TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 23 GIUGNO 2020

DOCENTI:
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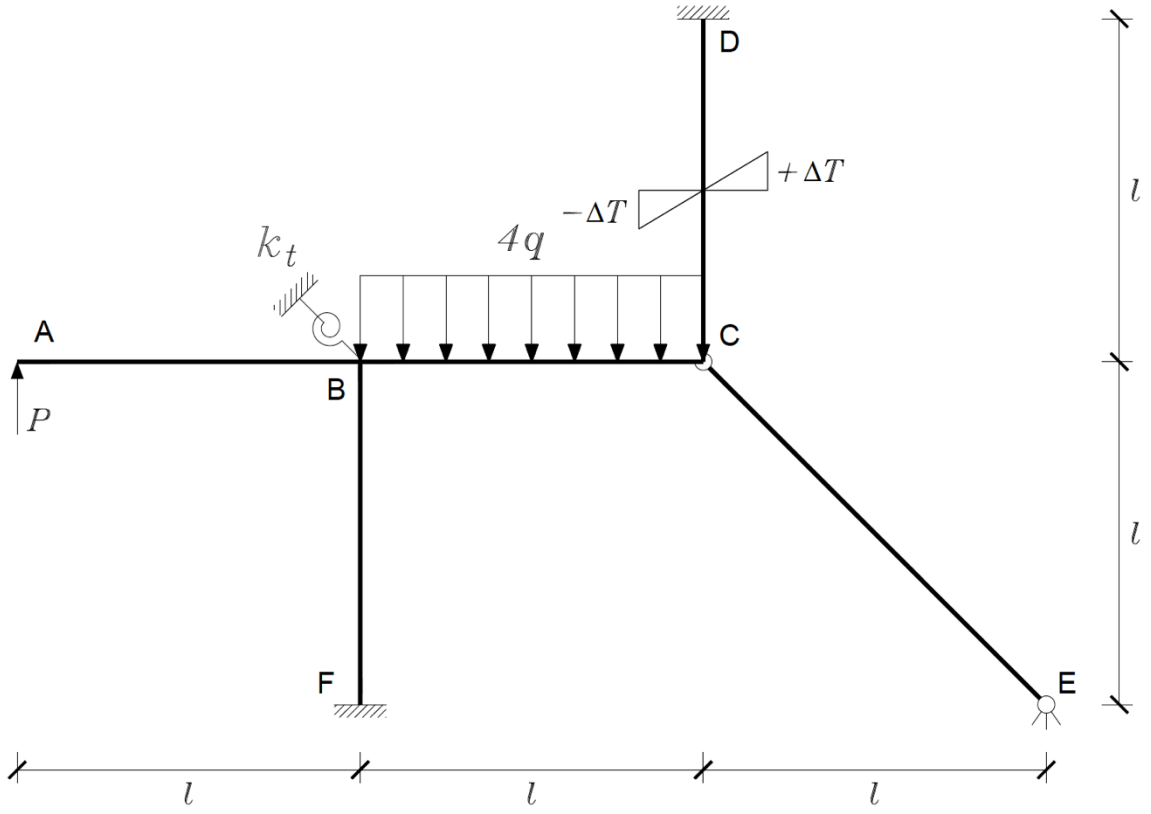
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**



$$k_t = 5\frac{EJ}{l} \qquad \frac{\alpha\Delta T}{h} = \frac{3}{4}\frac{ql^2}{EJ} \qquad P = ql$$

Esercizio

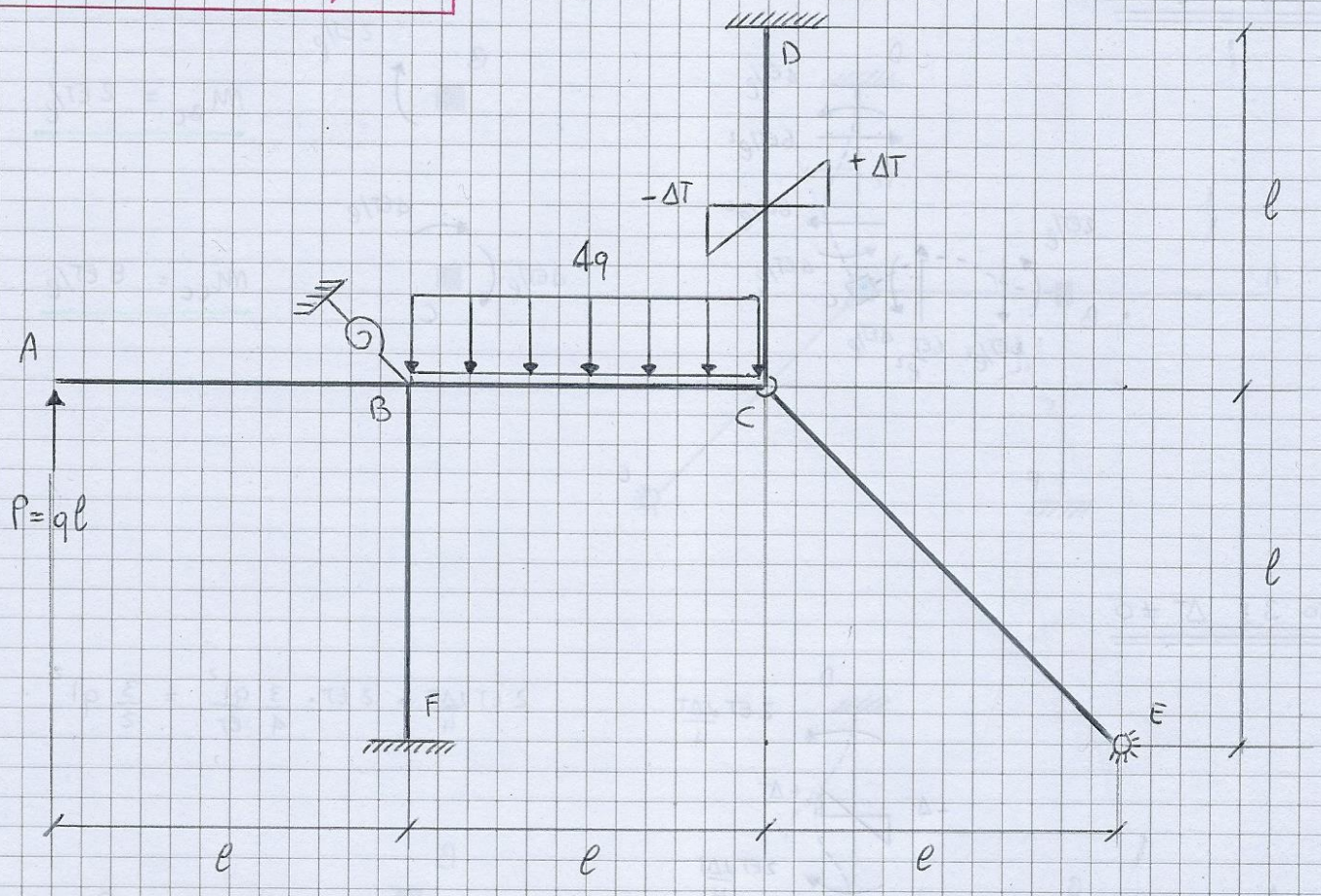
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TEMA ESAME 23/06/2020



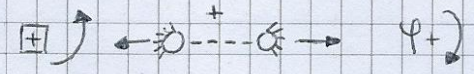
6 GRADI DI LIBERTÀ
11 GRADI DI VINCOLO (CONSIDERANDO ANCHE LA MOLLA TORSIONALE) → STRUTTURA 5 VOLTE IPERVINCOLATA

STRUTTURA A NODI FISSI; CURVETTI IN B E C, 2 INCOGNITE φ_B E φ_C

DATI: $k_\theta = 5\ EJ/\ell$

$$\frac{2\Delta T}{H} = \frac{3}{4}\frac{q\ell^2}{EJ}$$

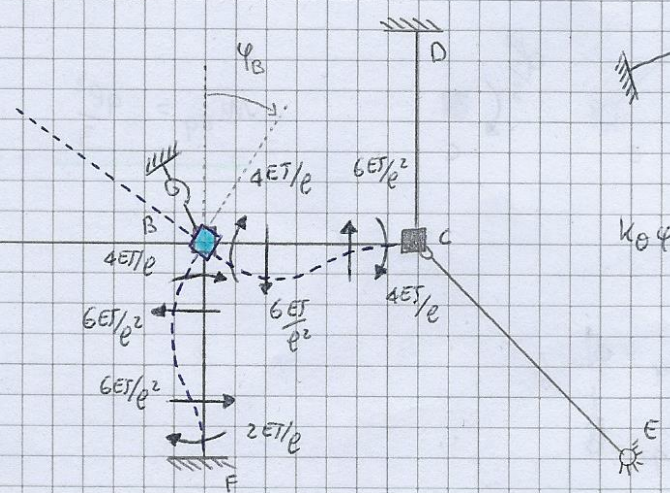
CONVENZIONI ADOTTATE:



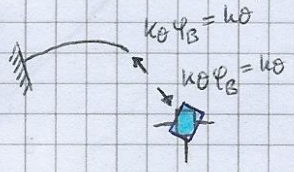
EQUAZIONI RISOLVENTI

$$\begin{cases} m_{BB}\varphi_B + m_{BC}\varphi_C + m_{B\frac{2\Delta T}{H}} + m_{Bq} = 0 \\ m_{CB}\varphi_B + m_{CC}\varphi_C + m_{C\frac{2\Delta T}{H}} + m_{Cq} = 0 \end{cases}$$

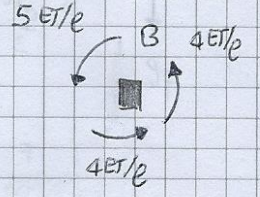
CASO 1: $\varphi_B = 1$



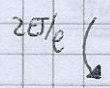
MOLLA TORSIONALE



$k_\theta \varphi_B = k_\theta = 5\ ET/\ell$

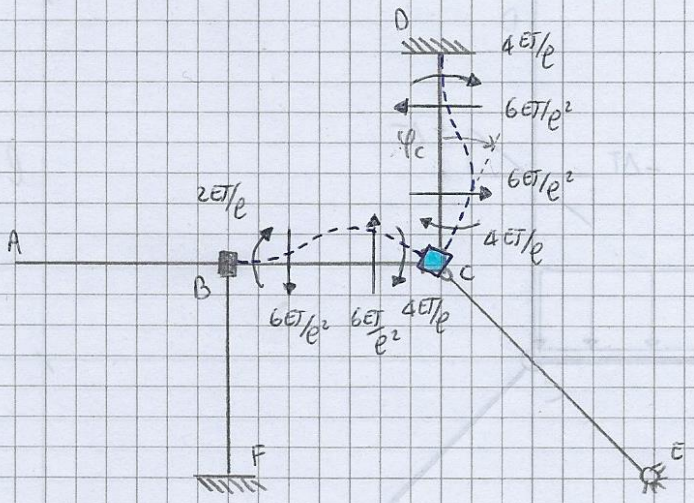


$m_{BB} = 13\ EJ/\ell$

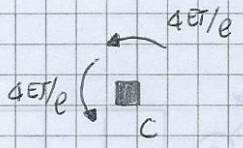


$m_{CB} = 2\ EJ/\ell$

CASO 2: $\varphi_C = 1$

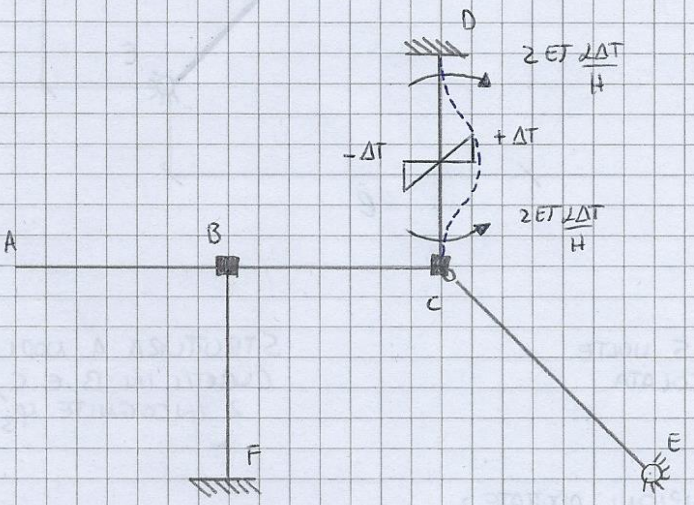


$$m_{BC} = 2EJ/\ell$$

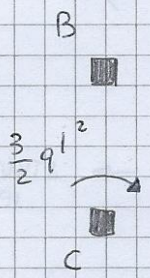


$$m_{CC} = 8EJ/\ell$$

CASO 3: $\Delta T \neq 0$



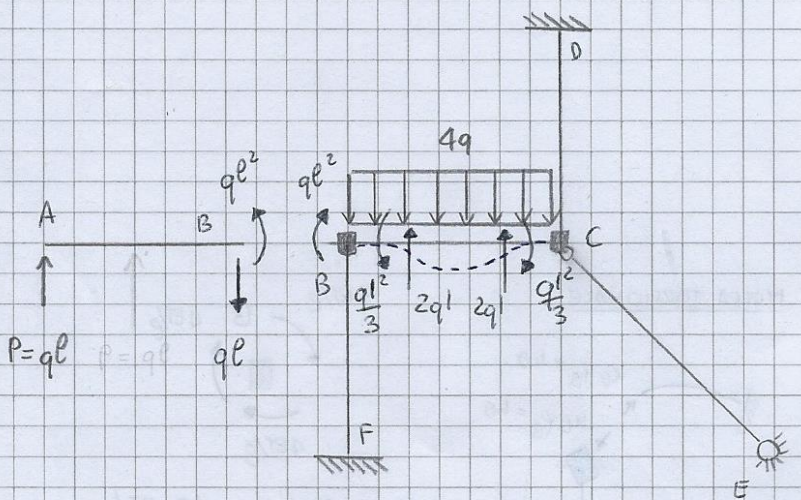
$$2EJ\frac{\alpha\Delta T}{H} = 2EJ \cdot \frac{3}{4}\frac{q\ell^2}{EJ} = \frac{3}{2}q\ell^2$$



$$m_{B\,\frac{\alpha\Delta T}{H}} = 0$$

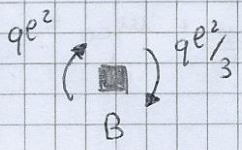
$$m_{C\,\frac{\alpha\Delta T}{H}} = -\frac{3}{2}q\ell^2$$

CASO 4: $q \neq 0$

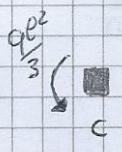


$$\frac{p\ell^2}{12} = \frac{4q\ell^2}{12} = \frac{q\ell^2}{3}$$ REAZIONE DATO DAL CARICO $4q$

$$\frac{p\ell}{2} = \frac{4q\ell}{2} = 2q\ell$$ AZIONE VERTICALE DATA DAL CARICO $4q$



$$m_{Bq} = -\frac{q\ell^2}{3} - q\ell^2 = -\frac{4}{3}q\ell^2$$



$$m_{Cq} = \frac{q\ell^2}{3}$$

SISTEMA RISOLVENTE

$$\begin{cases} m_{BB}\varphi_B + m_{BC}\varphi_C + m_{B\,\frac{\alpha\Delta T}{H}} + m_{Bq} = 0 \\ m_{CB}\varphi_B + m_{CC}\varphi_C + m_{C\,\frac{\alpha\Delta T}{H}} + m_{Cq} = 0 \end{cases}$$

$$\begin{cases} 13\,EJ/\ell\;\varphi_B + 2\,EJ/\ell\;\varphi_C + 0 - \frac{4}{3}q\ell^2 = 0 \\ 2\,EJ/\ell\;\varphi_B + 8\,EJ/\ell\;\varphi_C - \frac{3}{2}q\ell^2 + \frac{1}{3}q\ell^2 = 0 \end{cases}$$

$$\begin{cases} 13\,EJ/\ell\;\varphi_B + 2\,EJ/\ell\;\varphi_C - \frac{4}{3}q\ell^2 = 0 \\ 2\,EJ/\ell\;\varphi_B + 8\,EJ/\ell\;\varphi_C - \frac{7}{6}q\ell^2 = 0 \end{cases} \quad \text{(MOLTIPLICO PER (-4))}$$

$$\begin{cases} -52\,EJ/\ell\;\varphi_B - 8\,EJ/\ell\;\varphi_C + \frac{16}{3}q\ell^2 = 0 \\ 2\,EJ/\ell\;\varphi_B + 8\,EJ/\ell\;\varphi_C - \frac{7}{6}q\ell^2 = 0 \end{cases} \quad \text{(SOTTRAGO)}$$

$$-50\,EJ/\ell\;\varphi_B \;//\; + \frac{25}{6}q\ell^2 = 0 \rightarrow \varphi_B = \frac{25}{6}q\ell^2 \cdot \left(\frac{1}{50}\frac{\ell}{EJ}\right) \rightarrow \boxed{\varphi_B = \frac{1}{12}\frac{q\ell^3}{EJ}}$$

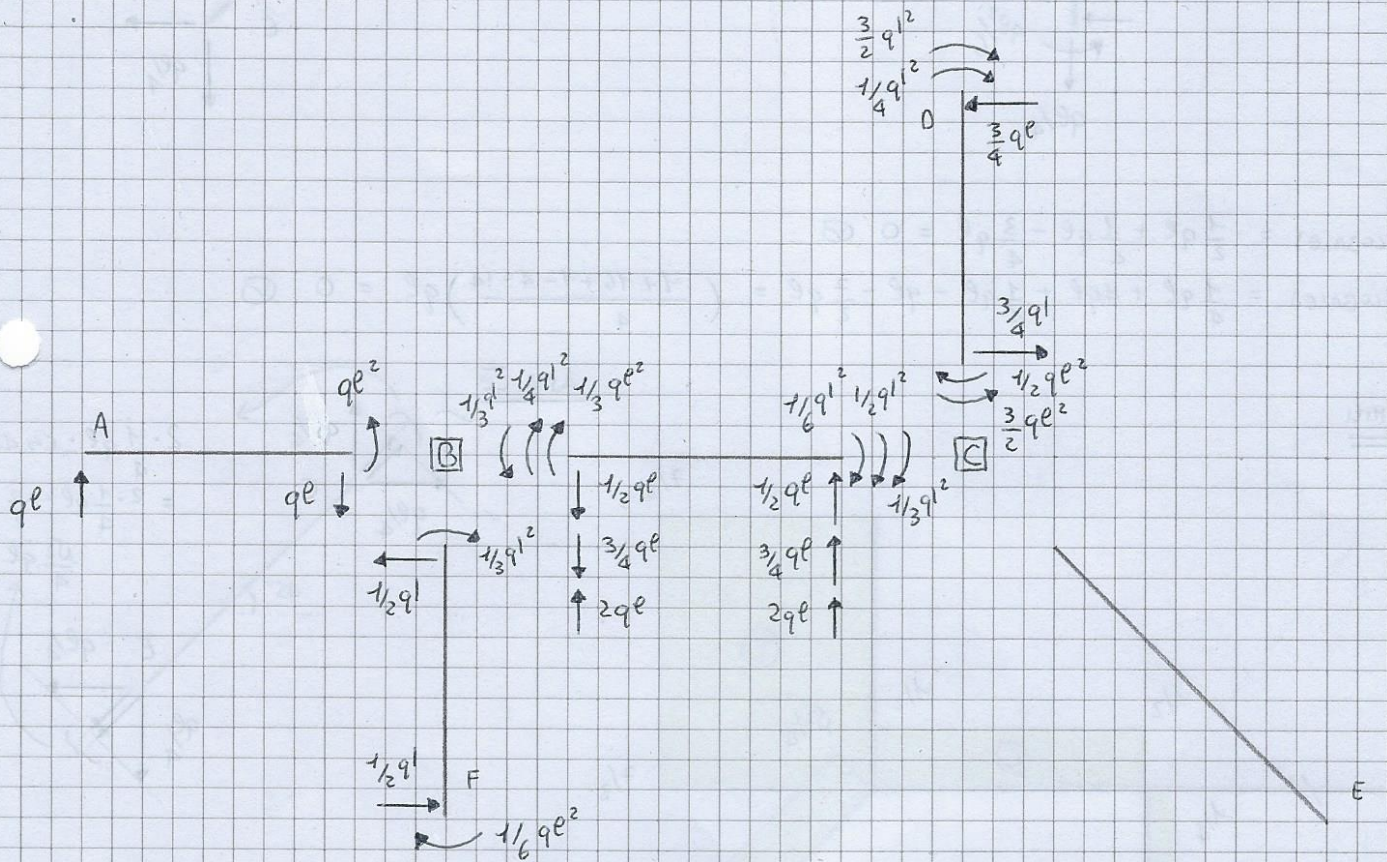
$$2\,EJ/\ell\;\varphi_B + 8\,EJ/\ell\;\varphi_C - \frac{7}{6}q\ell^2 = 0$$

$$2\,EJ/\ell \cdot \left(\frac{1}{12}\frac{q\ell^3}{EJ}\right) + 8\,EJ/\ell\;\varphi_C - \frac{7}{6}q\ell^2 = 0$$

$$\frac{1}{6}q\ell^2 + 8\,EJ/\ell\;\varphi_C - \frac{7}{6}q\ell^2 = 0$$

$$8\,EJ/\ell\;\varphi_C - q\ell^2 = 0 \rightarrow \boxed{\varphi_C = \frac{1}{8}\frac{q\ell^3}{EJ}}$$

AZIONI SULLE ASTE



AZIONI TOTALI SULLE ASTE ED EQUILIBRIO AI NODI

[MOLLA] $\frac{5}{12}qe^2$

$\Sigma' M_{(CB)} = 0$ (V)

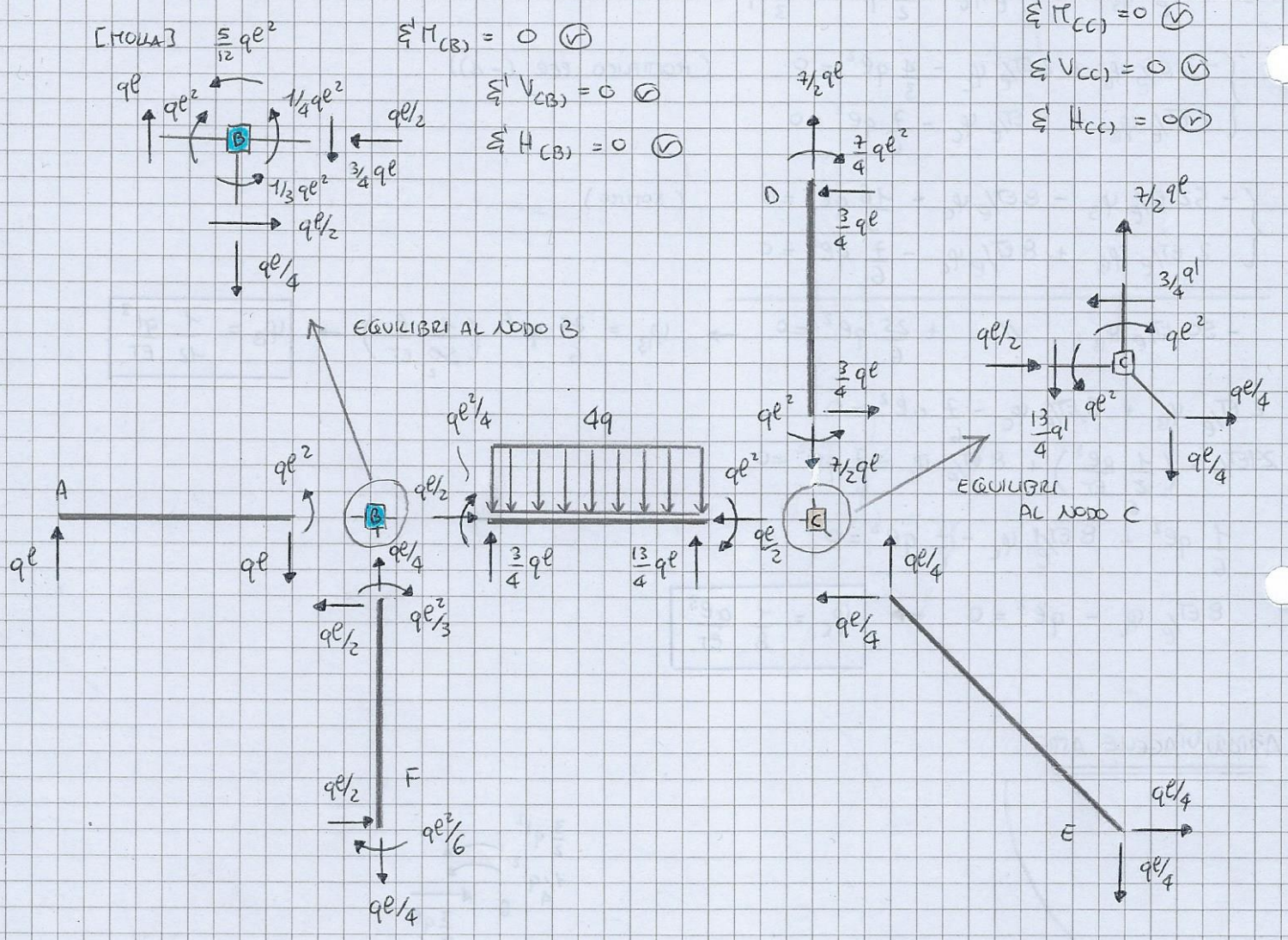
$\Sigma' V_{(CB)} = 0$ (V)

$\Sigma' H_{(CB)} = 0$ (V)

$\Sigma' M_{(CC)} = 0$ (V)

$\Sigma' V_{(CC)} = 0$ (V)

$\Sigma' H_{(CC)} = 0$ (R)

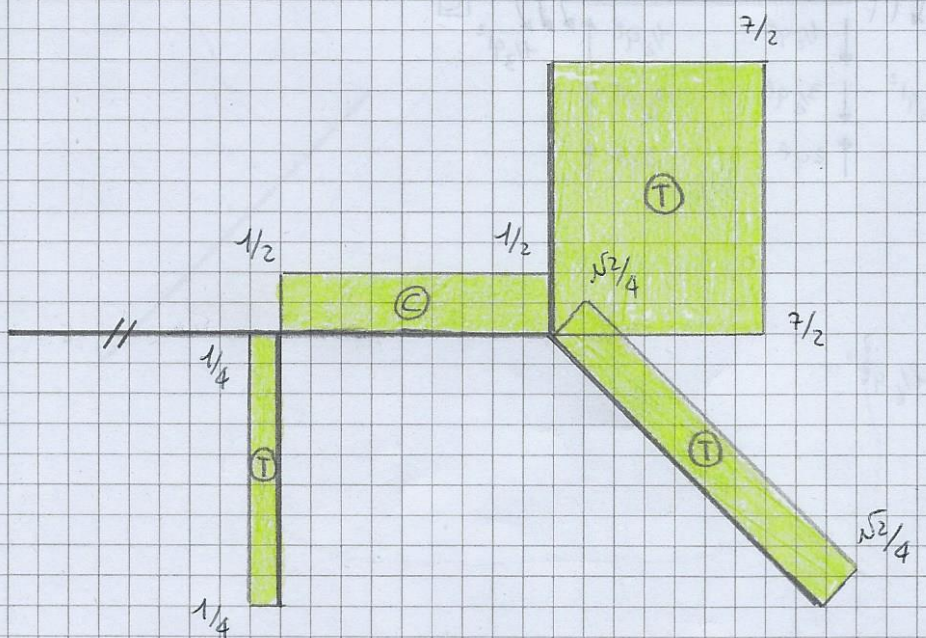


$\Sigma' H_{(GLOBALE)} = \frac{1}{2}qe + \frac{1}{4}qe - \frac{3}{4}qe = 0$ (V)

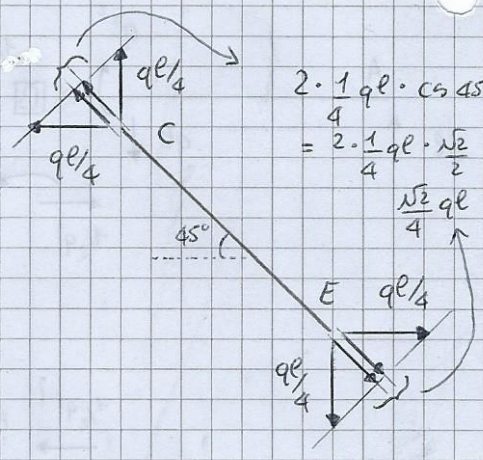
$\Sigma' V_{(GLOBALE)} = \frac{1}{4}qe + 4qe + \frac{1}{4}qe - qe - \frac{7}{2}qe = \left(\frac{1+16+1-4-14}{4}\right)qe = 0$ (V)

DIAGRAMMI

$\frac{N}{qe}$

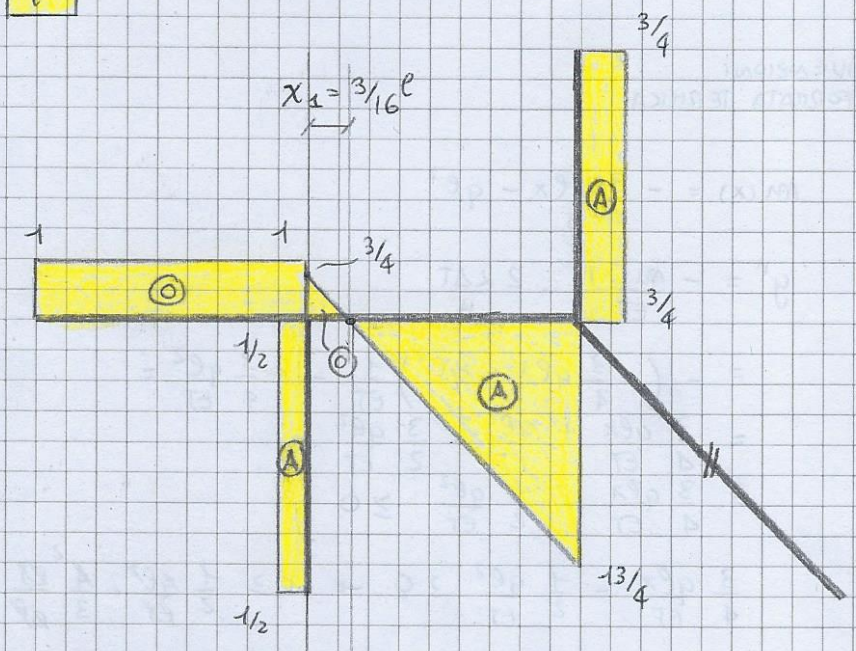


ASTA $\overline{CE}$



$2 \cdot \frac{1}{4}qe \cdot \cos 45° = 2 \cdot \frac{1}{4}qe \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}qe$

$\frac{V}{qe}$



▲ PUNTO DI ANNULLAMENTO DEL TAGLIO (x_1), PUNTO DI MOMENTO MASSIMO:

$$x_1 \rightarrow \frac{3}{4}ql - 4qx_1 = 0$$

$$x_1 = \frac{3}{16}e$$

▲ MOMENTO MASSIMO

$$M_{max} = \frac{1}{4}qe^2 + \frac{3}{4}qlx_1 - 4q\,x_1\frac{x_1}{2} =$$

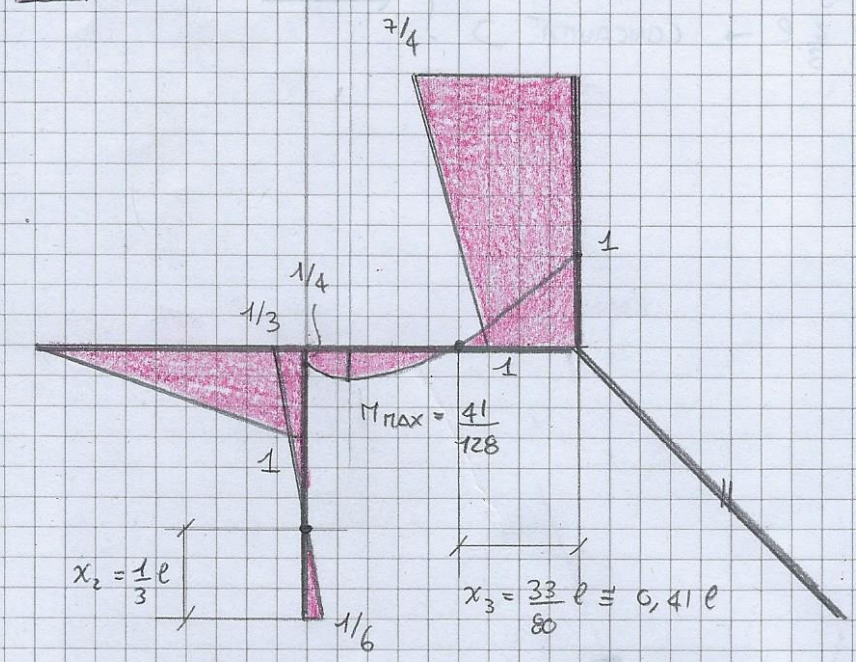
$$= \frac{1}{4}qe^2 + \frac{3}{4}ql\frac{3}{16}e - 4q\frac{3}{16}e\frac{3}{16}e\frac{1}{2} =$$

$$= \frac{1}{4}qe^2 + \frac{9}{64}qe^2 - \frac{9}{128}qe^2 =$$

$$= \left(\frac{32 + 18 - 9}{128}\right)qe =$$

$$= \frac{41}{128}qe^2$$

$\frac{M}{qe^2}$



▲ PUNTO DI ANNULLAMENTO DEL MOMENTO ASTA $\overline{BF}$ (FLESSO x_2)

$$x_2 \rightarrow \frac{1}{6}qe^2 - \frac{1}{2}qlx_2 = 0$$

$$x_2 = \frac{1}{3}e$$

▲ PUNTO DI ANNULLAMENTO DEL MOMENTO ASTA $\overline{BC}$ (FLESSO x_3):

$$x_3 \rightarrow qe^2 + 4qx_3\frac{x_3}{2} - \frac{13}{4}qlx_3 = 0 \rightarrow qe^2 + 2qx_3^2 - \frac{13}{4}qlx_3 = 0 \rightarrow$$

$$\rightarrow 4qe^2 + 8qx_3^2 - 13qlx_3 = 0 \rightarrow 8qx_3^2 - 13qlx_3 + 4qe^2 = 0$$

$$x_{3_{1,2}} = \frac{13 \pm \sqrt{169 - 128}}{16} = \frac{13 \pm \sqrt{41}}{16} \overset{\text{CIRCA}}{=} \frac{13 \pm 6,4}{16} \rightsquigarrow 6,4 \cong \frac{32}{5}$$

$$= \frac{13 \pm \frac{32}{5}}{16} = \frac{65 \pm 32}{80} \rightarrow x_{3,1} = \frac{97}{80}e > 1e \text{ NO}$$

$$\rightarrow x_{3,2} = \frac{33}{80}e \cong 0,41e$$

STUDIO DELLA DEFORMATA TERMICA

D
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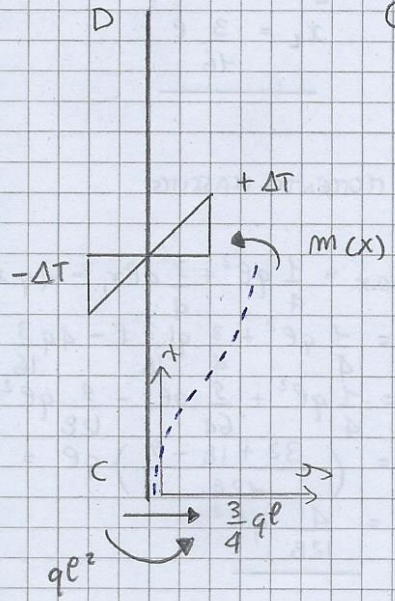
} CONVENZIONI DEFORMATA TERMICA



$$m(x) = -\frac{3}{4}ql\,x - qe^2$$

$$y'' = -\frac{m(x)}{EJ} - 2\,\alpha\frac{\Delta T}{H}$$

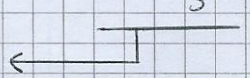
$$= -\left(-\frac{3}{4}ql\,x - qe^2\right)\frac{1}{EJ} - 2\cdot\frac{3}{4}\frac{qe^2}{EJ} =$$

$$= \frac{3}{4}\frac{qlx}{EJ} + \frac{qe^2}{EJ} - \frac{3}{2}\frac{qe^2}{EJ}$$

$$= \frac{3}{4}\frac{qlx}{EJ} - \frac{1}{2}\frac{qe^2}{EJ} \geq 0$$

$$\frac{3}{4}\frac{qlx}{EJ} - \frac{1}{2}\frac{qe^2}{EJ} \geq 0 \rightarrow x \geq \frac{1}{2}\frac{qe^2}{EJ}\cdot\frac{4}{3}^{2}\frac{EJ}{ql}$$

$$x \geq \frac{2}{3}e$$



- PER $x \geq \frac{2}{3}e \rightarrow$ CONCAVITÀ (
- PER $x < \frac{2}{3}e \rightarrow$ CONCAVITÀ)

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DEFORMATA QUALITATIVA