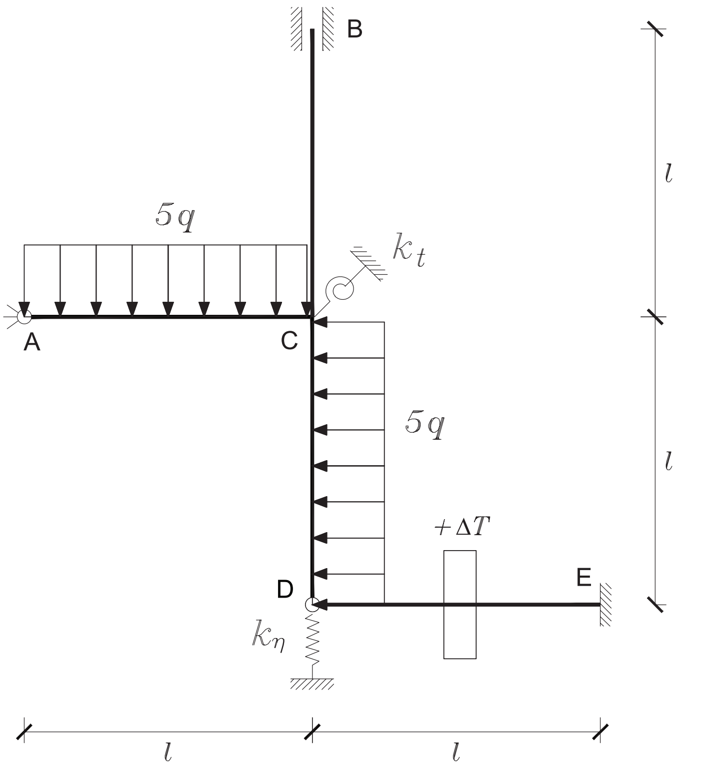
# TECNICA DELLE COSTRUZIONI

## TEMA ESAME DEL 03 FEBBRAIO 2020

DOCENTI:
**PROF. GIOVANNI A. PLIZZARI**

DURATA: **2 ORE.**




$$k_\eta = \frac{31}{2}\frac{EJ}{l^3} \qquad k_t = 3\frac{EJ}{l} \qquad \alpha\Delta T = 3\frac{ql^3}{EJ}$$

**Esercizio**

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

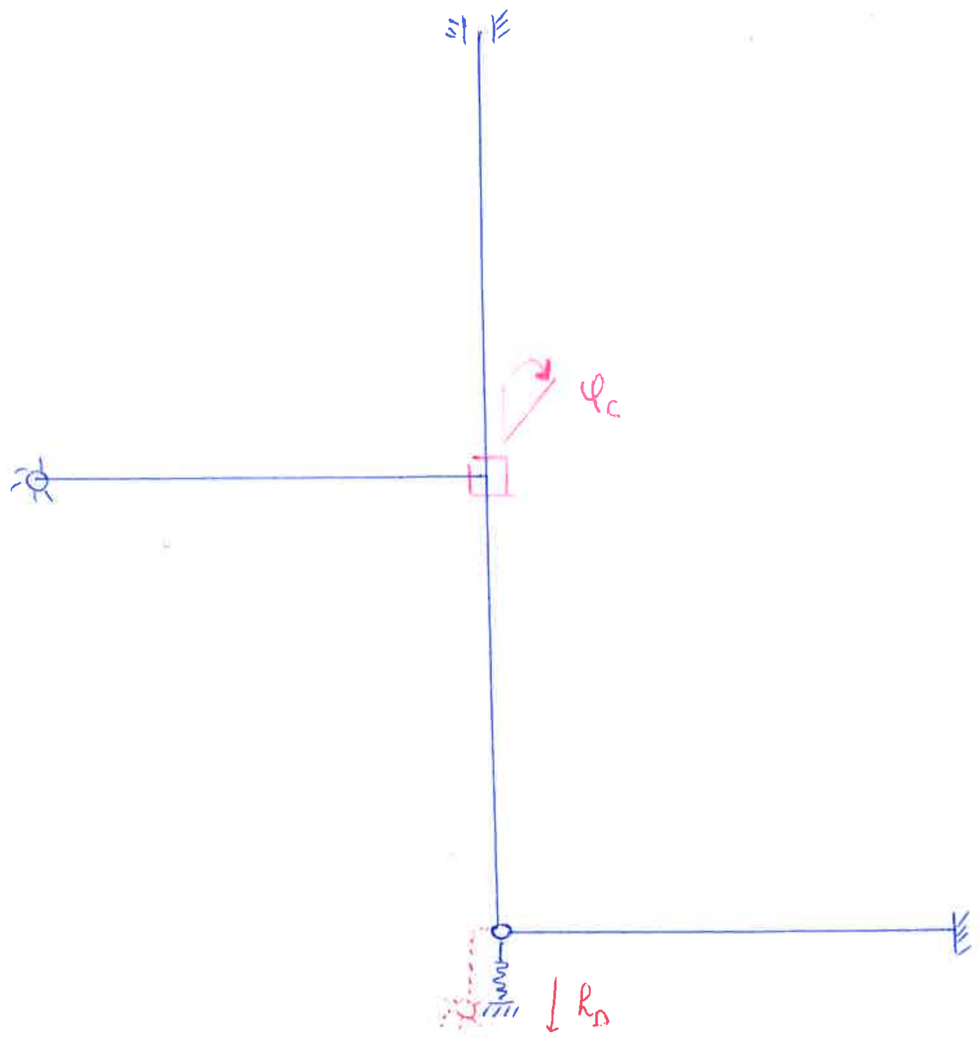
I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

## ANALISI CINEMATICA

6 GDL
11 GDV }

STRUTTURA A NODI SPOSTABILI

## STRUTTURA DI SERVIZIO E SCELTA DELLE INCOGNITE



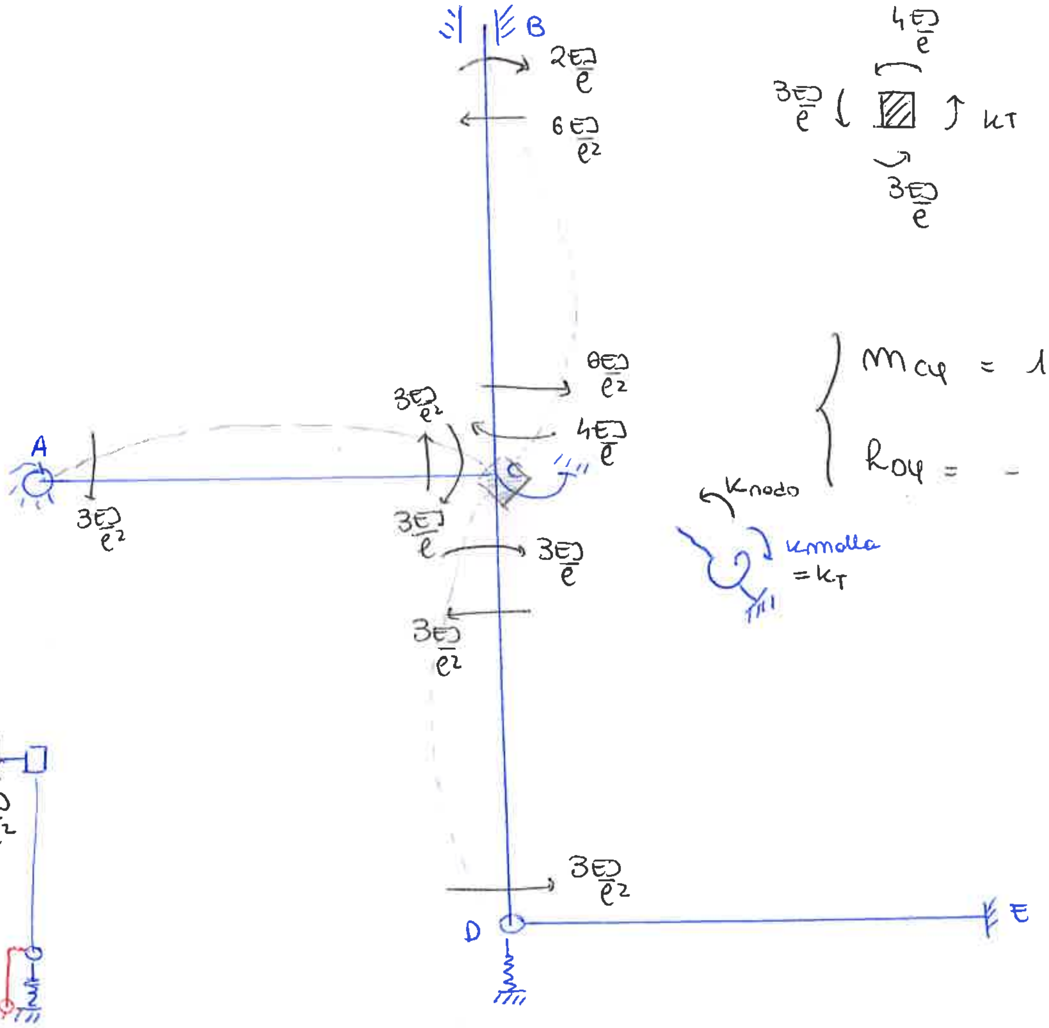

## EQUAZIONI DI EQUILIBRIO

$$
\begin{cases}
m_{C\varphi}\,\varphi_C + m_{Cr}\,r_D + m_{C\bar{q}} + m_{C\Delta T} = 0 \\
R_{D\varphi}\,\varphi_C + R_{Dr}\,r_D + R_{D\bar{q}} + R_{D\Delta T} = 0
\end{cases}
$$

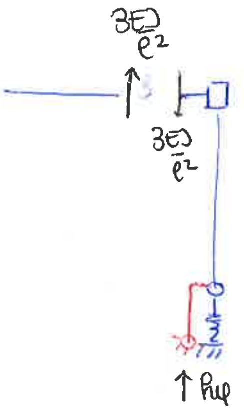
RISOLUZIONE

CONVENZIONE $\varphi$ +↺ $M$ +↻ ← ⊕ →

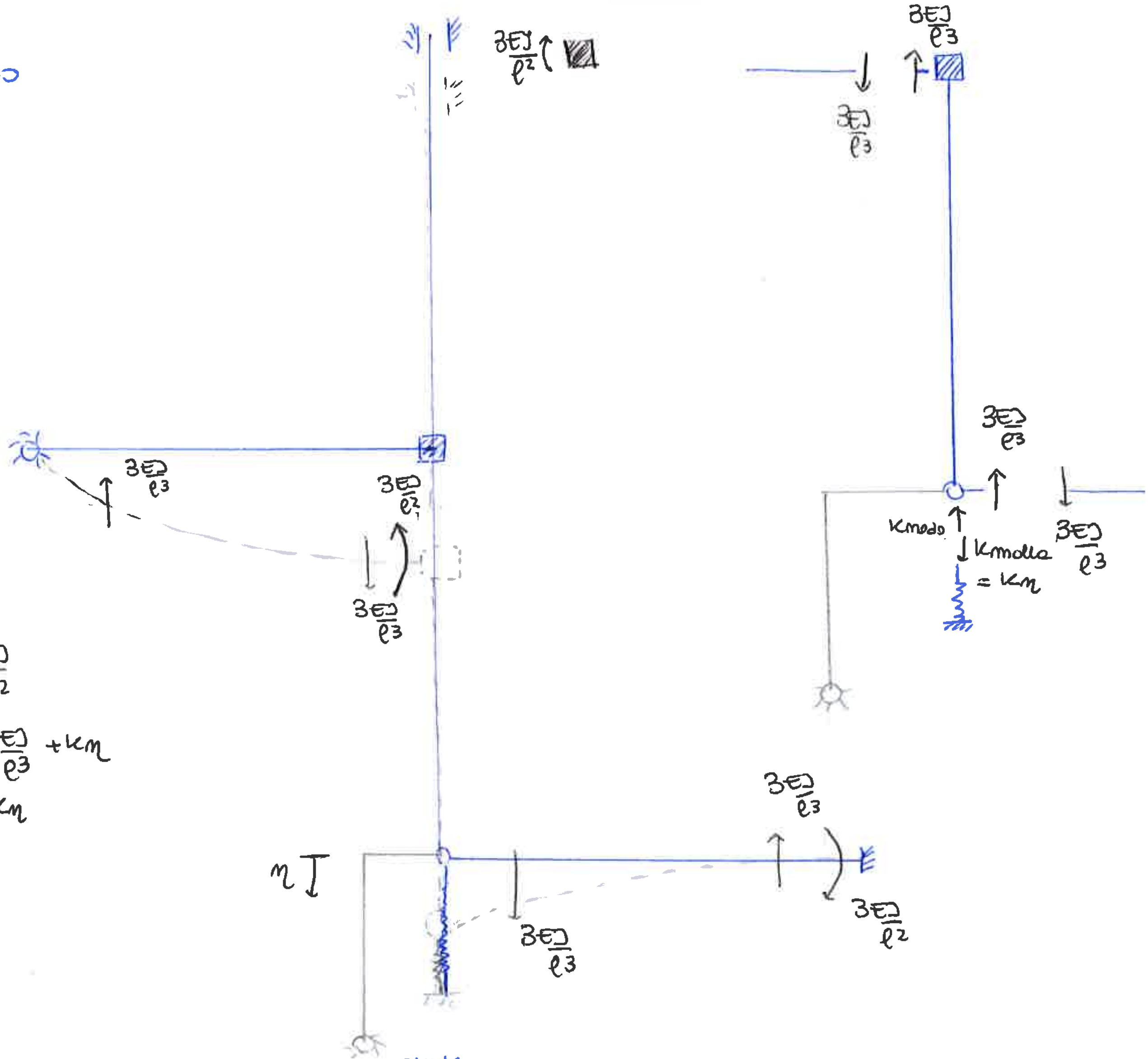
$$\begin{cases} \varphi_C = 1 \\ R_D = \Delta T = q = 0 \end{cases}$$




$$\begin{cases} m_{C\varphi} = 10\,\frac{EJ}{l} + k_T \\ R_{D\varphi} = -\frac{3EJ}{l^2} \end{cases}$$

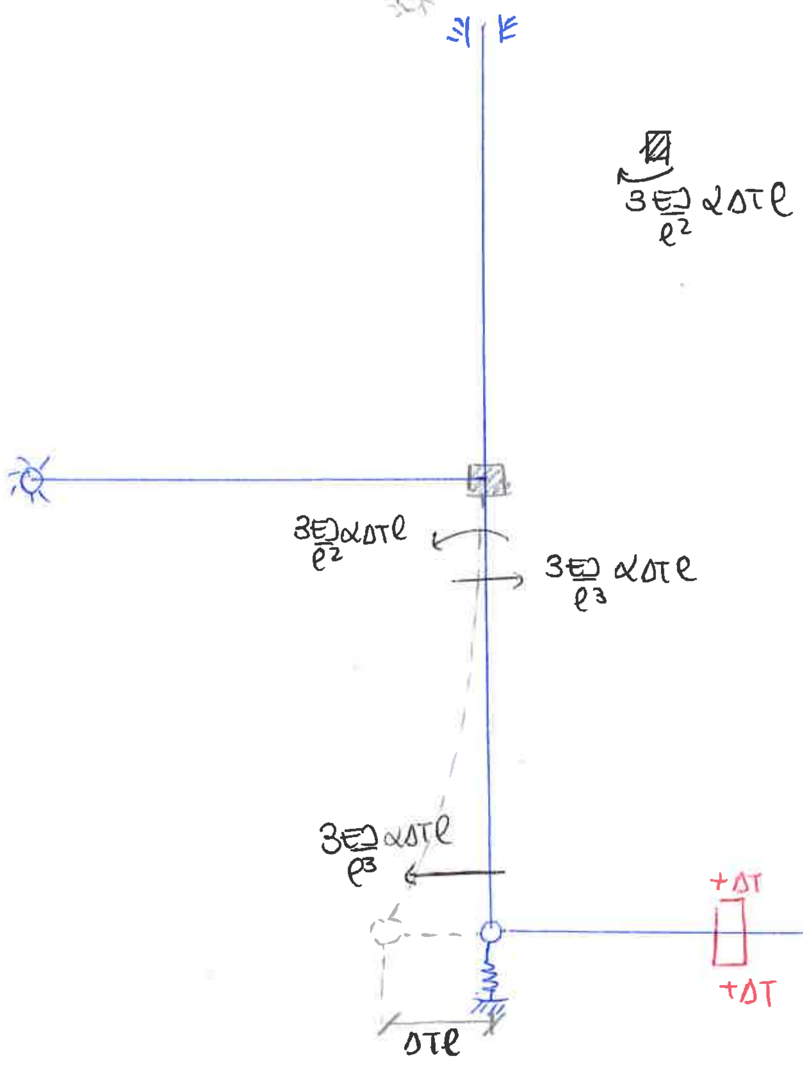


$$\begin{cases} \eta_0 = 1 \\ \varphi_c = q = \Delta T = 0 \end{cases}$$



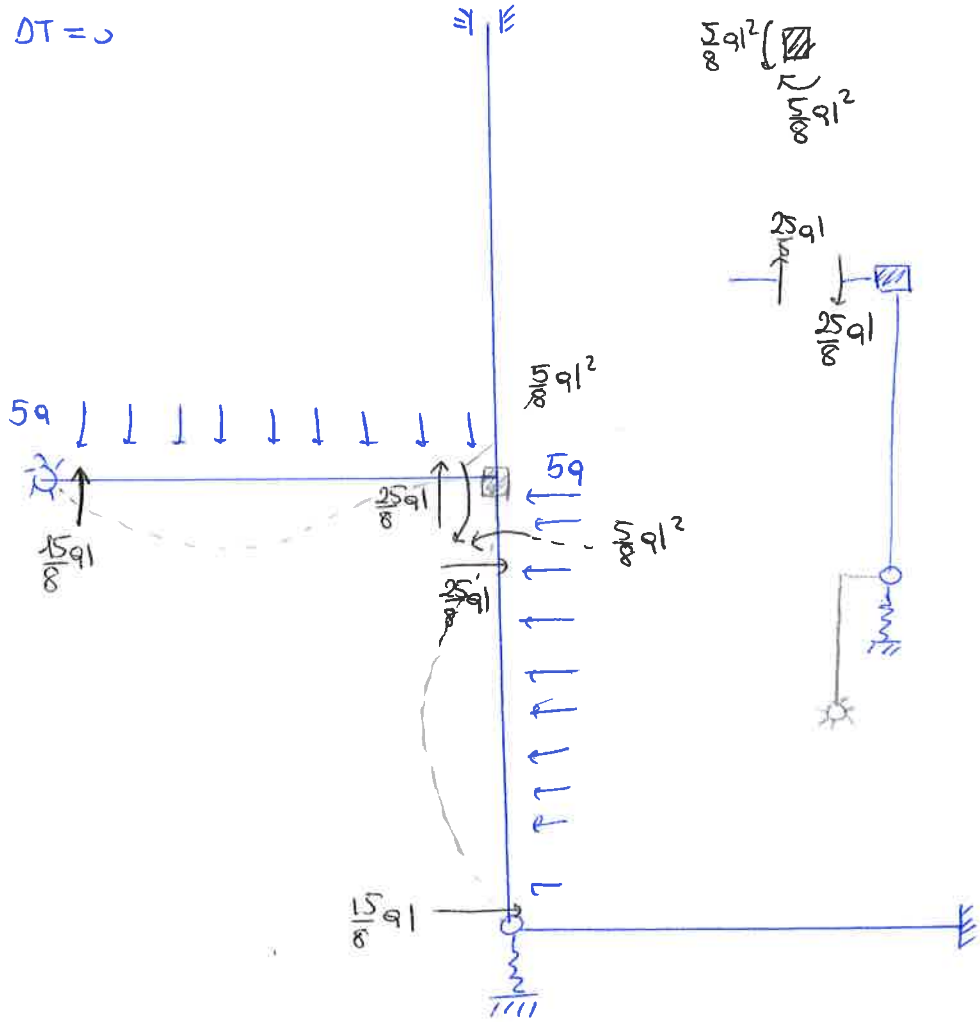

$$\begin{cases} m_{c\eta} = -\dfrac{3EJ}{\ell^2} \\ R_{\eta\eta} = \dfrac{3EJ}{\ell^3} + \dfrac{3EJ}{\ell^3} + k_\eta \\ \quad = \dfrac{6EJ}{\ell^3} + k_\eta \end{cases}$$

$$\begin{cases} \Delta T \neq 0 \\ \varphi_c = \eta_0 = q = 0 \end{cases}$$




$$\begin{cases} m_{c\Delta T} = -\dfrac{3EJ}{\ell^2}\alpha\Delta T\ell \\ R_{\eta\Delta T} = 0 \end{cases}$$

$$\begin{cases} q \neq 0 \\ \varphi_C = R_D = \Delta T = 0 \end{cases}$$




$$\begin{cases} M_{cq} = 0 \\ R_{Dq} = -\frac{25}{8}ql \end{cases}$$

EQUAZIONI RISOLVENTI

$$\begin{cases} \left(10\frac{EJ}{l} + K_T\right)\varphi - \frac{3EJ}{l^2} \qquad - \frac{3EJ}{l^2}\alpha\Delta T l = 0 \\ -\frac{3EJ}{l^2}\varphi + \left(6\frac{EJ}{l^2} + K_\eta\right)R - \frac{25}{8}ql = 0 \end{cases}$$

$$\begin{cases} \left(10\frac{EJ}{l} + \frac{3EJ}{l}\right)\varphi - \frac{3EJ}{l^2}\eta - \frac{3EJ}{l}\cdot\frac{3ql^3}{EJ} = 0 \\ -\frac{3EJ}{l^2}\varphi + \left(\frac{6EJ}{l^3} + \frac{31}{2}\frac{EJ}{l^3}\right)R - \frac{25}{8}ql = 0 \end{cases}$$

$$\begin{cases} 13\frac{EJ}{l}\varphi - \frac{3EJ}{l^2}R - 9ql^2 = 0 \\ -\frac{3EJ}{l^2}\varphi + \frac{43}{2}\frac{EJ}{l^3}R - \frac{25}{8}ql = 0 \end{cases}$$

$$\begin{cases} \varphi = \left( \frac{3EJ}{l^2} R + 9ql^2 \right) \frac{1}{13} \frac{l}{EJ} = \frac{3}{13} \frac{R}{l} + \frac{9}{13} \frac{ql^3}{EJ} \\ -\frac{3EJ}{l^2} \left( \frac{3}{13} \frac{R}{l} + \frac{9}{13} \frac{ql^3}{EJ} \right) + \frac{43}{2} \frac{EJ}{l^3} R - \frac{25}{8} ql = 0 \end{cases}$$

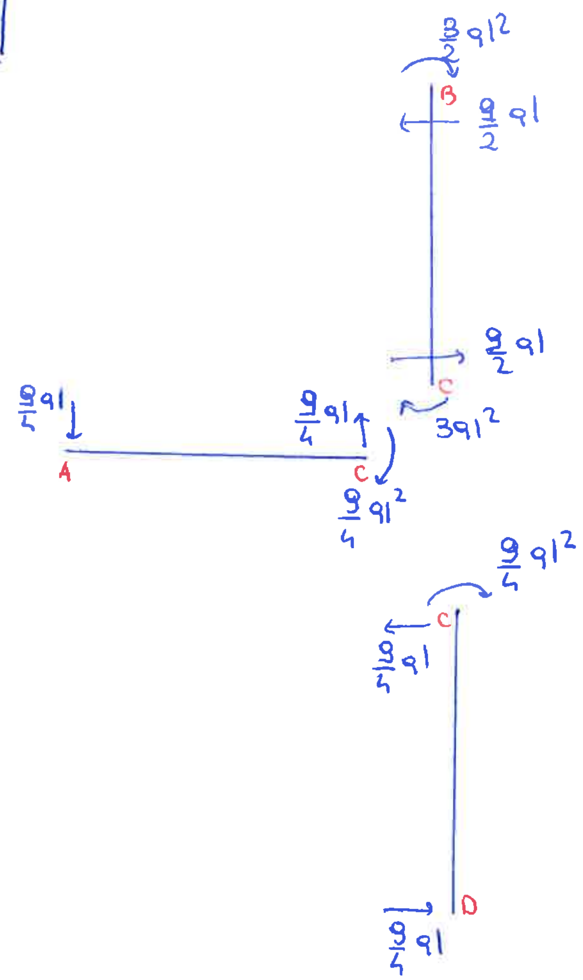
$$\hookrightarrow \quad -\frac{9}{13} \frac{EJ}{l^3} R - \frac{27}{13} ql + \frac{43}{2} \frac{EJ}{l^3} R - \frac{25}{8} ql = 0$$

$$\frac{541}{26} \frac{EJ}{l^3} R = \frac{541}{104} ql \quad \rightarrow \quad \boxed{R_D = \frac{1}{4} \frac{ql^4}{EJ}}$$

$$\varphi = \frac{3}{13} \cdot \frac{1}{4} \frac{ql^3}{EJ} + \frac{9}{13} \frac{ql^3}{EJ} = \left( \frac{3}{52} + \frac{9}{13} \right) \frac{ql^3}{EJ} \Rightarrow \boxed{\varphi_C = \frac{3}{4} \frac{ql^3}{EJ}}$$

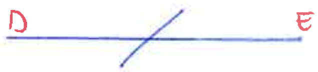
CARICHI

$$\boxed{\varphi_C = \frac{3}{4} \frac{ql^3}{EJ}}$$

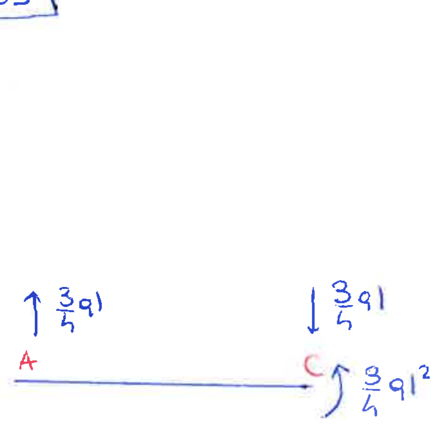



$$R_{molla} = k_T \cdot \varphi = 3 \frac{EJ}{l} \cdot \frac{3}{4} \frac{ql^3}{EJ} = \frac{9}{4} ql^2$$

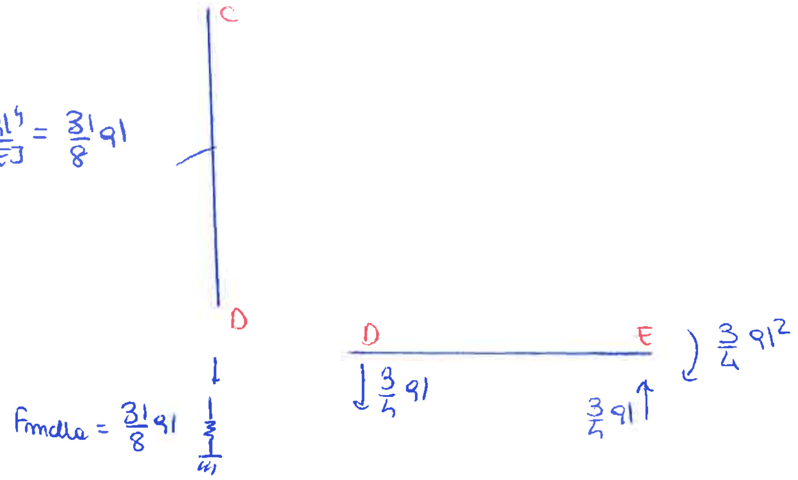
$$R_{molla} = \frac{9}{4} ql^2$$



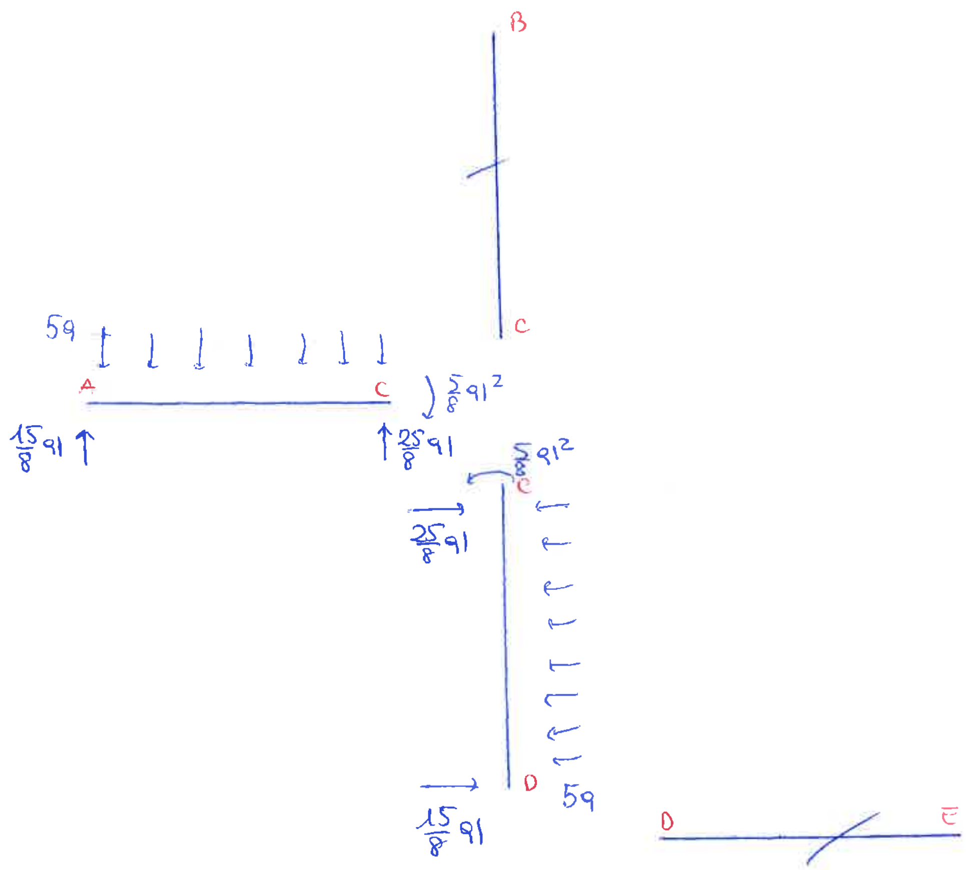
$$\boxed{\delta_D = \frac{1}{4}\frac{ql^4}{EJ}}$$



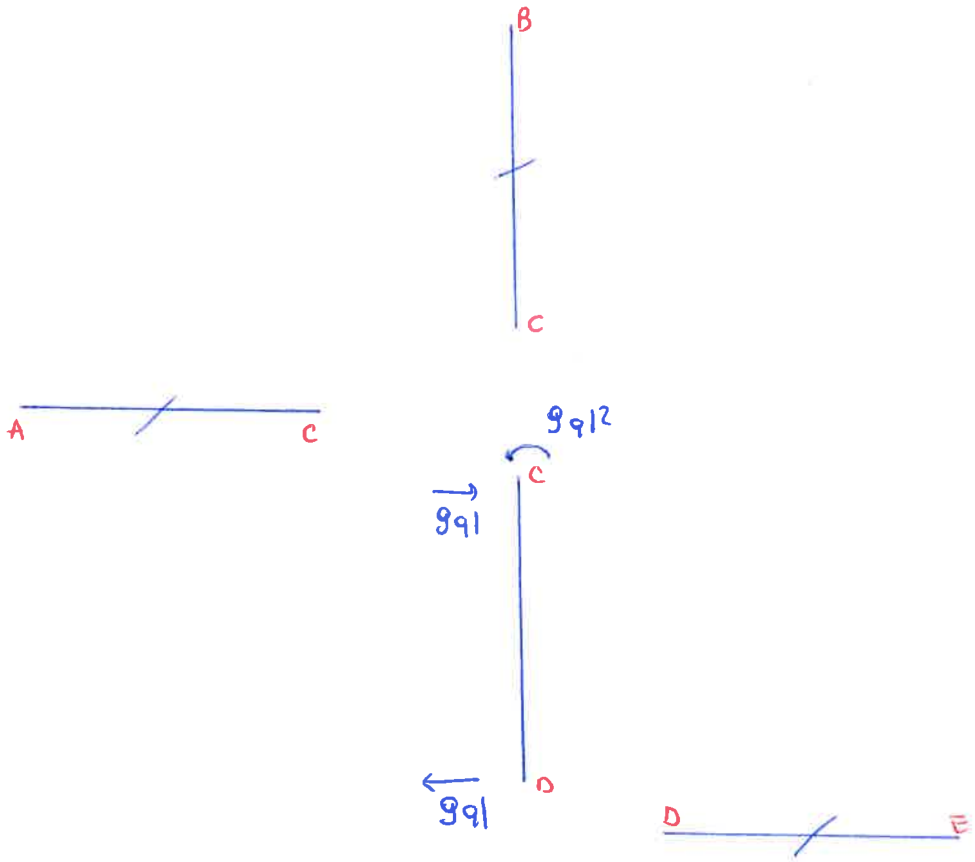

$$F_{molla} = k_m \cdot \delta = \frac{3}{2}\frac{EJ}{l^3}\cdot\frac{1}{4}\frac{ql^4}{EJ} = \frac{3}{8}ql$$



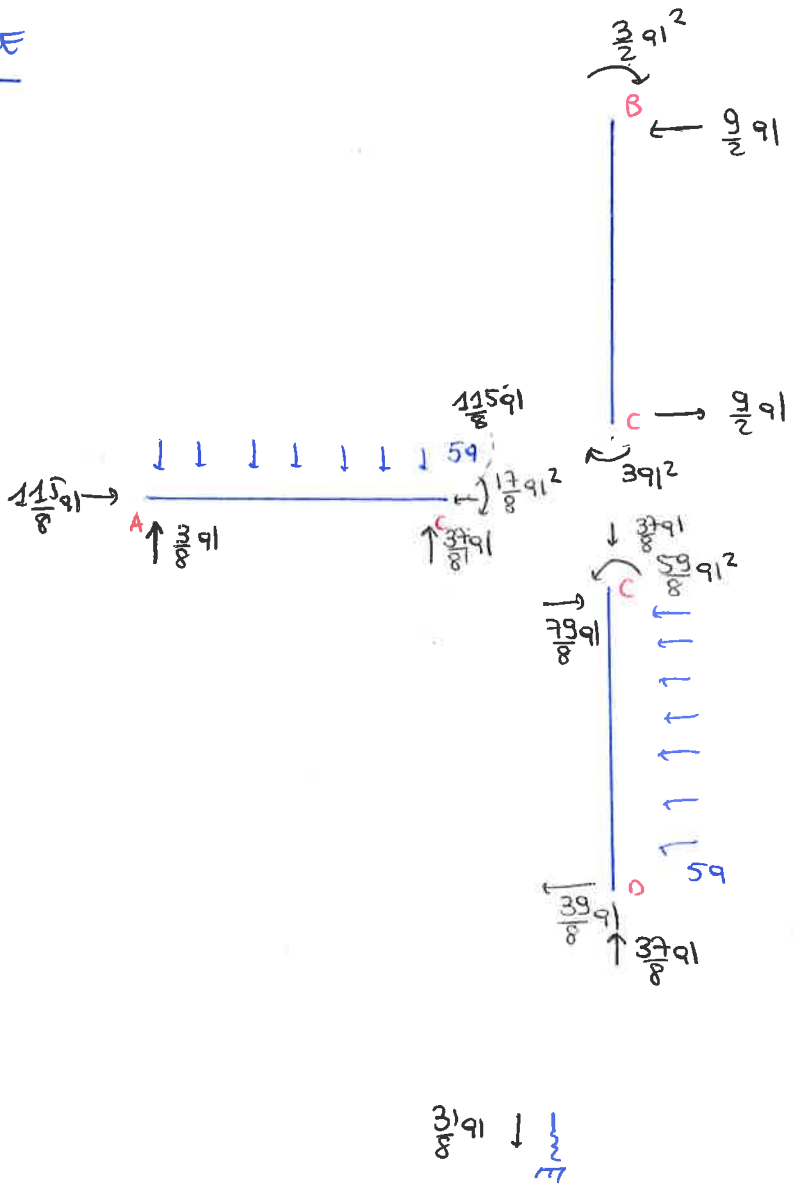

$\boxed{9}$

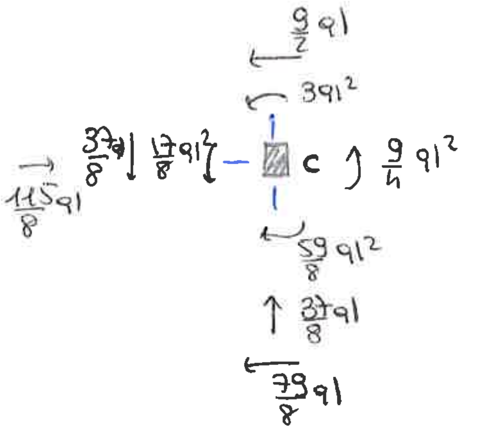


$\boxed{\Delta T}$

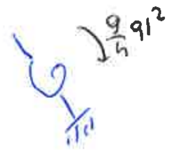


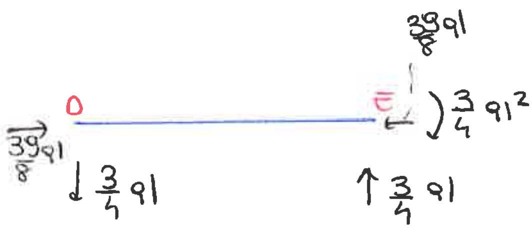

GLOBALE




EQUILIBRATO ✓



$X_1 \rightarrow \frac{3}{8} ql - 5q x_1 = 0 \longrightarrow x_1 = \frac{3}{40} l$

$$M_{max} = \frac{3}{8} ql \cdot \frac{3}{40} l - 5q \cdot \frac{3}{40} l \cdot \frac{3}{80} l = \frac{9}{320} ql^2 - \frac{45}{3200} ql^2$$

$$= \frac{45}{3200} ql^2$$

$$\longrightarrow M_{max} = \frac{9}{640} ql^2$$

FLESSI

$X_2 \rightarrow \frac{3}{2} ql^2 - \frac{9}{2} ql x_2 = 0 \longrightarrow x_2 = \frac{1}{3} l$

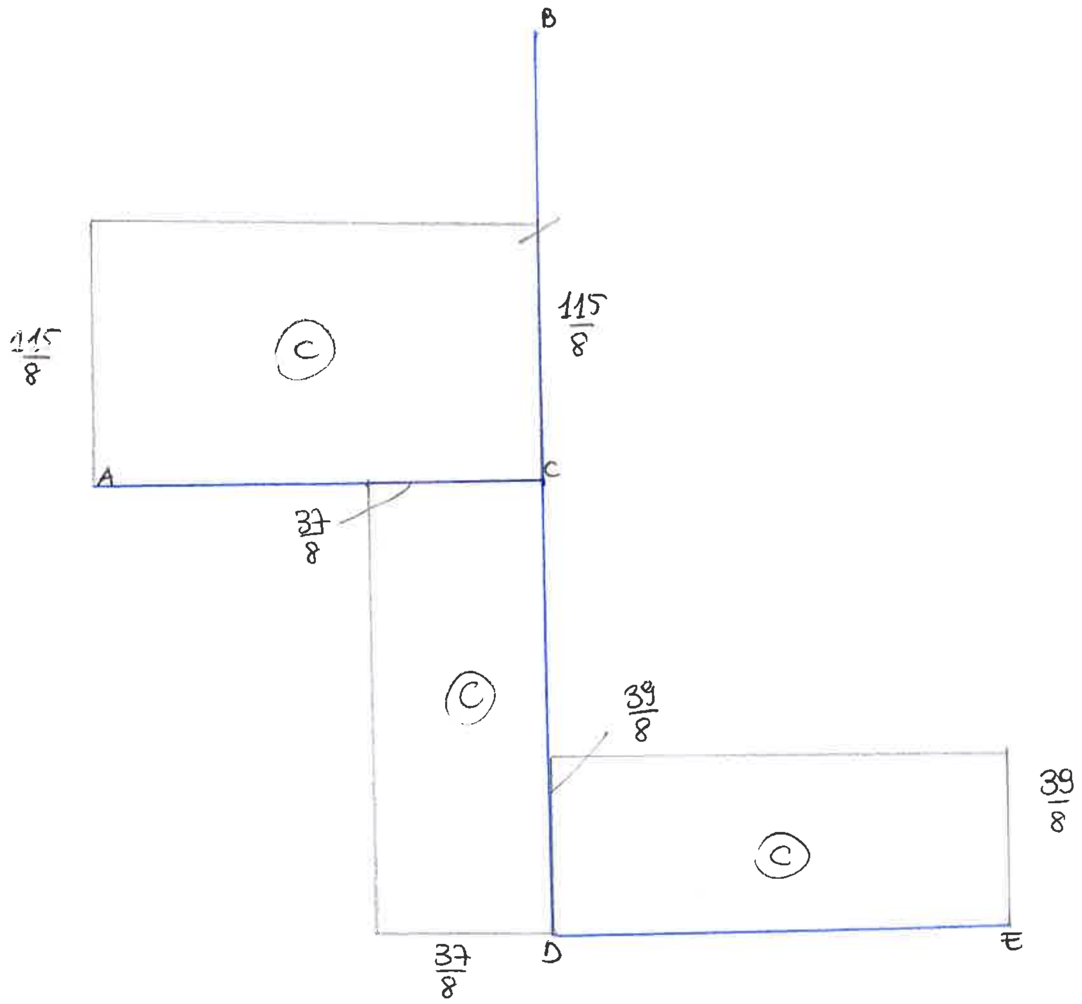
$X_3 \rightarrow \frac{3}{8} ql x - 5q x \frac{x}{2} = 0 \qquad \frac{3}{8} qlx - \frac{5qx^2}{2} \begin{cases} x_3 = 0 \\ x_3 = \frac{3}{20} l \end{cases}$

ALLUNGAMENTO DATO DA $\Delta T$

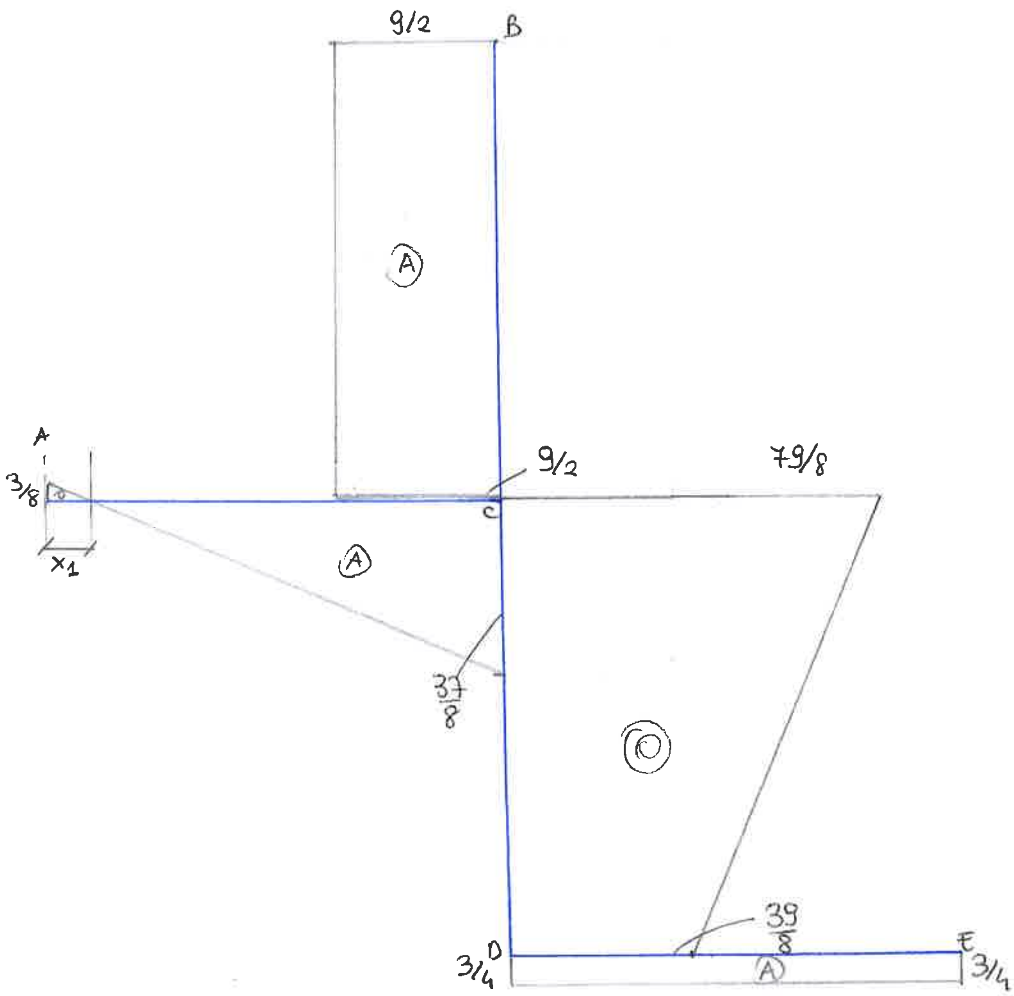
$$\Delta l = \alpha \Delta T l = \frac{3ql^4}{EJ}$$

DIAGRAMMI

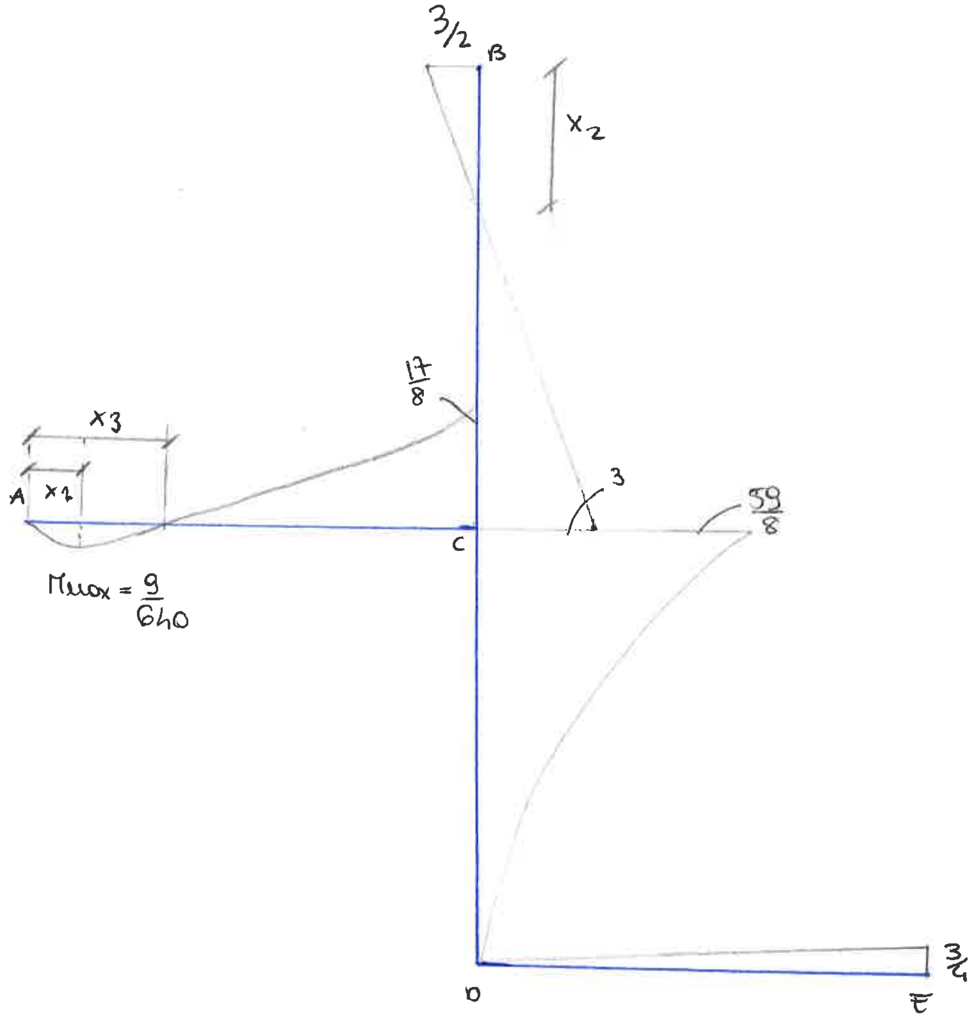
$\frac{N}{ql}$




$\frac{V}{ql}$



$\boxed{\frac{M}{q\ell^2}}$




DEFORMATA