

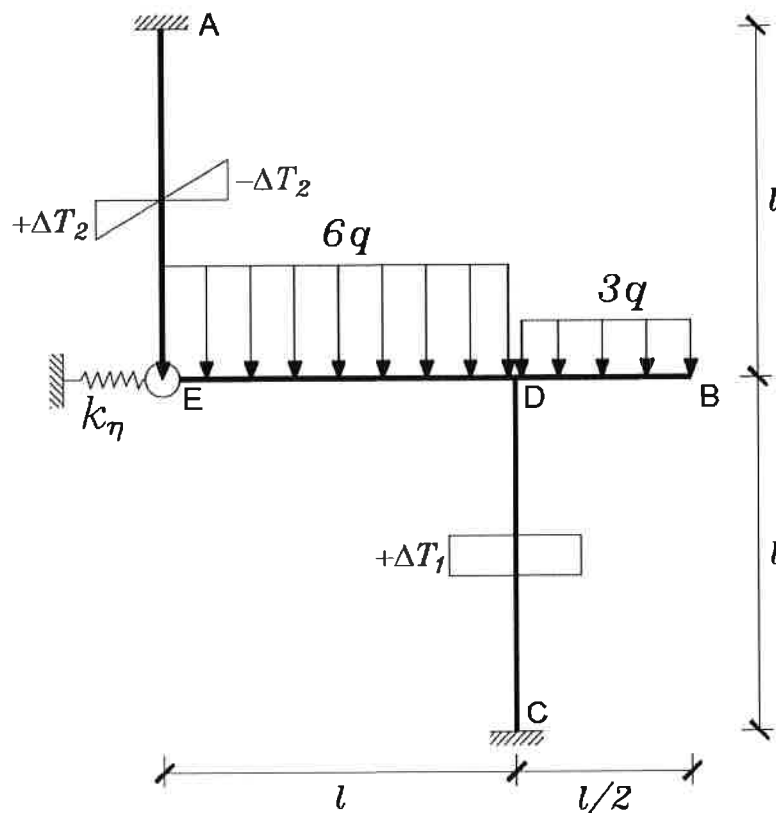
# TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 19 NOVEMBRE 2019

DOCENTI:

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DURATA: 2 ORE e 30 MIN.



$$k_{\eta} = 8 \frac{EJ}{l^3} \quad \frac{\alpha \Delta T_2}{h} = \frac{37 q l^2}{8 EJ}$$

$$\alpha \Delta T_1 = \frac{1}{16} \frac{q l^3}{EJ}$$

## Esercizio

Dato il telaio in figura, si richiedono i grafici di:

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma  $EA \rightarrow \infty$ ,  $EJ = \text{costante}$ .

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.



# TEMA ESAME TECNICA DELLE COSTRUZIONI 19/11/19

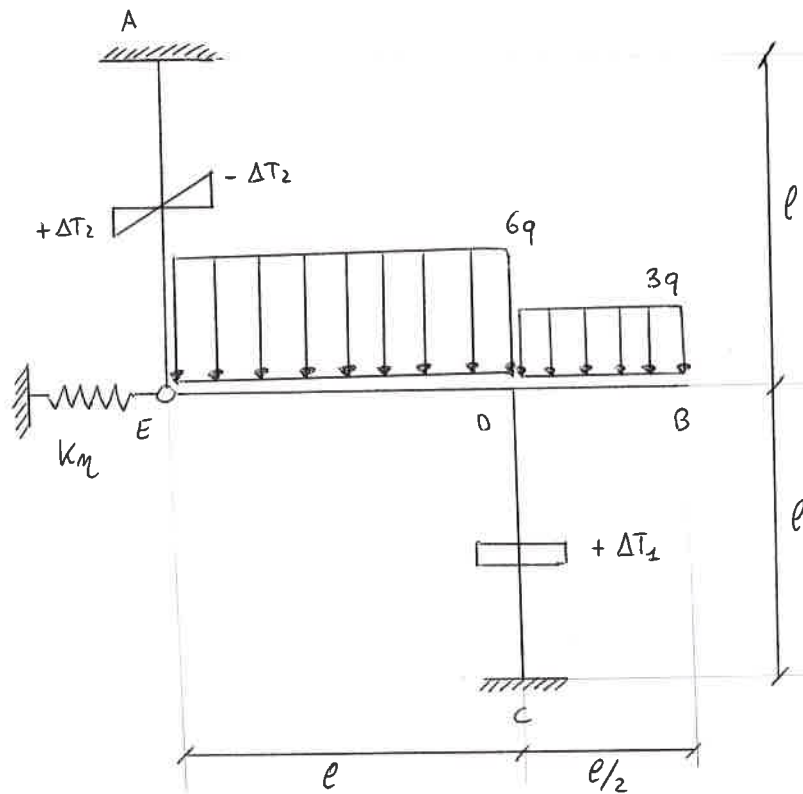
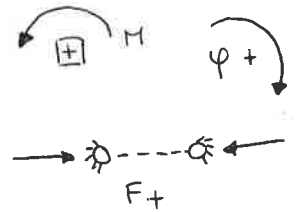
DATI

$$\bullet \Delta T_1 = \frac{1}{16} \frac{q l^3}{EJ}$$

$$\bullet \frac{\Delta T_2}{H} = \frac{37}{8} \frac{q l^2}{EJ}$$

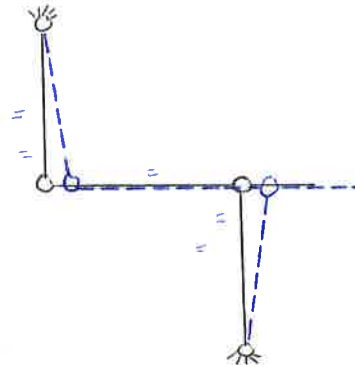
$$\bullet K_M = 8 \frac{EJ}{l^3}$$

CONVENZIONI ADOTATE



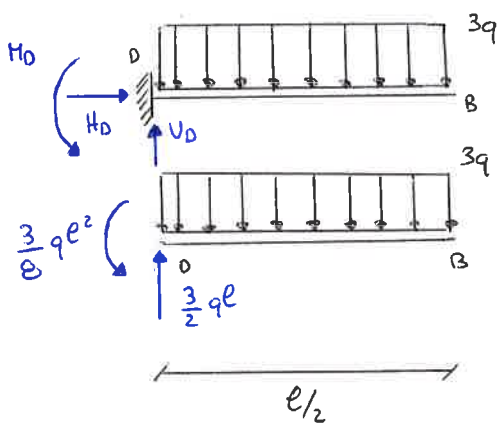
## ANALISI PRELIMINARE

- 8 GRADI DI VINCOLO } STRUTTURA 2 VOLTE IPERVINCOLATA
- 6 DOF
- STRUTTURA A NODI SPORSTABILI



## RISOLUZIONE

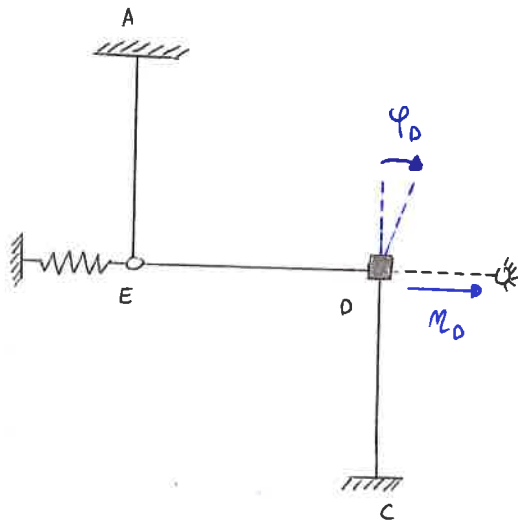
ASTA DB → APPENDICE ISOSTATICA



$$\left\{ \begin{aligned} \sum H_{(D)} = 0 &\rightarrow H_D = 0 \\ \sum V_{(D)} = 0 &\rightarrow V_D = \frac{3}{2} q l \\ \sum M_{(D)} = 0 &\rightarrow M_D = 3q \frac{l}{2} \cdot \frac{l}{4} = \frac{3}{8} q l^2 \end{aligned} \right.$$

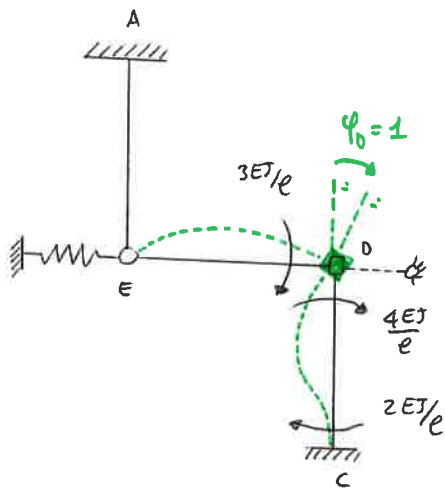
SCELTA INCOGNITE

EQUAZIONI DI EQUILIBRIO



$$\begin{cases} \sum M_{CD} = 0 \rightarrow m_{D\varphi} \cdot \varphi_D + m_{DM} \cdot M_D + m_{D\phi} = 0 \\ \sum H_{CD} = 0 \rightarrow h_{M\varphi} \cdot \varphi_D + h_{MM} \cdot M_D + h_{M\phi} = 0 \end{cases}$$

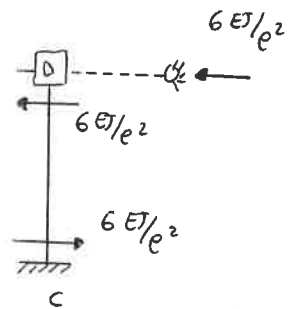
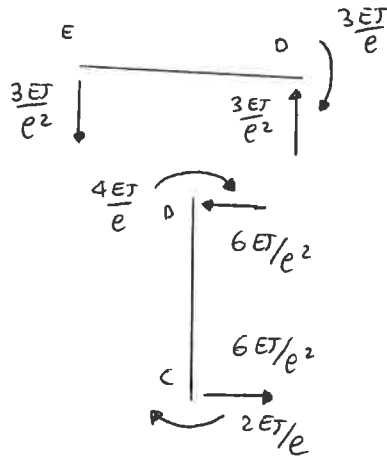
$\varphi_D = 1$



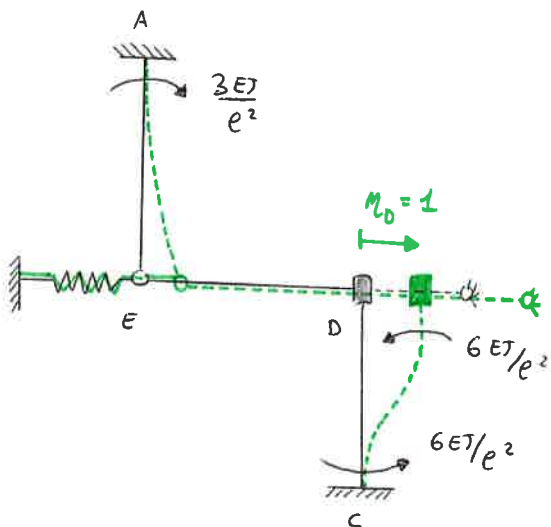
$\frac{3ET}{e}$  (curved arrow)

$$m_{D\varphi} = \frac{3ET}{e} + \frac{4ET}{e} = \frac{7ET}{e}$$

$$h_{M\varphi} = \frac{6ET}{e^2}$$

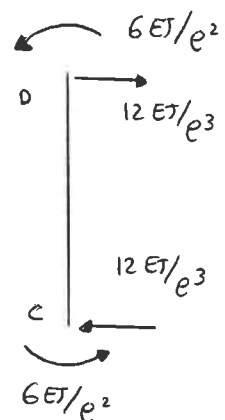
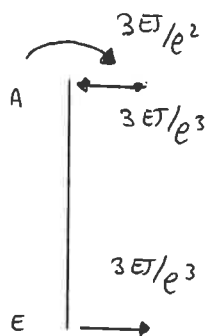


$M_D = 1$

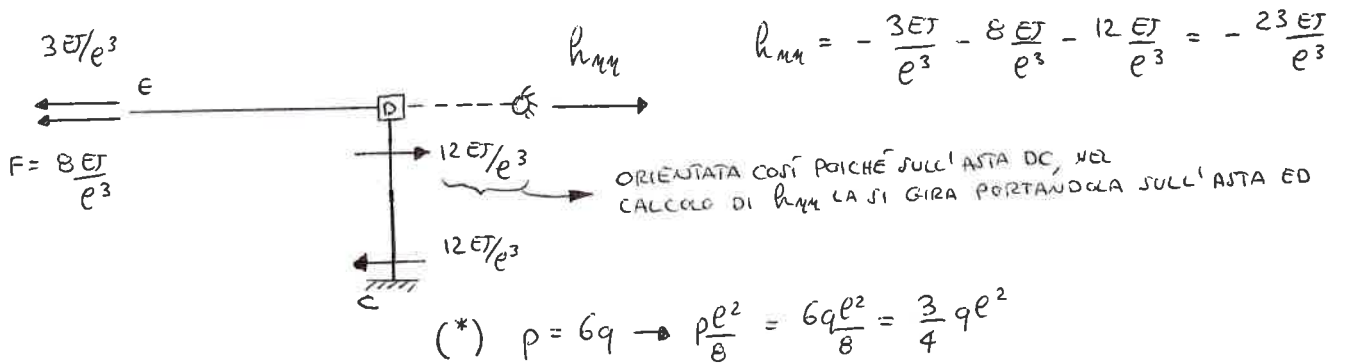


$\frac{6ET}{e^2}$  (curved arrow)

$$m_{DM} = -\frac{6ET}{e^2}$$

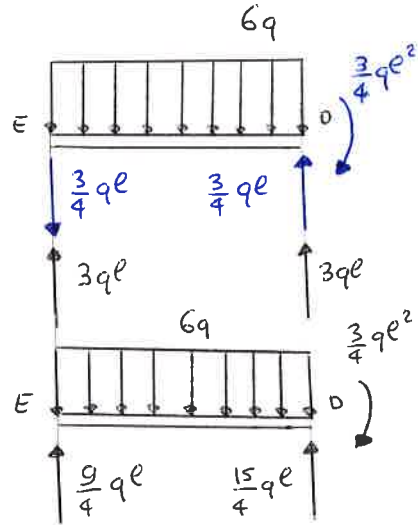
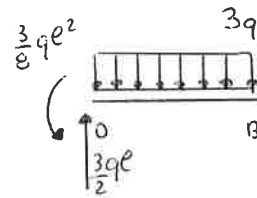
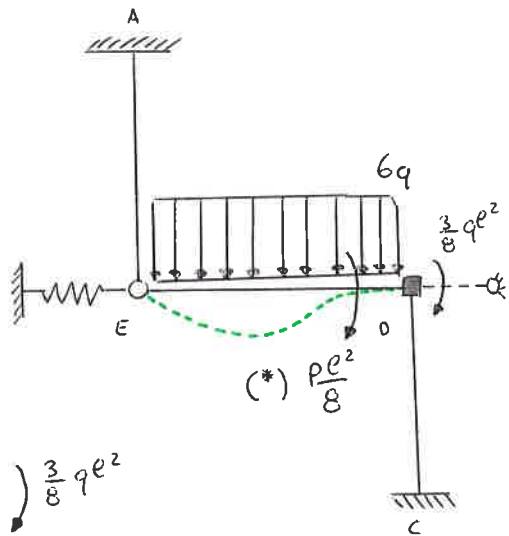


$F = k_M \cdot M_D = k_M = \frac{6ET}{e^3}$



CARICHI

$q \neq 0$

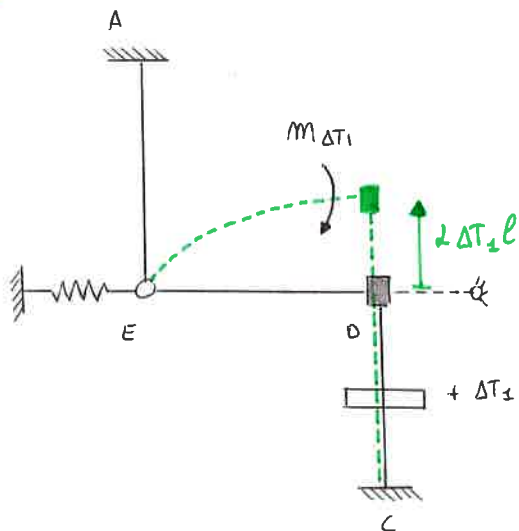


$\frac{3}{4}qe^2$  ( )  $\frac{3}{8}qe^2$

$m_{D\phi}^q = \frac{3}{4}qe^2 - \frac{3}{8}qe^2 = \frac{3}{8}qe^2$

$h_{M\phi}^q = 0$

$\Delta T_{\pm} = 0$

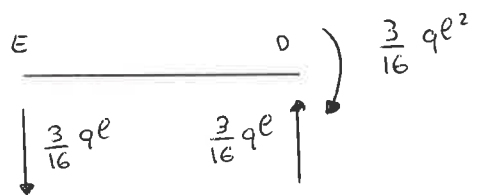


$m_{\Delta T_1} = \frac{3EJ}{e^2} \cdot 2\Delta T_1 e = \frac{3EJ}{e} \cdot 2\Delta T_1 = \frac{3EJ}{e} \cdot \frac{1}{16} \frac{qe^3}{EJ} = \frac{3}{16}qe^2$



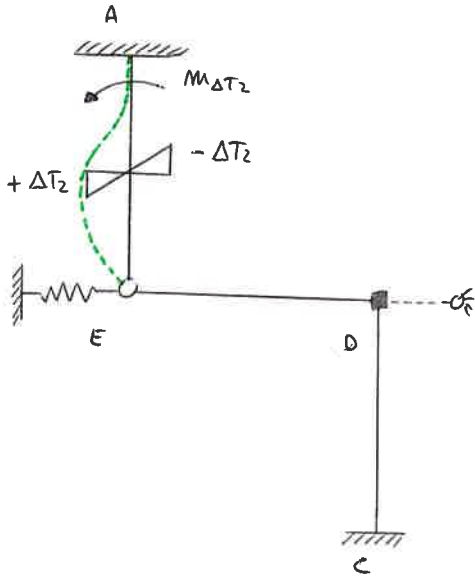
$m_{D\phi}^{\Delta T_1} = \frac{3}{16}qe^2$

$h_{M\phi}^{\Delta T_1} = 0$

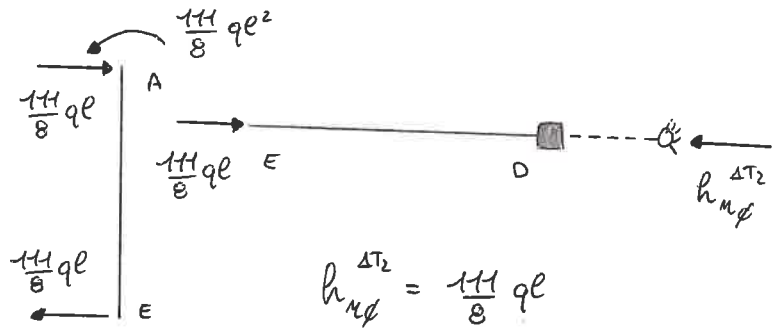


$$\underline{\underline{\Delta T_2 \neq 0}}$$

$$M_{\Delta T_2} = 3ET \frac{\Delta T_2}{H} = 3ET \cdot \frac{37}{8} \frac{qe^2}{ET} = \frac{111}{8} qe^2$$



$$\boxed{D} \quad M_{D\phi}^{\Delta T_2} = 0$$



$$h_{M\phi}^{\Delta T_2} = \frac{111}{8} ql$$

### SISTEMA RISOLVENTE

$$\begin{cases} \sum M_{(D)} = 0 \rightarrow M_{D\phi} \cdot \varphi_0 + M_{D\psi} \cdot \psi_0 + M_{D\phi}^q + M_{D\phi}^{\Delta T_1} + M_{D\phi}^{\Delta T_2} = 0 \\ \sum H_{(D)} = 0 \rightarrow h_{M\phi} \cdot \varphi_0 + h_{M\psi} \cdot \psi_0 + h_{M\phi}^q + h_{M\phi}^{\Delta T_1} + h_{M\phi}^{\Delta T_2} = 0 \end{cases}$$

$$\begin{cases} \frac{7ET}{e} \cdot \varphi_0 + \left(-\frac{6ET}{e^2}\right) \cdot \psi_0 + \frac{3}{8} qe^2 + \frac{3}{16} qe^2 + 0 = 0 \\ \frac{6ET}{e^2} \cdot \varphi_0 + \left(-\frac{23ET}{e^3}\right) \cdot \psi_0 + 0 + 0 + \frac{111}{8} ql = 0 \end{cases}$$

$$\frac{7ET}{e} \cdot \varphi_0 - \frac{6ET}{e^2} \cdot \psi_0 + \frac{9}{16} qe^2 = 0$$

$$\frac{7ET}{e} \cdot \varphi_0 + \frac{9}{16} qe^2 = \frac{6ET}{e^2} \psi_0 \rightarrow \psi_0 = \frac{e^2}{6ET} \left( \frac{7ET}{e} \cdot \varphi_0 + \frac{9}{16} qe^2 \right)$$

$$\psi_0 = \frac{7}{6} e \cdot \varphi_0 + \frac{3}{32} qe^4$$

$$\frac{6ET}{e^2} \varphi_0 - \frac{23ET}{e^3} \cdot \left( \frac{7}{6} e \varphi_0 + \frac{3}{32} \frac{qe^4}{ET} \right) + \frac{111}{8} ql = 0$$

$$\frac{6ET}{e^2} \varphi_0 - \frac{161}{6} \frac{ET}{e^2} \varphi_0 - \frac{69}{32} ql + \frac{111}{8} ql = 0$$

$$-\frac{125}{6} \frac{ET}{e^2} \varphi_0 + \frac{375}{32} ql = 0 \rightarrow \frac{125}{6} \frac{ET}{e^2} \varphi_0 = \frac{375}{32} ql \rightarrow \varphi_0 = \frac{9}{16} \frac{qe^3}{ET}$$

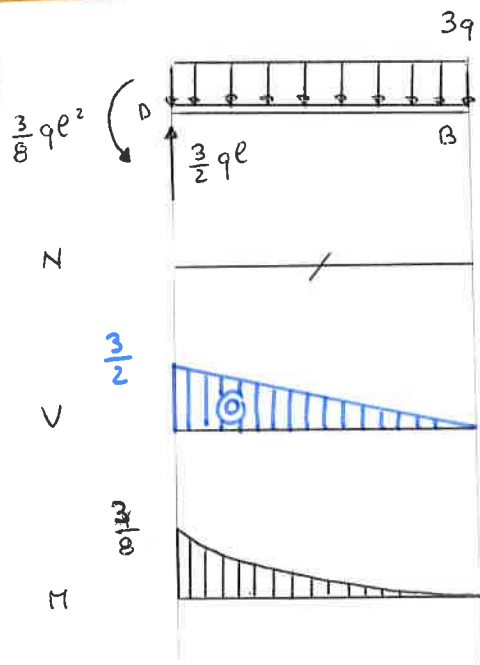
$$\boxed{\varphi_0 = \frac{9}{16} \frac{qe^3}{ET}}$$

$$\psi_0 = \frac{7}{6} e \varphi_0 + \frac{3}{32} qe^4 \rightarrow \psi_0 = \frac{7}{6} e \cdot \frac{9}{16} \frac{qe^3}{ET} + \frac{3}{32} \frac{qe^4}{ET} \rightarrow$$

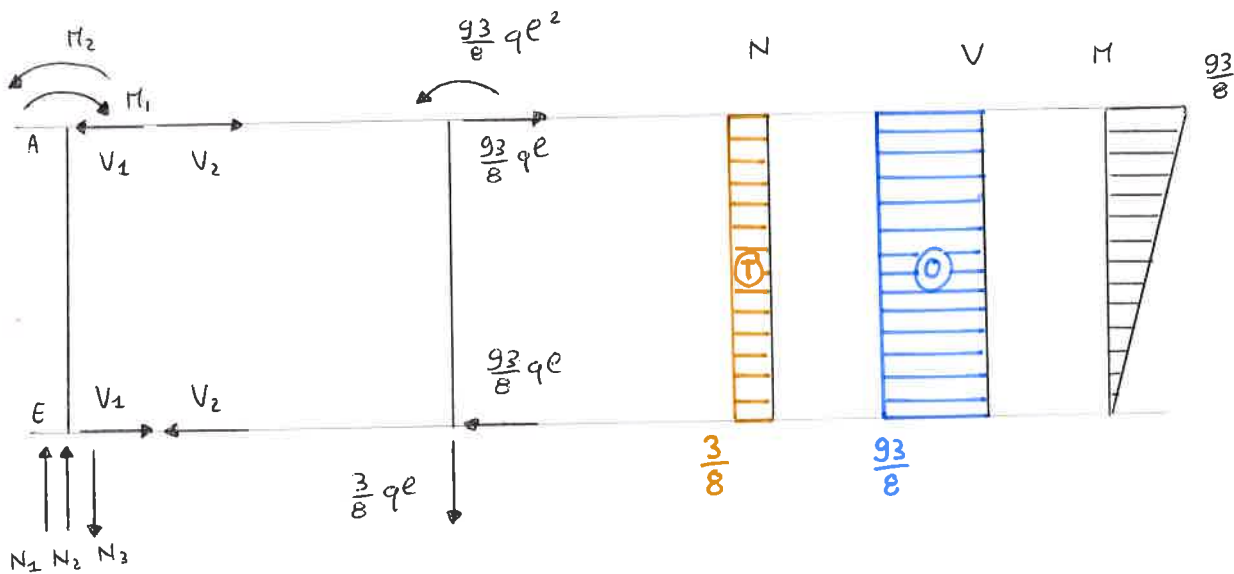
$$\rightarrow \psi_0 = \frac{21}{32} \frac{qe^4}{ET} + \frac{3}{32} \frac{qe^4}{ET} \rightarrow \boxed{\psi_0 = \frac{3}{4} \frac{qe^4}{ET}}$$

# ASTE E DIAGRAMMI

## ASTA DB



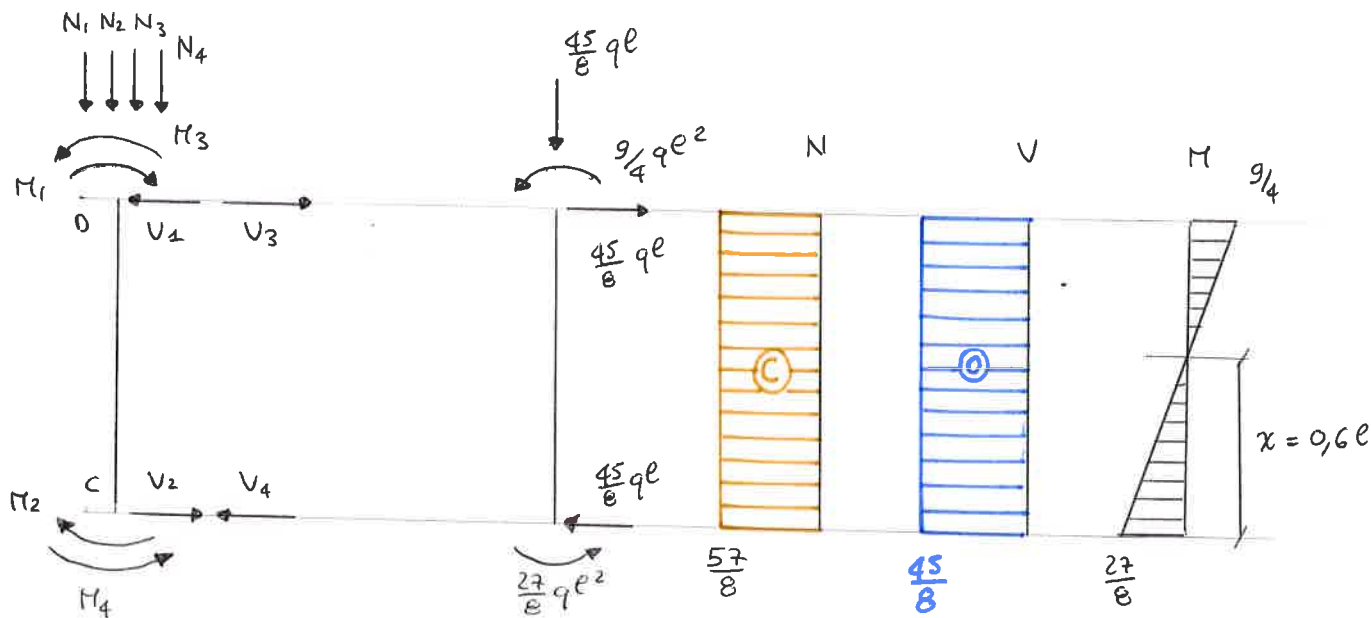
## ASTA AE



$$\left. \begin{aligned} M_1 &= \frac{3ET}{e^2} \cdot \mu_0 = \frac{3ET}{e^2} \cdot \frac{3}{4} \frac{qe^4}{ET} = \frac{9}{4} qe^2 \\ M_2 &= \frac{111}{8} qe^2 \end{aligned} \right\} M_1 + M_2 = \frac{111}{8} qe^2 - \frac{9}{4} qe^2 = \frac{93}{8} qe^2$$

$$\left. \begin{aligned} V_1 &= \frac{3ET}{e^2} \cdot \mu_0 = \frac{3ET}{e^2} \cdot \frac{3}{4} \frac{qe^4}{ET} = \frac{9}{4} qe \\ V_2 &= \frac{111}{8} qe \end{aligned} \right\} V_1 + V_2 = \frac{111}{8} qe - \frac{9}{4} qe = \frac{93}{8} qe$$

$$\left. \begin{aligned} N_1 &= \frac{3ET}{e^2} \cdot \varphi_0 = \frac{3ET}{e^2} \cdot \frac{9}{16} \frac{qe^3}{ET} = \frac{27}{16} qe \\ N_2 &= \frac{3}{16} qe \\ N_3 &= \frac{9}{4} qe \end{aligned} \right\} N_1 + N_2 - N_3 = \left( \frac{27}{16} + \frac{3}{16} - \frac{9}{4} \right) qe = -\frac{6}{16} qe = -\frac{3}{8} qe$$



$$M_1 = \frac{4EJ}{e} \cdot \varphi_0 = \frac{4EJ}{e} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{9}{4} qe^2$$

$$M_2 = \frac{2EJ}{e} \cdot \varphi_0 = \frac{2EJ}{e} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{9}{8} qe^2$$

$$M_3 = \frac{6EJ}{e^2} \cdot M_0 = \frac{6EJ}{e^2} \cdot \frac{3}{4} \frac{qe^4}{EJ} = \frac{9}{2} qe^2$$

$$M_4 = \frac{6EJ}{e^2} \cdot M_0 = \frac{9}{2} qe^2$$

$$V_1 = \frac{6EJ}{e^2} \cdot \varphi_0 = \frac{6EJ}{e^2} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{27}{8} qe$$

$$V_2 = \frac{6EJ}{e^2} \cdot \varphi_0 = \frac{27}{8} qe$$

$$V_3 = \frac{12EJ}{e^3} \cdot M_0 = \frac{12EJ}{e^3} \cdot \frac{3}{4} \frac{qe^4}{EJ} = 9 qe$$

$$V_4 = \frac{12EJ}{e^3} \cdot M_0 = 9 qe$$

$$N_1 = \frac{3EJ}{e^2} \cdot \varphi_0 = \frac{3EJ}{e^2} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{27}{16} qe$$

$$N_2 = \frac{15}{4} qe$$

$$N_3 = \frac{3}{16} qe \quad N_4 = \frac{3}{2} qe$$

$$M_1 + M_3 = \frac{9}{2} qe^2 - \frac{9}{4} qe^2 = \frac{9}{4} qe^2$$

$$M_2 + M_4 = \frac{9}{2} qe^2 - \frac{9}{8} qe^2 = \frac{27}{8} qe^2$$

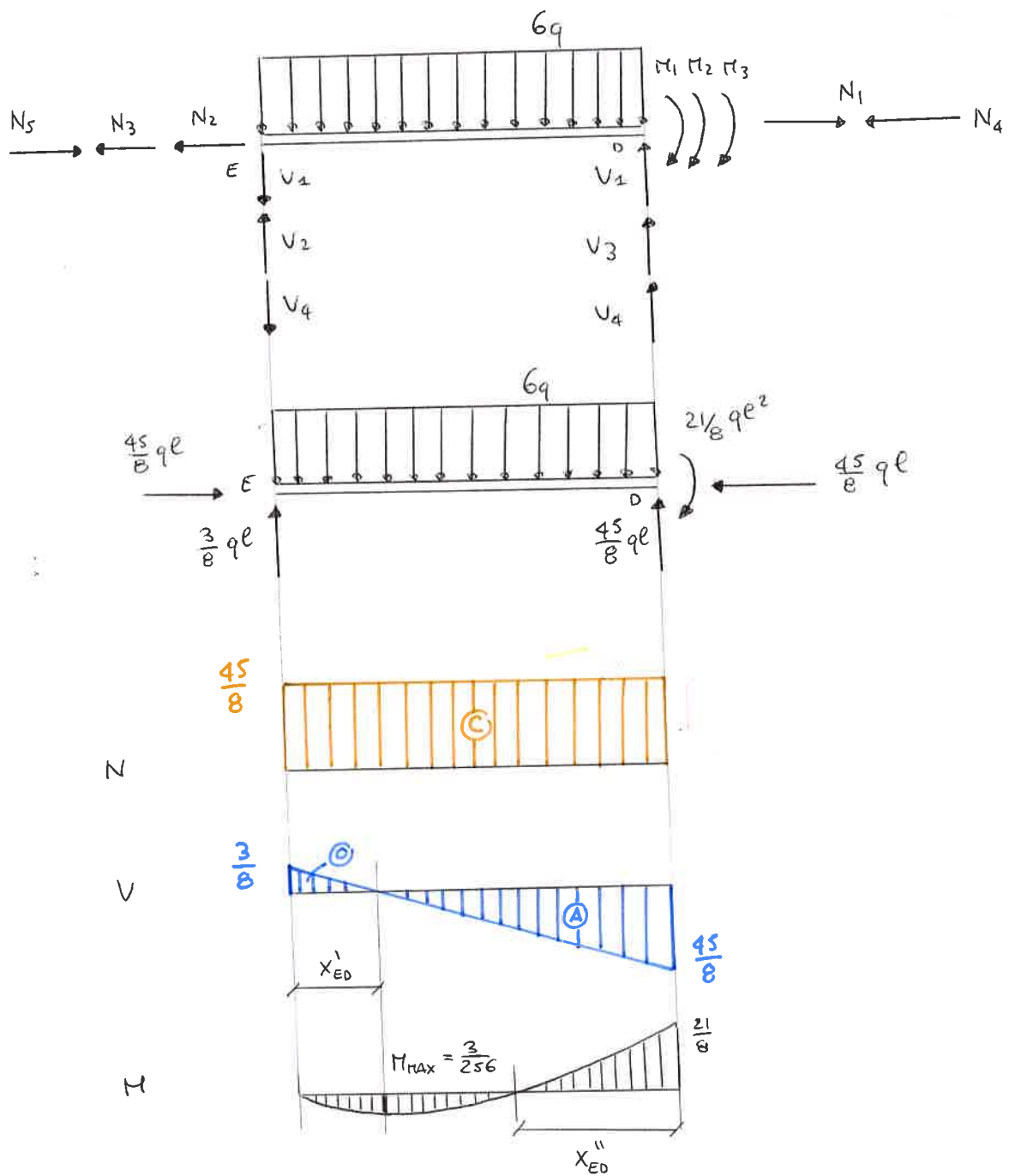
$$V_1 + V_3 = V_2 + V_4 = \frac{27}{8} qe - 9 qe = -\frac{45}{8} qe$$

$$N_1 + N_2 + N_3 + N_4 = \left( \frac{27}{16} + \frac{15}{4} + \frac{3}{16} + \frac{3}{2} \right) qe = \frac{114}{16} qe = \frac{57}{8} qe$$

$$x \rightarrow M = 0 \rightarrow \frac{27}{8} qe^2 - \frac{45}{8} qe x = 0 \rightarrow x = \frac{27}{8} \cdot \frac{8}{45} e = \frac{3}{5} e = 0,6 e$$



ASTA ED



$$\left. \begin{aligned} M_1 &= \frac{3EJ}{e} \cdot \varphi_0 = \frac{3EJ}{e} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{27}{16} qe^2 \\ M_2 &= \frac{3}{4} qe^2 \\ M_3 &= \frac{3}{16} qe^2 \end{aligned} \right\} M_1 + M_2 + M_3 = \left( \frac{27}{16} + \frac{3}{4} + \frac{3}{16} \right) qe^2 = \frac{42}{16} qe^2 = \frac{21}{8} qe^2$$

$$\left. \begin{aligned} V_1 &= \frac{3EJ}{e^2} \cdot \varphi_0 = \frac{3EJ}{e^2} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{27}{16} qe \\ V_2 &= \frac{9}{4} qe \\ V_3 &= \frac{15}{4} qe \\ V_4 &= \frac{3}{16} qe \end{aligned} \right\} \begin{aligned} V_1 + V_2 + V_4 &= \left( \frac{27}{16} - \frac{9}{4} + \frac{3}{16} \right) qe = -\frac{6}{16} qe = -\frac{3}{8} qe \\ V_1 + V_3 + V_4 &= \left( \frac{27}{16} + \frac{15}{4} + \frac{3}{16} \right) qe = \frac{96}{16} qe = \frac{45}{8} qe \end{aligned}$$

$$N_1 = \frac{6EJ}{e^2} \cdot \varphi_0 = \frac{6EJ}{e^2} \cdot \frac{9}{16} \frac{qe^3}{EJ} = \frac{27}{8} qe$$

$$N_2 = \frac{3EJ}{e^3} \cdot \varphi_0 = \frac{3EJ}{e^3} \cdot \frac{3}{4} \frac{qe^4}{EJ} = \frac{9}{4} qe$$

$$N_3 = \frac{8 EJ}{e^3} \cdot M_0 = \frac{8 EJ}{e^3} \cdot \frac{3}{4} \frac{q e^4}{EJ} = 6 q e$$

$$N_4 = \frac{12 EJ}{e^3} \cdot M_0 = \frac{12 EJ}{e^3} \cdot \frac{3}{4} \frac{q e^4}{EJ} = 9 q e$$

$$N_5 = \frac{111}{8} q e$$

$$N_2 + N_3 + N_5 = \left( \frac{9}{4} + 6 - \frac{111}{8} \right) q e = - \frac{45}{8} q e$$

$$N_1 + N_4 = \left( \frac{27}{8} q e - 9 q e \right) = - \frac{45}{8} q e$$

$$x'_{ED} \rightarrow V=0 \rightarrow M=M_{\max} \rightarrow \frac{3}{8} q e - 6 q x'_{ED} = 0 \rightarrow x'_{ED} = \frac{3}{48} e = \frac{1}{16} e$$

$$\begin{aligned} M_{\max} \rightarrow M_{\max} &= \frac{3}{8} q e x'_{ED} - 6 q x'_{ED} \frac{x'_{ED}}{2} = \frac{3}{8} q e \frac{1}{16} e - 6 q \frac{1}{16} e \frac{1}{32} e = \frac{3}{128} q e^2 - \frac{3}{256} q e^2 = \\ &= \frac{3}{256} q e^2 \end{aligned}$$

$$x''_{ED} \rightarrow M=0 \rightarrow \frac{21}{8} q e^2 + 6 q x''_{ED} \frac{x''_{ED}}{2} - \frac{45}{8} q e x''_{ED} = 0$$

$$\frac{21}{8} q e^2 + 3 q x''_{ED}^2 - \frac{45}{8} q e x''_{ED} = 0$$

$$0,875 q e^2 + q x''_{ED}^2 - 1,875 q e x''_{ED} = 0$$

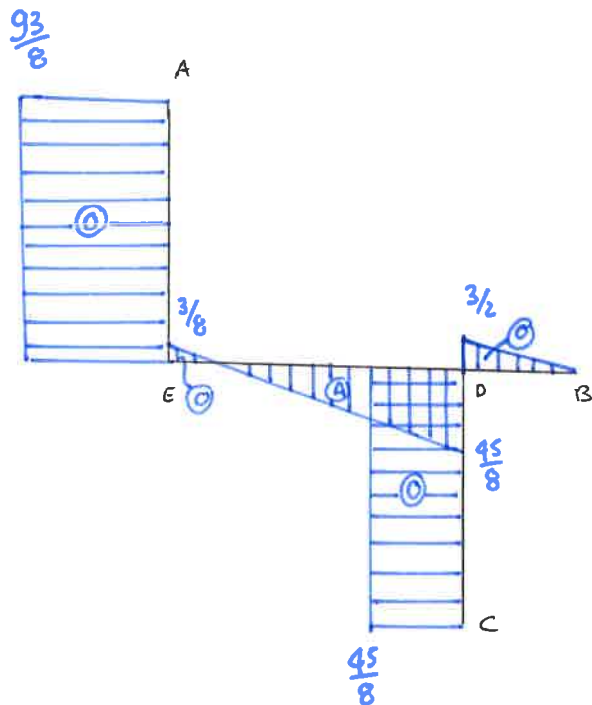
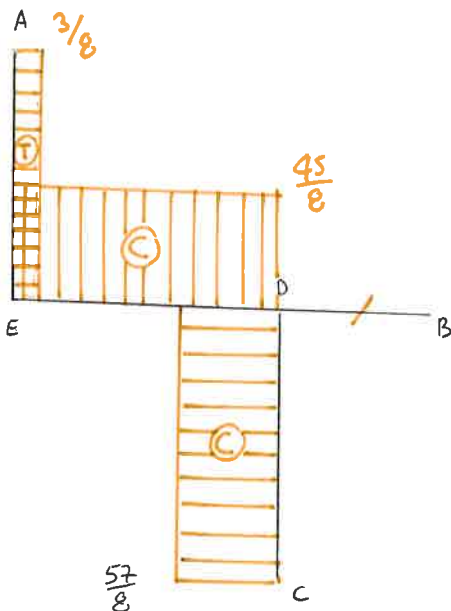
$$q x''_{ED}^2 - 1,875 q e x''_{ED} + 0,875 q e^2 = 0$$

$$x''_{ED} = \frac{1,875 \pm \sqrt{3,516 - 3,5}}{2} = \frac{1,875 \pm 0,126}{2} \begin{cases} x''_{ED_1} \approx 1 e \\ x''_{ED_2} = 0,874 e = \frac{7}{8} e \end{cases}$$

DIAGRAMMI

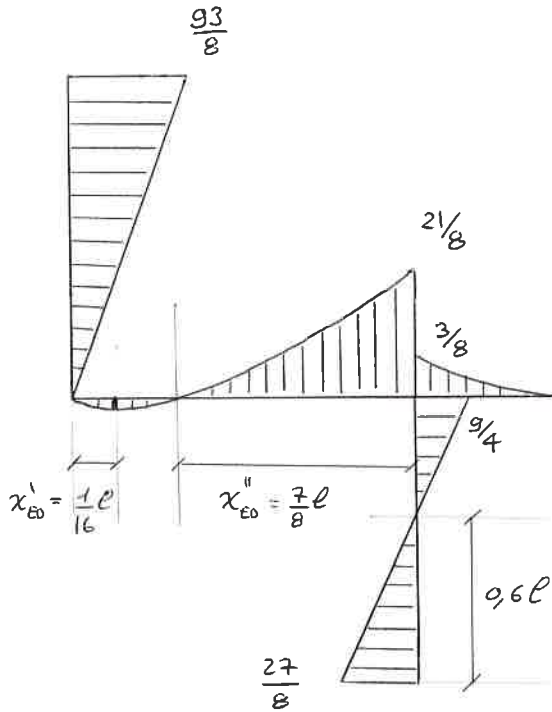
(N)

(V)



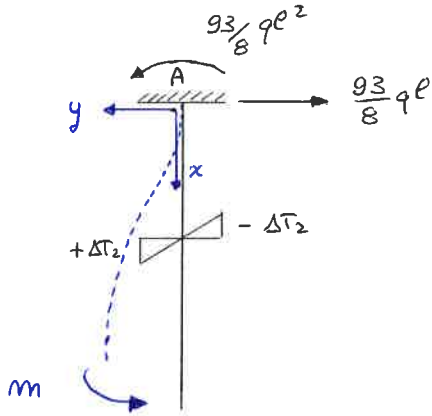
(8)

(M)



STUDIO DELLA DEFORMATA TERMICA

ASTA AE



$$m = \frac{93}{8} qlx - \frac{93}{8} ql^2 \quad \left\{ \begin{array}{l} \text{(+)} \end{array} \right.$$

$$y'' = - \frac{M(x)}{EI} - \frac{2 \alpha \Delta T_2}{H}$$

$$= - \frac{1}{EI} \left( \frac{93}{8} qlx - \frac{93}{8} ql^2 \right) - 2 \cdot \frac{37}{8} \frac{ql^2}{EI}$$

$$= - \frac{93}{8} \frac{qlx}{EI} + \frac{93}{8} \frac{ql^2}{EI} - \frac{74}{8} \frac{ql^2}{EI} \geq 0 \rightarrow$$

$$\rightarrow - \frac{93}{8} \frac{qlx}{EI} + \frac{19}{8} \frac{ql^2}{EI} \geq 0 \rightarrow$$

$$\frac{93}{8} \frac{qlx}{EI} \leq \frac{19}{8} \frac{ql^2}{EI} \rightarrow x \leq \frac{19}{93} l \approx 0,2l$$

DEFORMATA QUALITATIVA

