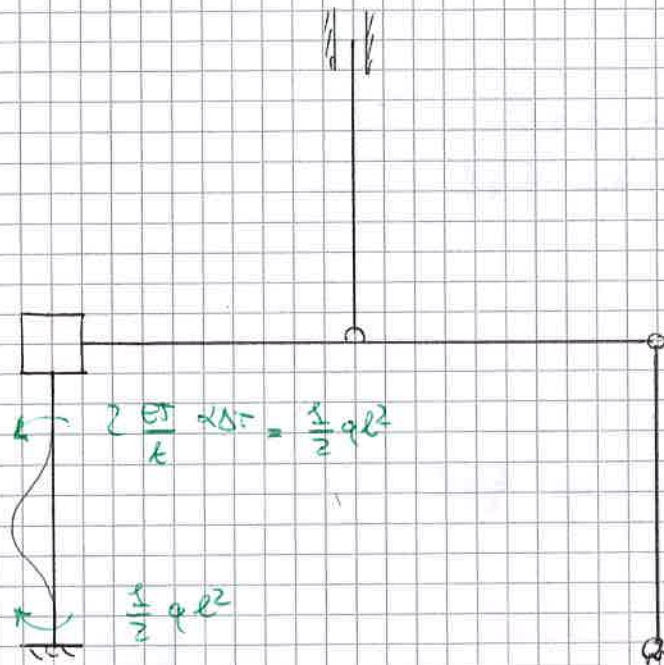
• Carichi $\neq 0$, resto 0



$$\begin{matrix} \square B \\ \uparrow \frac{q l^2}{2} \end{matrix} \quad m_{B, \text{carichi}} = -\frac{q l^2}{2}$$

$$k_{D, \text{carichi}} = \frac{q l}{2}$$

• $\Delta T \neq 0$, resto = 0



$$m_{B, \Delta T} = -\frac{1}{2} q l^2$$

$$h_{B, \Delta T} = 0$$

$$\textcircled{1} \quad \left\{ \begin{aligned} 6 \frac{ES}{l} \varphi_B - 6 \frac{ES}{l^2} M_D - \frac{q l^2}{2} - \frac{q l^2}{2} &= 0 \end{aligned} \right.$$

$$\textcircled{2} \quad \left\{ \begin{aligned} 6 \frac{ES}{l^2} \varphi_B - 15 \frac{ES}{l^3} M_D + \frac{q l}{2} &= 0 \end{aligned} \right.$$

$$\textcircled{1} \quad \varphi_B = \frac{l}{6ES} \left[6 \frac{ES}{l^2} M_D + q l^2 \right]$$

$$\textcircled{2} \quad \cancel{6 \frac{ES}{l^2}} \frac{l}{\cancel{6ES}} \left[\cancel{6 \frac{ES}{l^2}} M_D + q l^2 \right] - 15 \frac{ES}{l^3} M_D + \frac{q l}{2} = 0$$

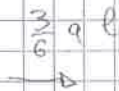
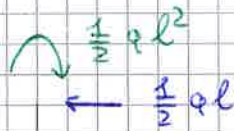
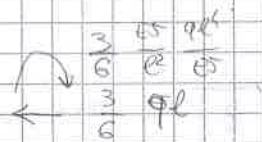
$$6 \frac{ES}{l^3} M_D + q l - 15 \frac{ES}{l^3} M_D + \frac{q l}{2} = 0$$

$$3 \frac{ES}{l^3} M_D = \frac{3}{2} q l \quad M_D = \frac{1}{6} \frac{q l^3}{ES}$$

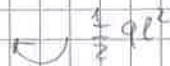
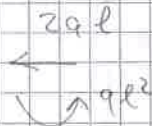
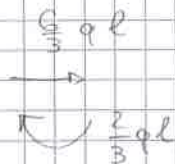
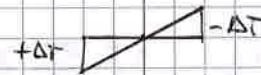
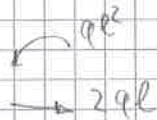
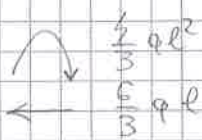
$$\textcircled{1} \quad \varphi_B = \frac{l}{6ES} \left[\cancel{6 \frac{ES}{l^2}} \frac{1}{\cancel{6 \frac{ES}{l^2}}} \frac{q l^3}{ES} + q l^2 \right] = \frac{1}{3} \frac{q l^3}{ES}$$

$$\boxed{\varphi_B = \frac{1}{3} \frac{q l^3}{ES}}$$

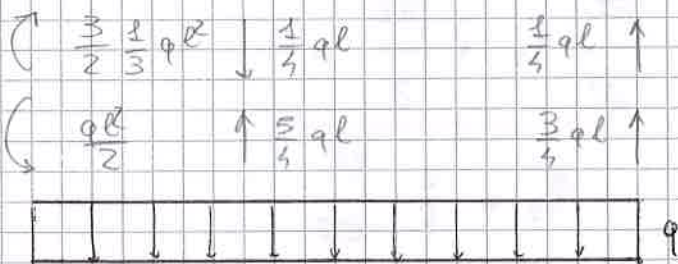
Axle AD



Axle BF

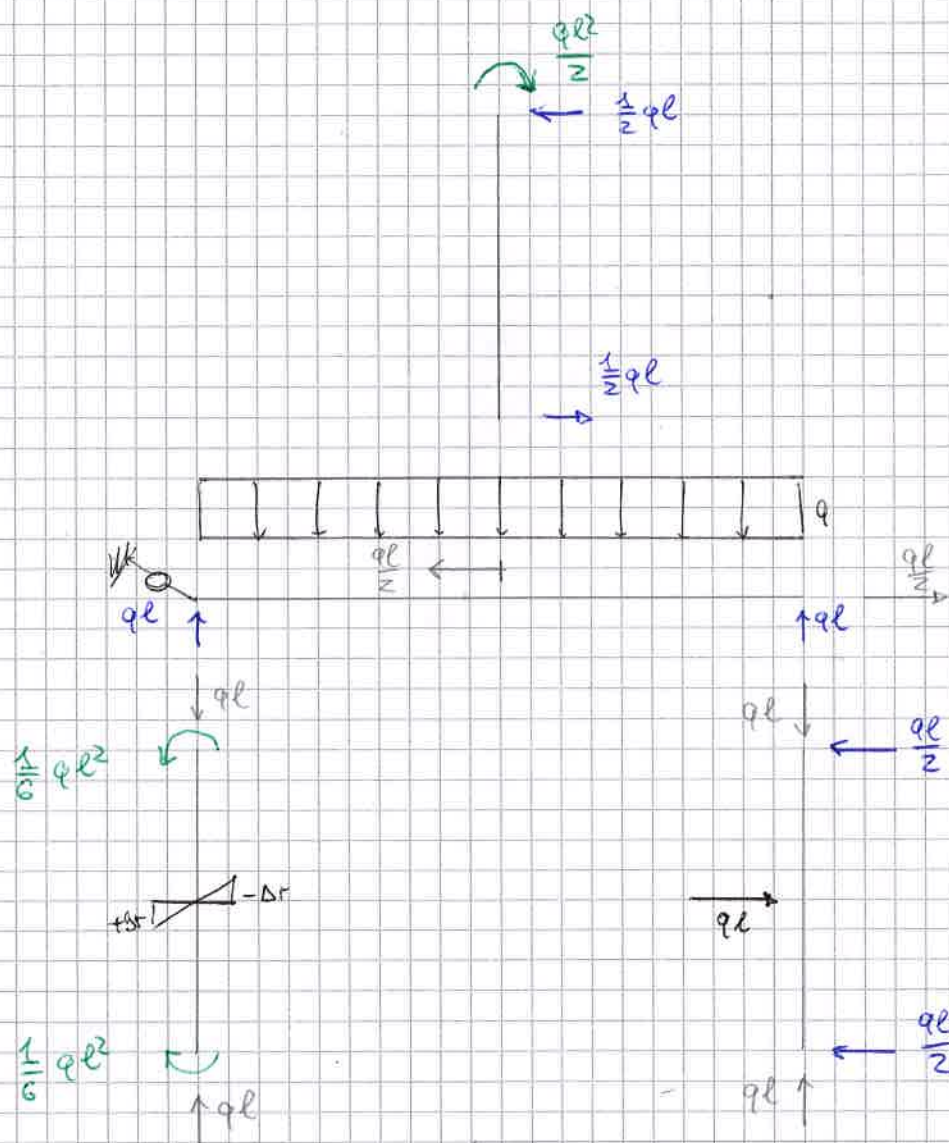


Axle BD



Axle DE





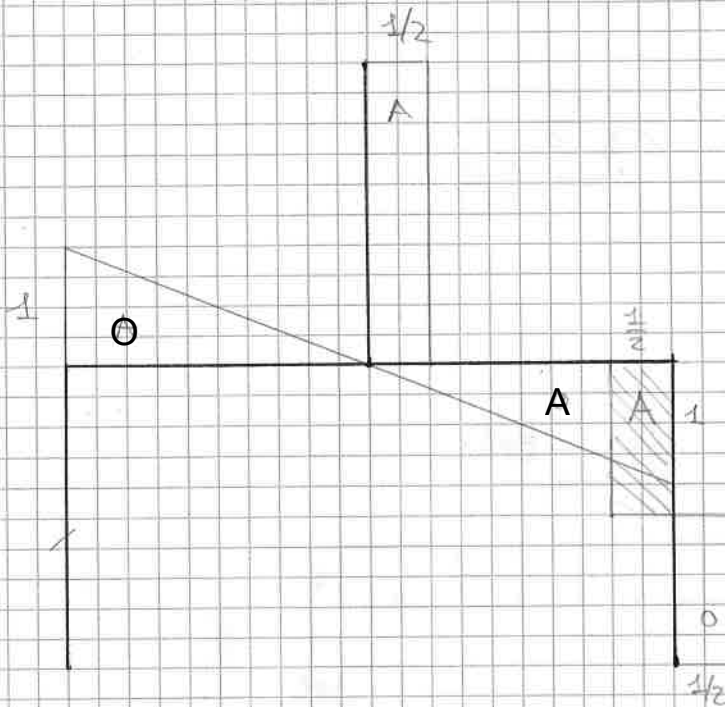
Controllo

$$\sum V = 0 \quad ql + ql - 2ql = 0 \quad \text{OK!}$$

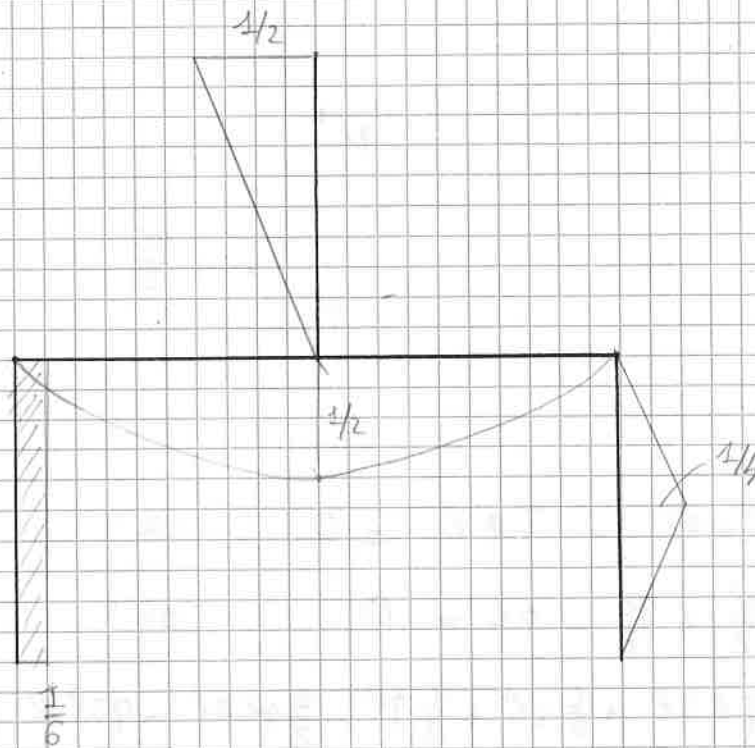
$$\sum H = 0 \quad \frac{ql}{2} + \frac{ql}{2} - ql = 0 \quad \text{OK!}$$

$$\sum M(e) = 0 \quad ql \cdot 2l + \frac{1}{6} ql^2 + \frac{1}{2} ql^2 - \frac{1}{2} ql \cdot 2l - ql \cdot l + ql \cdot \frac{l}{2} - \frac{1}{2} \cdot \frac{1}{3} \frac{ql^3}{l} = 0$$

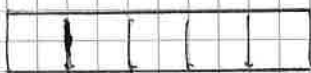
$V [ql]$



$M [ql^2]$



Area BC

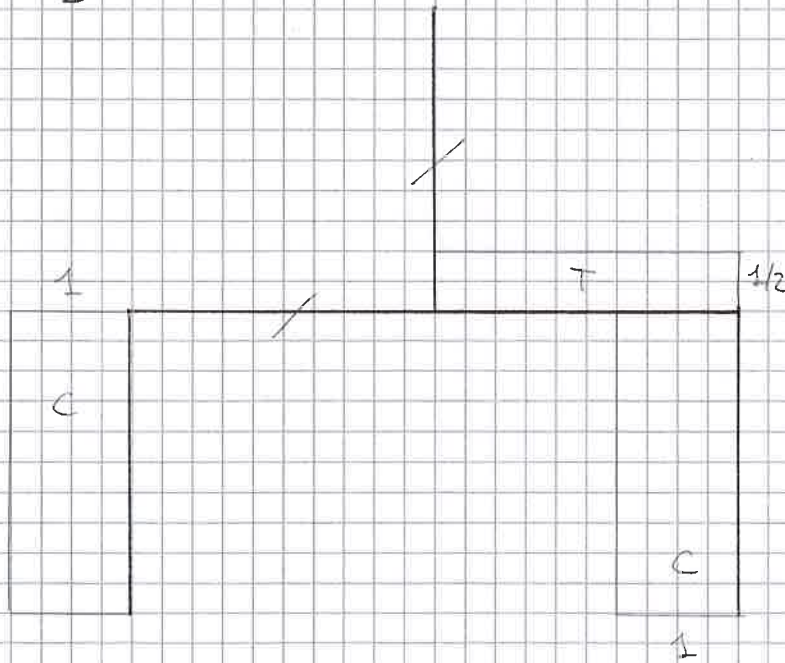


$ql \uparrow$

M_{max}

$$M_{max} = ql \cdot l - ql \cdot \frac{l}{2} = \frac{1}{2} ql^2$$

N [qL]



Studio deformata Termica

$$\frac{qL^2}{6}$$

$$+\Delta T \quad -\Delta T$$

$$\frac{qL^2}{6}$$

DEFORMATA

$$\begin{aligned} v'' &= -\frac{M(x)}{EI} + 2 \frac{\alpha \Delta T}{L} = \\ &= -\frac{qL^2}{6} + 2 \frac{1}{3} qL^2 = \\ &= \frac{1}{3} qL^2 \geq 0 \quad \forall x \end{aligned}$$

