

# TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 25 NOVEMBRE 2020

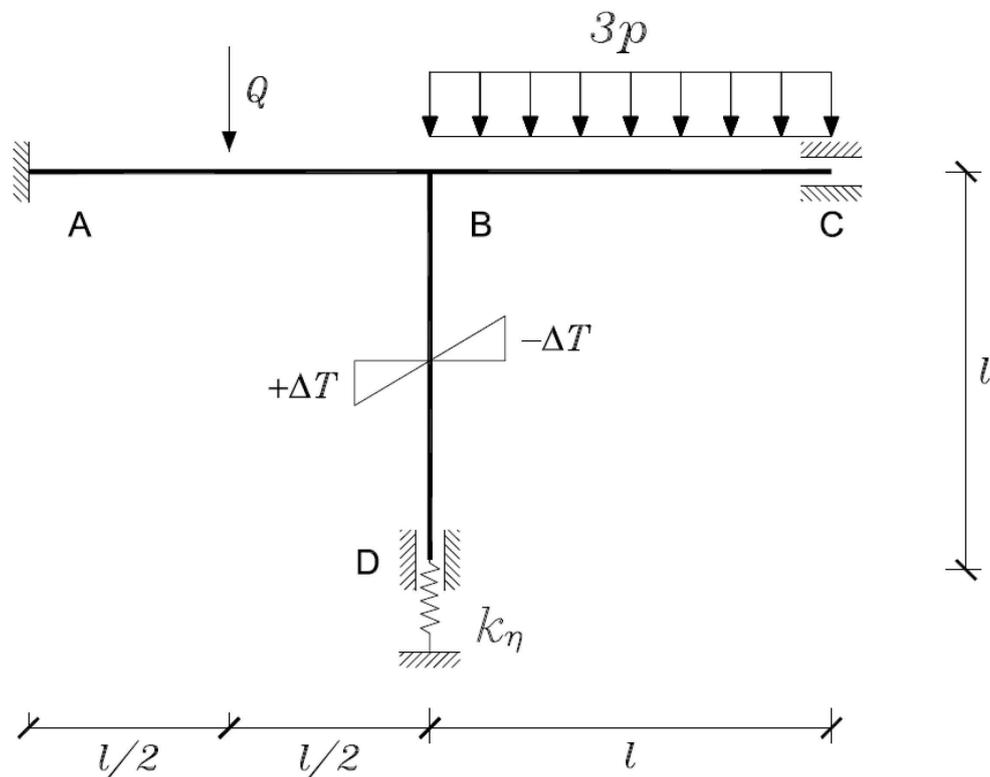
DOCENTI:

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PROF. FAUSTO MINELLI

DURATA: 2 ORE e 30 MIN.

## Esercizio



$$k_{\eta} = 36 \frac{EJ}{l^3} \quad Q = 2 \cdot p \cdot l \quad \frac{\alpha \Delta T}{h} = \frac{3pl^2}{4EJ}$$

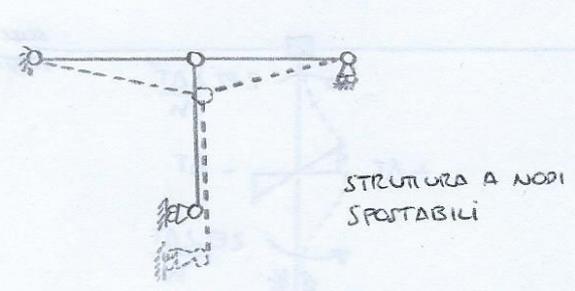
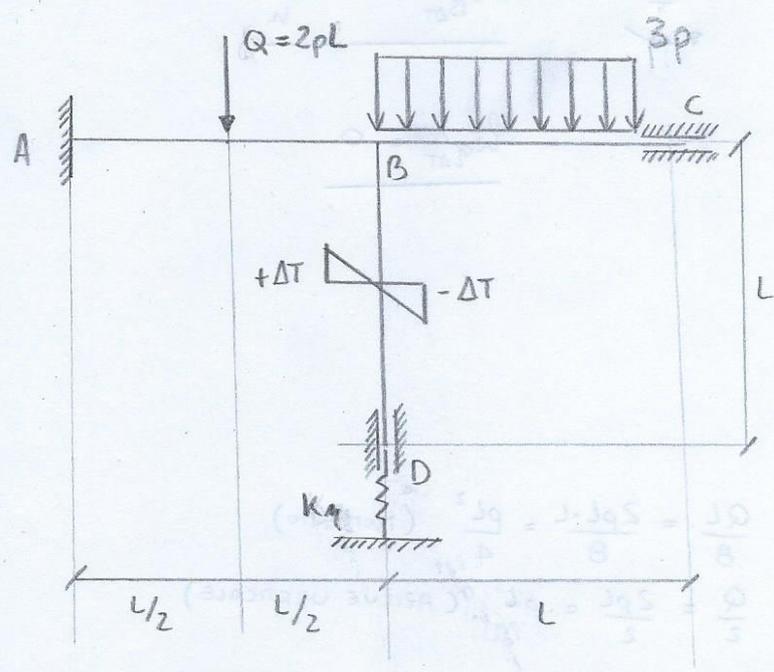
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma  $EA \rightarrow \infty$ ,  $EJ = \text{costante}$ .

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

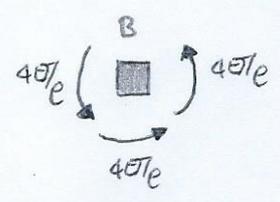
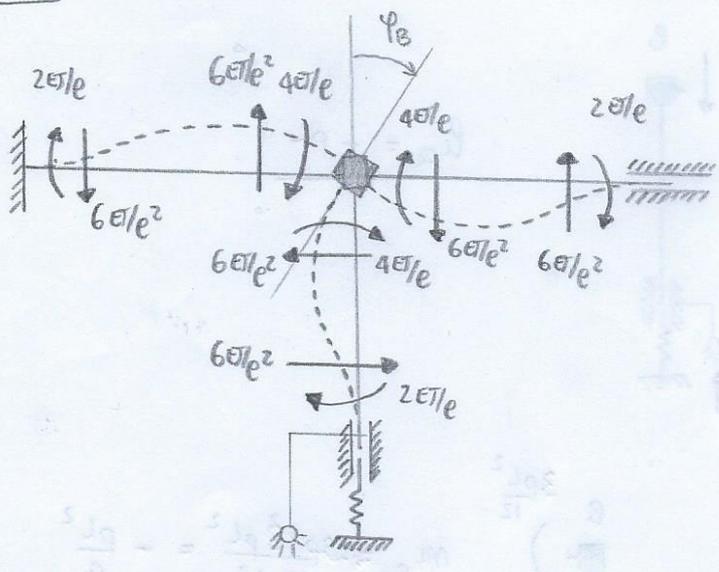
3 g.d.l. } → 5 VOLTE  
8 g.d.l. } IPERUNCOLATA



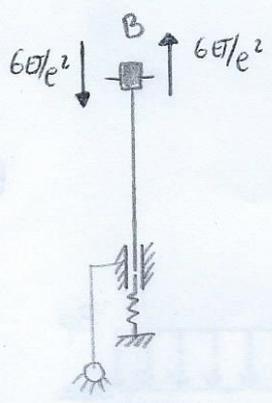
CONVENZIONI:  $\oplus \rightarrow \varphi +$   
 $\leftarrow \oplus \rightarrow$

INCOGNITE:  $\varphi_B; M_D$       $\frac{\Delta T}{h} = \frac{3}{4} \frac{pl^2}{ET}$ ;  $k_M = 36 ET/L^3$

$\varphi_B = 1$

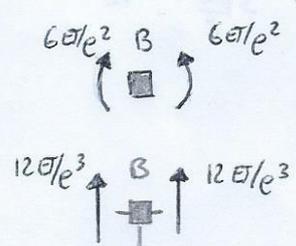
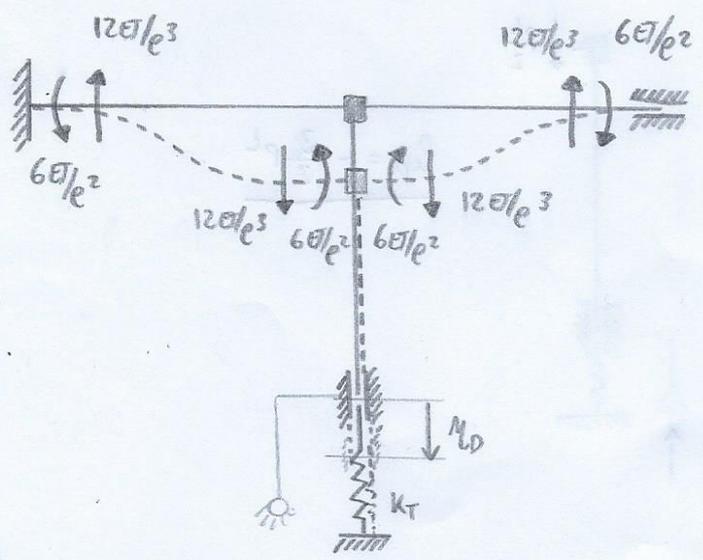


$M_{B\varphi} = -12 EI/L$

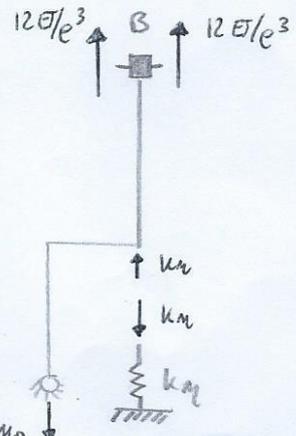


$h_{\varphi_B} = 0$

$M_D = 1$

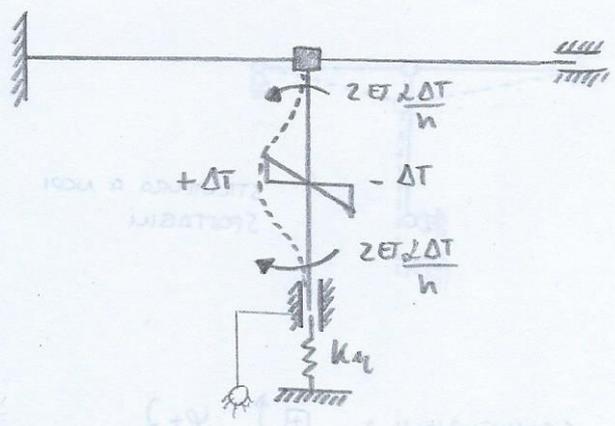


$M_{B\varphi} = 0$



$h_{M_D} = 24 EI/L^3 + k_M$

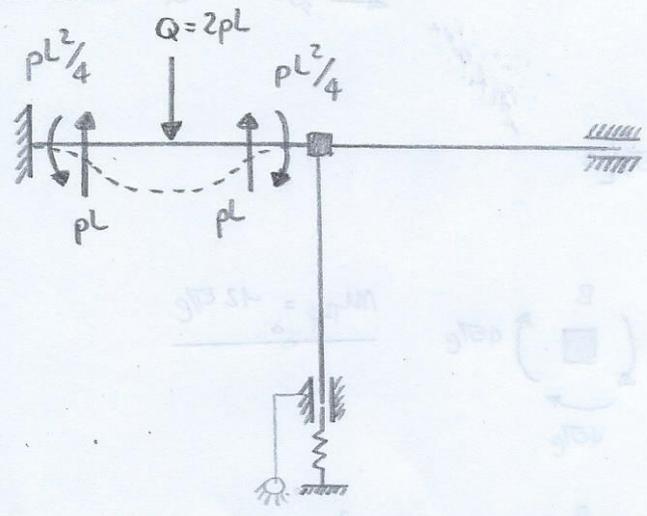
$\Delta T \neq 0$



$$M_{B\Delta T} = -\frac{2EI\Delta T}{h}$$

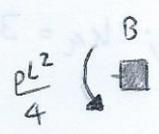
$$h_{M\Delta T} = 0$$

$Q \neq 0$

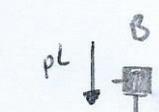


$$\frac{QL}{8} = \frac{2pL \cdot L}{8} = \frac{pL^2}{4} \quad (\text{MOMENTO})$$

$$\frac{Q}{2} = \frac{2pL}{2} = pL \quad (\text{AZIONE VERTICALE})$$

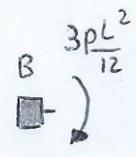
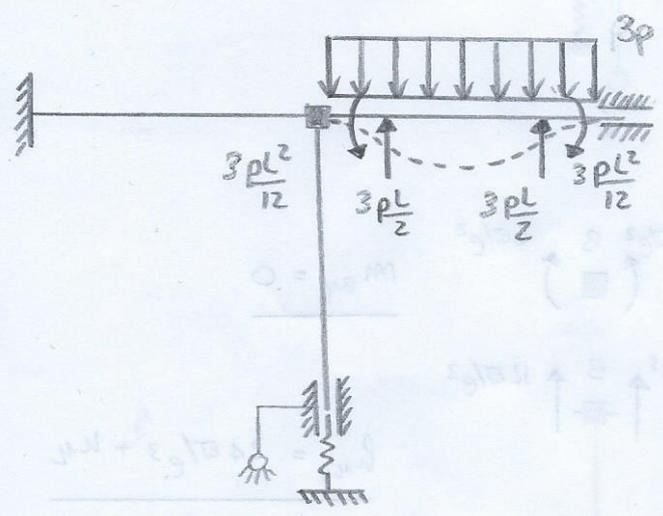


$$M_{BQ} = \frac{pL^2}{4}$$

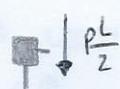


$$h_Q = -pL$$

$p \neq 0$



$$M_{Bp} = -\frac{3pL^2}{12} = -\frac{pL^2}{4}$$



$$h_p = -\frac{3}{2}pL$$

**SISTEMA RISOLVENTE**

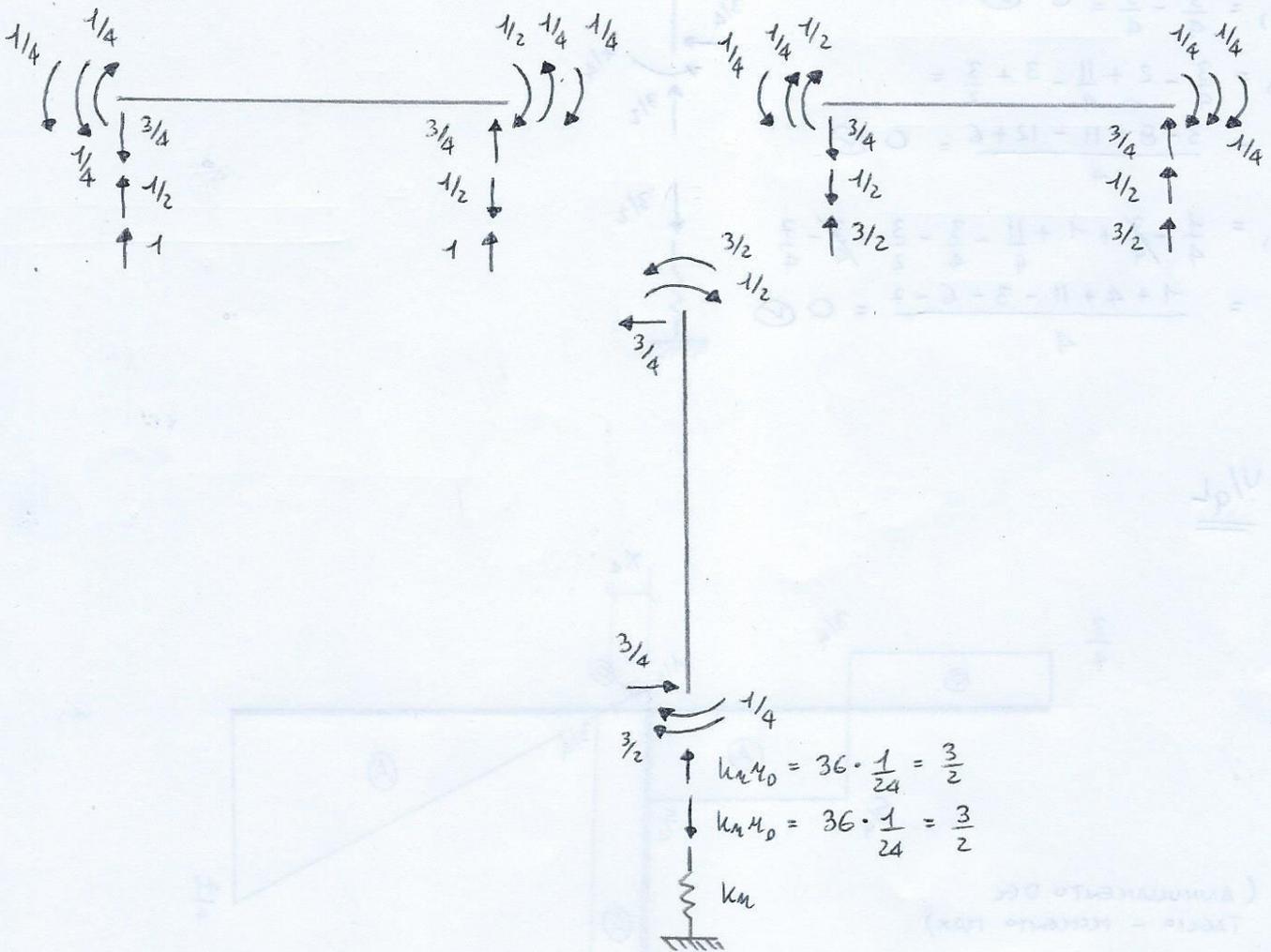
$$\begin{cases} -12 \frac{ET}{e} \varphi_B + 0 - 2 \frac{ET}{h} \frac{2\Delta T}{h} + \frac{\rho L^2}{4} - \frac{\rho L^2}{4} = 0 \\ 0 + (24 \frac{ET}{e^3} + k_n) M_D + 0 - \rho L - \frac{3}{2} \rho L = 0 \end{cases}$$

$$\begin{cases} -12 \frac{ET}{e} \varphi_B - 2 \frac{ET}{h} \frac{2\Delta T}{h} + 0 = 0 \\ 0 + (24 \frac{ET}{e^3} + 36 \frac{ET}{e^3}) M_D + 0 - \frac{5}{2} \rho L = 0 \end{cases}$$

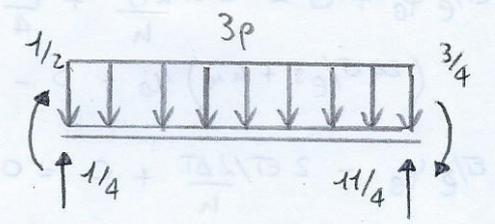
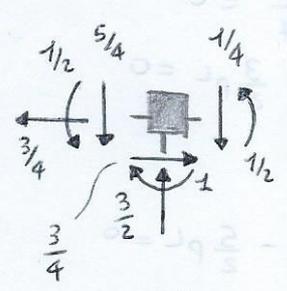
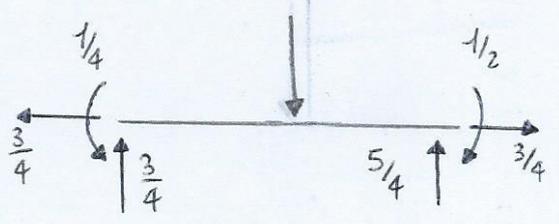
$$\begin{cases} -12 \frac{ET}{e} \varphi_B - 2 \frac{ET}{h} \cdot \frac{3}{4} \frac{\rho L^2}{ET} = 0 \\ 60 \frac{ET}{e^3} M_D - \frac{5}{2} \rho L = 0 \end{cases} \rightarrow \begin{cases} -12 \frac{ET}{e} \varphi_B = \frac{3}{2} \rho L^2 \rightarrow \\ 60 \frac{ET}{e^3} M_D = \frac{5}{2} \rho L \rightarrow \end{cases}$$

$\varphi_B = \frac{1}{8} \frac{\rho L^3}{ET}$
$M_D = \frac{1}{24} \frac{\rho L^4}{ET}$

**AZIONI AGENTI SULLA STRUTTURA**



$Q = 2pl$



AL NODO B

$$\sum V_{(B)} = \frac{5}{4} + \frac{1}{4} - \frac{3}{2} = 0 \quad \checkmark$$

$$\sum M_{(B)} = \frac{1}{2} + \frac{1}{2} - 1 = 0 \quad \checkmark$$

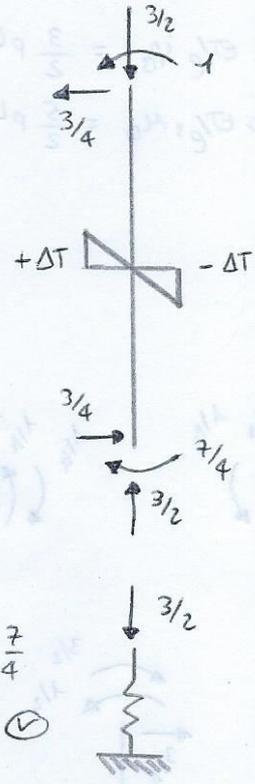
$$\sum H_{(B)} = 0 \quad \checkmark$$

EQUILIBRIO GLOBALE

$$\sum H_{(GLOB)} = \frac{3}{4} - \frac{3}{4} = 0 \quad \checkmark$$

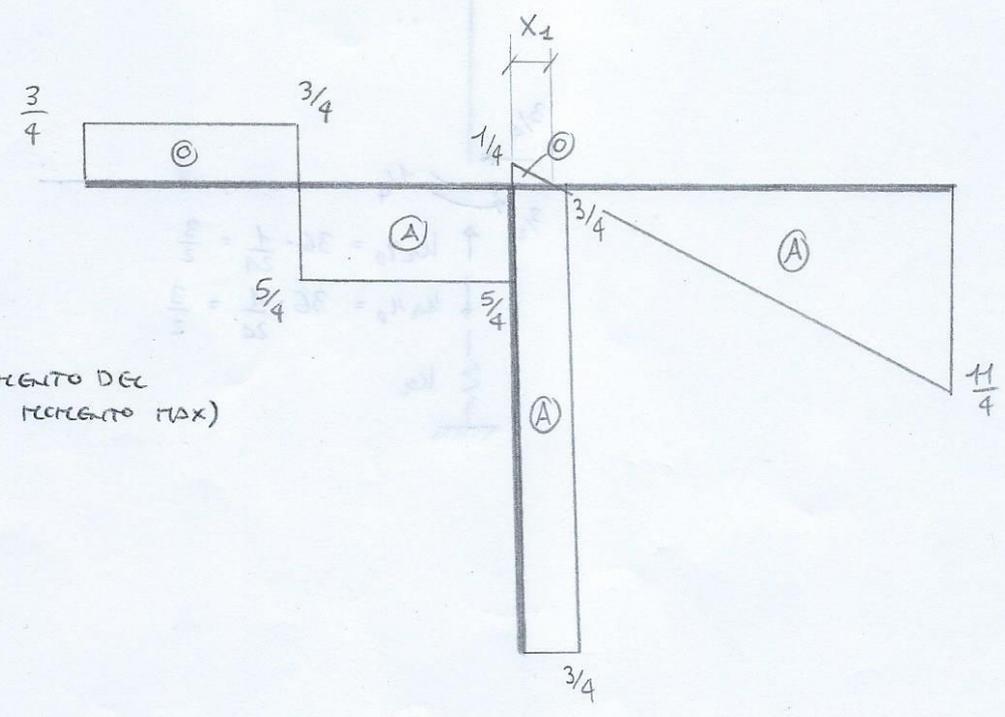
$$\sum V_{(GLOB)} = \frac{3}{4} - 2 + \frac{11}{4} - 3 + \frac{3}{2} = \frac{3 - 8 + 11 - 12 + 6}{4} = 0 \quad \checkmark$$

$$\sum M_{(GLOB)}^{(B)} = \frac{1}{4} - \frac{3}{4} + 1 + \frac{11}{4} - \frac{3}{4} - \frac{3}{2} + \frac{3}{4} - \frac{7}{4} = \frac{-1 + 4 + 11 - 3 - 6 - 7}{4} = 0 \quad \checkmark$$



GRAFICA

TAGLIO  $\frac{V}{qL}$

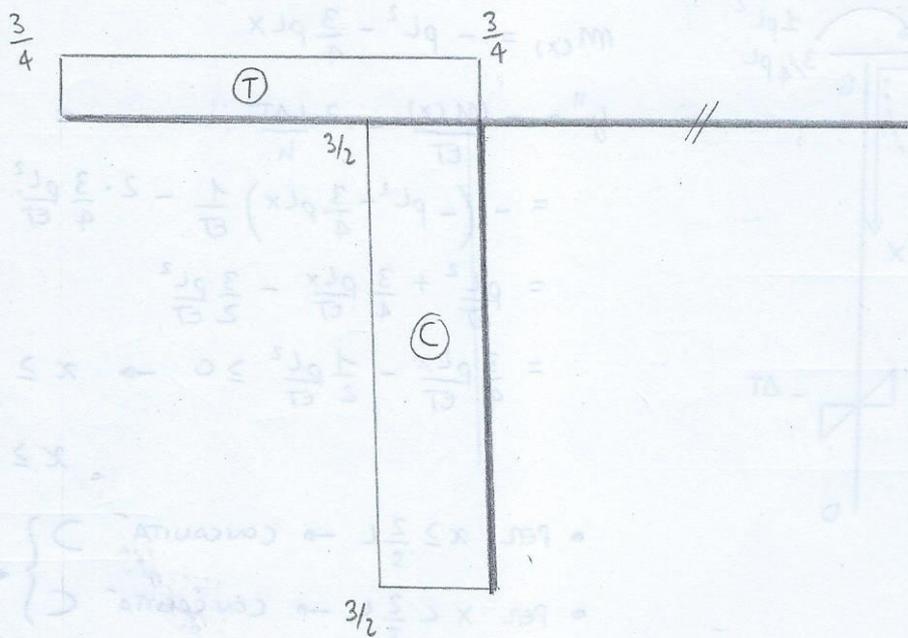


PUNTO  $X_1$  (ANNULLAMENTO DEL TAGLIO - RICERCA MAX)

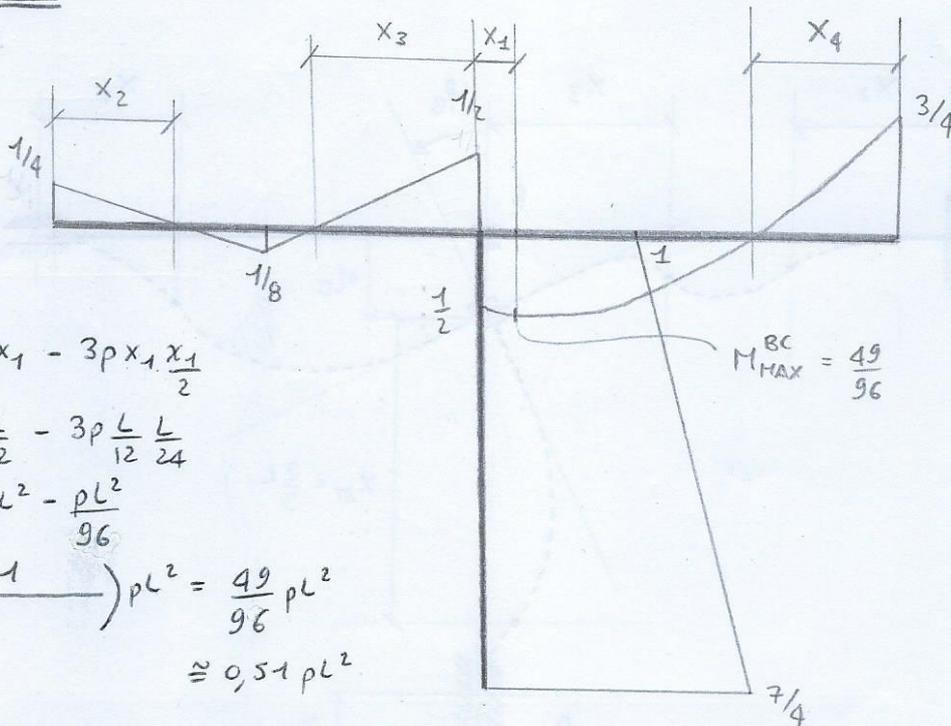
$$\frac{1}{4}pl - 3px_1 = 0$$

$$x_1 = \frac{1}{12}L$$

AZIONE ASSIALE  $\frac{N}{qL}$



MOMENTO FLETTENTE  $\frac{M}{qL^2}$



$$\begin{aligned}
 M_{(BC)}^{(MAX)} &= \frac{1}{2} pL^2 + \frac{1}{4} pLx_1 - 3px_1 \frac{x_1}{2} \\
 &= \frac{1}{2} pL^2 + \frac{1}{4} pL \frac{L}{12} - 3p \frac{L}{12} \frac{L}{24} \\
 &= \frac{1}{2} pL^2 + \frac{1}{48} pL^2 - \frac{pL^2}{96} \\
 &= \left( \frac{48 + 2 - 1}{96} \right) pL^2 = \frac{49}{96} pL^2 \\
 &\approx 0,51 pL^2
 \end{aligned}$$

PUNTI DI FLESSO:

$$x_2 \rightarrow \frac{1}{4} pL^2 - \frac{3}{4} pLx_2 = 0 \rightarrow \underline{x_2 = \frac{L}{3}}$$

$$x_3 \rightarrow \frac{1}{2} pL^2 - \frac{5}{4} pLx_3 = 0 \rightarrow \underline{x_3 = \frac{2L}{5}}$$

$$x_4 \rightarrow \frac{3}{4} pL^2 - \frac{11}{4} pLx_4 + 3p \frac{x_4^2}{2} = 0$$

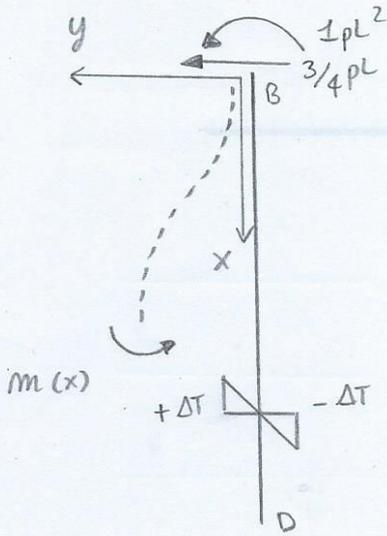
$$\frac{3}{4} pL^2 - \frac{11}{4} pLx_4 + \frac{3}{2} p x_4^2 = 0 \rightarrow 3pL^2 - 11pLx_4 + 6p x_4^2 = 0 \rightarrow 6p x_4^2 - 11pLx_4 + 3pL^2 = 0$$

$$\begin{aligned}
 x_{1,2} &= \frac{-11 \pm \sqrt{121 - 72}}{12} = \frac{-11 \pm \sqrt{49}}{12} = \frac{-11 \pm 7}{12} \\
 &\rightarrow x_4' = \frac{-11+7}{12} = \frac{-4}{12} < 0 \quad \text{NG} \\
 &\rightarrow x_4'' = \frac{-11-7}{12} = \frac{-18}{12} = -\frac{3}{2} < 0 \quad \text{NG}
 \end{aligned}$$

STUDIO DELLA DEFORMATA TERMICA (ASTA BD)

CONVENZIONI DEF. TERMICA

{ ( )  
(+)



$$M(x) = -pL^2 - \frac{3}{4}pLx$$

$$y'' = -\frac{M(x)}{EI} - \frac{2\alpha\Delta T}{h}$$

$$= -\left(-pL^2 - \frac{3}{4}pLx\right)\frac{1}{EI} - 2 \cdot \frac{3}{4}\frac{pL^2}{EI}$$

$$= \frac{pL^2}{EI} + \frac{3}{4}\frac{pLx}{EI} - \frac{3}{2}\frac{pL^2}{EI}$$

$$= \frac{3}{4}\frac{pLx}{EI} - \frac{1}{2}\frac{pL^2}{EI} \geq 0 \rightarrow x \geq \frac{1}{2}\frac{pL^2}{EI} \cdot \frac{4}{3}\frac{EI}{pL}$$

$$x \geq \frac{2}{3}L$$

- PER  $x \geq \frac{2}{3}L \rightarrow$  CONCAVITA' )
- PER  $x < \frac{2}{3}L \rightarrow$  CONCAVITA' (

DEFORMATA QUADRATA

