TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 27 GENNAIO 2021

DOCENTI:
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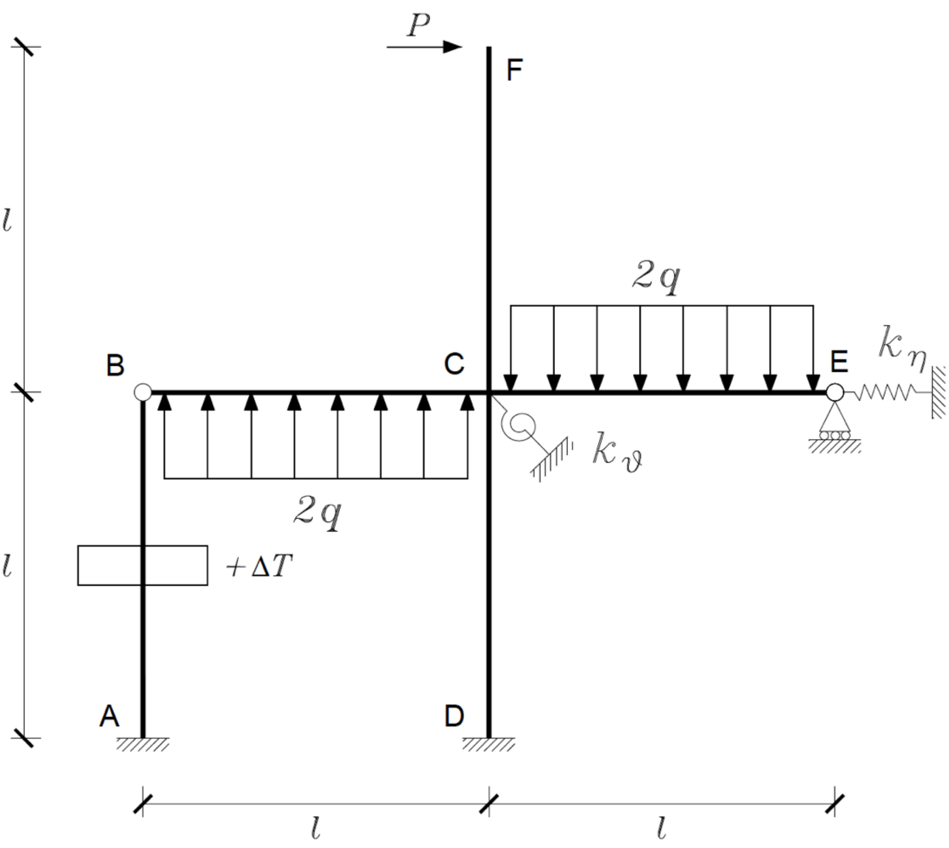
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**

Esercizio



$$k_\eta = 5\frac{EJ}{l^3} \qquad k_\theta = \frac{1}{2}\frac{EJ}{l} \qquad P = q \cdot l \qquad \alpha\Delta T = \frac{1}{8}\frac{ql^3}{EJ}$$

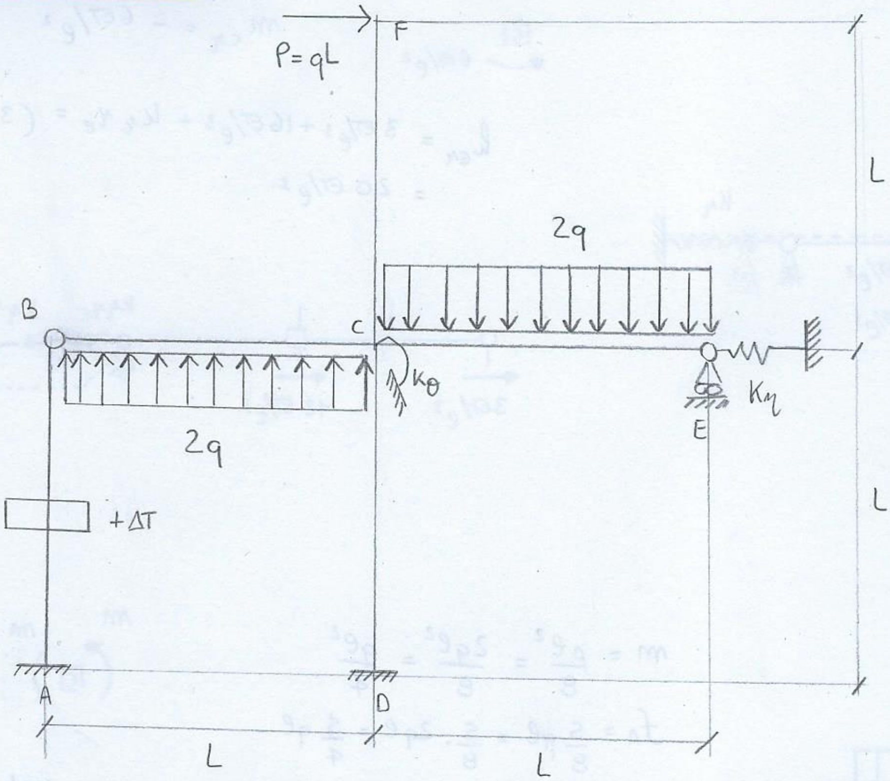
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \rightarrow \infty$, $EJ = costante$.

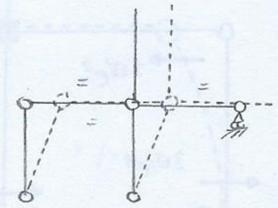
I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere riportati in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TELAIO 27/01/2021



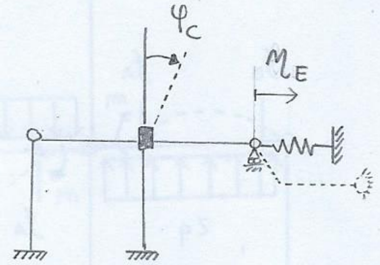
ANALISI CINEMATICA

$$\begin{cases} 6 \text{ gdl} \\ 11 \text{ gdv} \end{cases} \rightarrow \text{5 VOLTE IPERVINCOLATA}$$



- TELAIO A NODI SPOSTABILI

- SCELTA DELLE INCOGNITE :



- CONVENZIONI

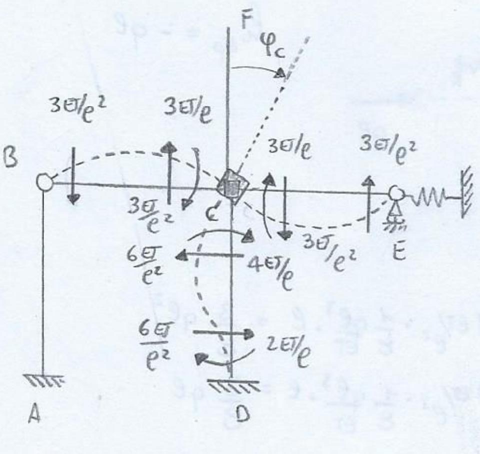
- DATI : $k_\theta = \frac{1}{2}\, EI/\ell$
  $k_\eta = 5\, EI/\ell^3$
  $2\Delta T = 1/8\; q\ell^3/EI$

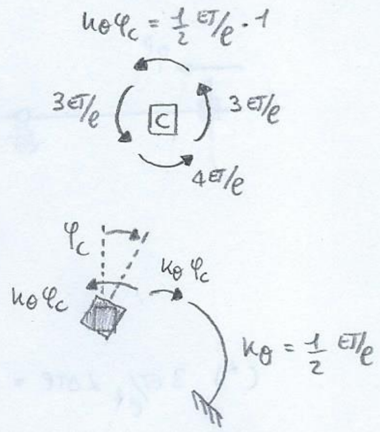
EQUAZIONI DI EQUILIBRIO

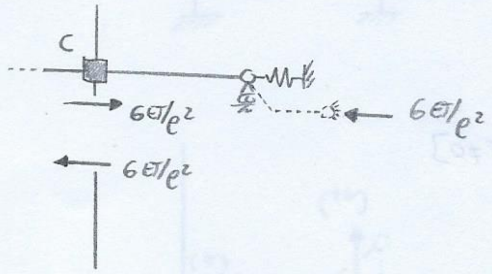
$$\begin{cases} \Sigma' M_{(C)} = 0 \rightarrow m_{C\varphi}\varphi_C + m_{C\eta}\eta_E + m_{Cq} + m_{C\Delta T} = 0 \\ \Sigma' h_{(E)} = 0 \rightarrow h_{E\varphi}\varphi_C + h_{E\eta}\eta_E + h_{Eq} + h_{E\Delta T} = 0 \end{cases}$$

RISOLUZIONE

$[\varphi_C = 1]$



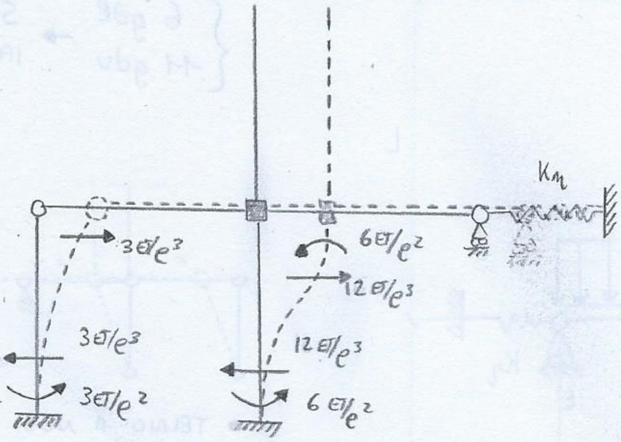


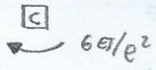


$$m_{C\varphi} = \left(3 + 4 + 3 + \tfrac{1}{2}\right) EI/\ell = \frac{21}{2}\, EI/\ell$$

$$h_{E\varphi} = -\,6\, EI/\ell^2$$

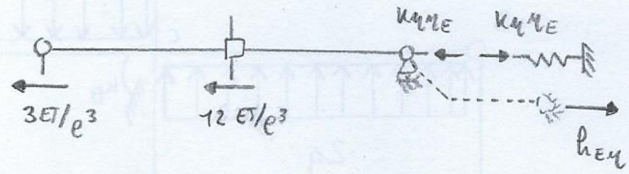
$[\eta_E = 1]$



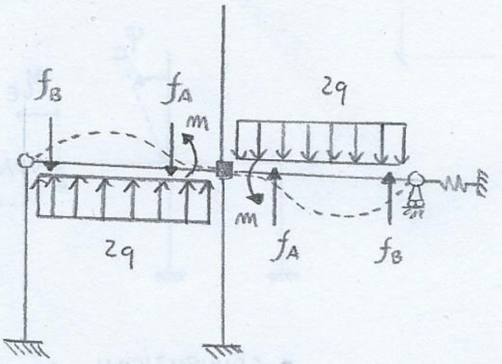


$$m_{C\eta} = -6EI/\ell^2$$

$$h_{E\eta} = 3EI/\ell^3 + 12EI/\ell^3 + k_\eta \eta_E = (3+12+5)\,EI/\ell^3 = 20\,EI/\ell^3$$



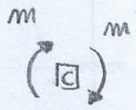
$[q \neq 0]$



$$m = \frac{p\ell^2}{8} = \frac{2q\ell^2}{8} = \frac{q\ell^2}{4}$$

$$f_A = \frac{5}{8}p\ell = \frac{5}{8}\cdot 2q\ell = \frac{5}{4}q\ell$$

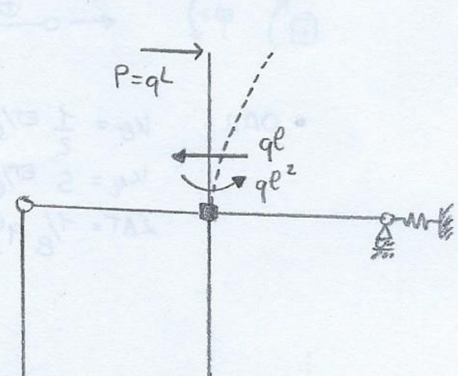
$$f_B = \frac{3}{8}p\ell = \frac{3}{8}\cdot 2q\ell = \frac{3}{4}q\ell$$

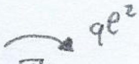


$$m_{Cq} = -\frac{1}{4}q\ell^2 - \frac{1}{4}q\ell^2 = -\frac{1}{2}q\ell^2$$

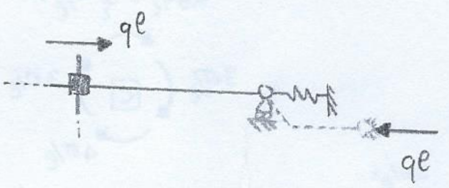
$$h_{Cq} = 0$$

$[P \neq 0]$



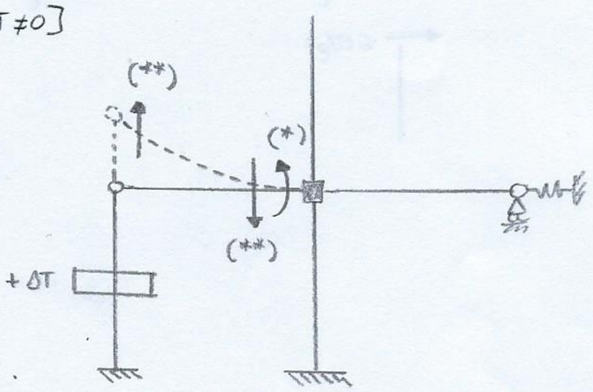


$$m_{Cp} = -q\ell^2$$



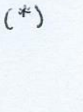
$$h_{Ep} = -q\ell$$

$[\Delta T \neq 0]$



$$(*)\quad 3EI/\ell^2\, \alpha \Delta T \ell = 3EI/\ell^2 \cdot \frac{1}{8}\frac{q\ell^3}{EI}\cdot \ell = \frac{3}{8}q\ell^2$$

$$(**)\quad 3EI/\ell^3\, \alpha \Delta T \ell = 3EI/\ell^3 \cdot \frac{1}{8}\frac{q\ell^3}{EI}\cdot \ell = \frac{3}{8}q\ell$$



$$m_{C\Delta T} = -\frac{3}{8}q\ell^2$$

$$h_{E\Delta T} = 0$$

SISTEMA RISOLVENTE

$$\begin{cases} \sum M_{(C)} = 0 \rightarrow \frac{21}{2} EI/l\, \varphi_C - 6 EI/l^2\, \eta_E - \frac{1}{2} ql^2 - ql^2 - \frac{3}{8} ql^2 = 0 \quad (\times 5) \\ \sum h_{(CE)} = 0 \rightarrow -6 EI/l^2\, \varphi_C + 20 EI/l^3\, \eta_E - ql = 0 \quad (\times 3/2\, l) \end{cases}$$

$$\begin{cases} \frac{105}{2} EI/l\, \varphi_C - 30 EI/l^2\, \eta_E - \frac{5}{2} ql^2 - 5ql^2 - \frac{15}{8} ql^2 = 0 \\ -9 EI/l\, \varphi_C + 30 EI/l^2\, \eta_E - \frac{3}{2} ql^2 = 0 \end{cases}$$

$$\frac{87}{2} EI/l\, \varphi_C \quad // \quad - \frac{87}{8} ql^2 = 0 \rightarrow \varphi_C = \frac{\cancel{87}}{\cancel{8}_4} ql^2 \cdot \frac{\cancel{2}^1\, l}{\cancel{87}_1\, EI} \rightarrow \boxed{\varphi_C = \frac{ql^3}{4EI}}$$

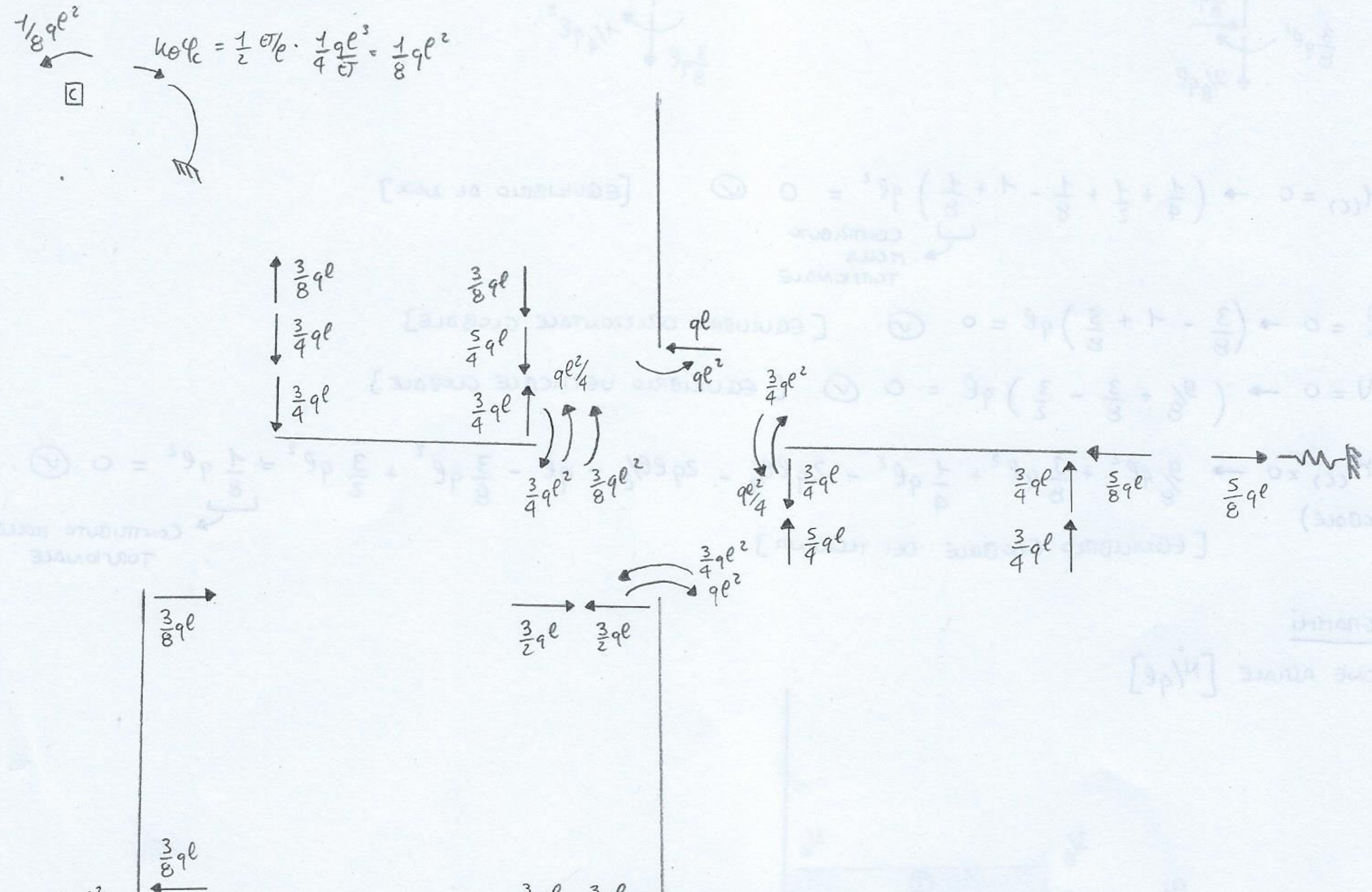
$$- 6 EI/l^2 \cdot \frac{1}{4} \frac{ql^3}{EI} + 20 EI/l^3\, \eta_E - ql = 0$$

$$- \frac{3}{2} ql + 20 EI/l^3\, \eta_E - ql = 0$$

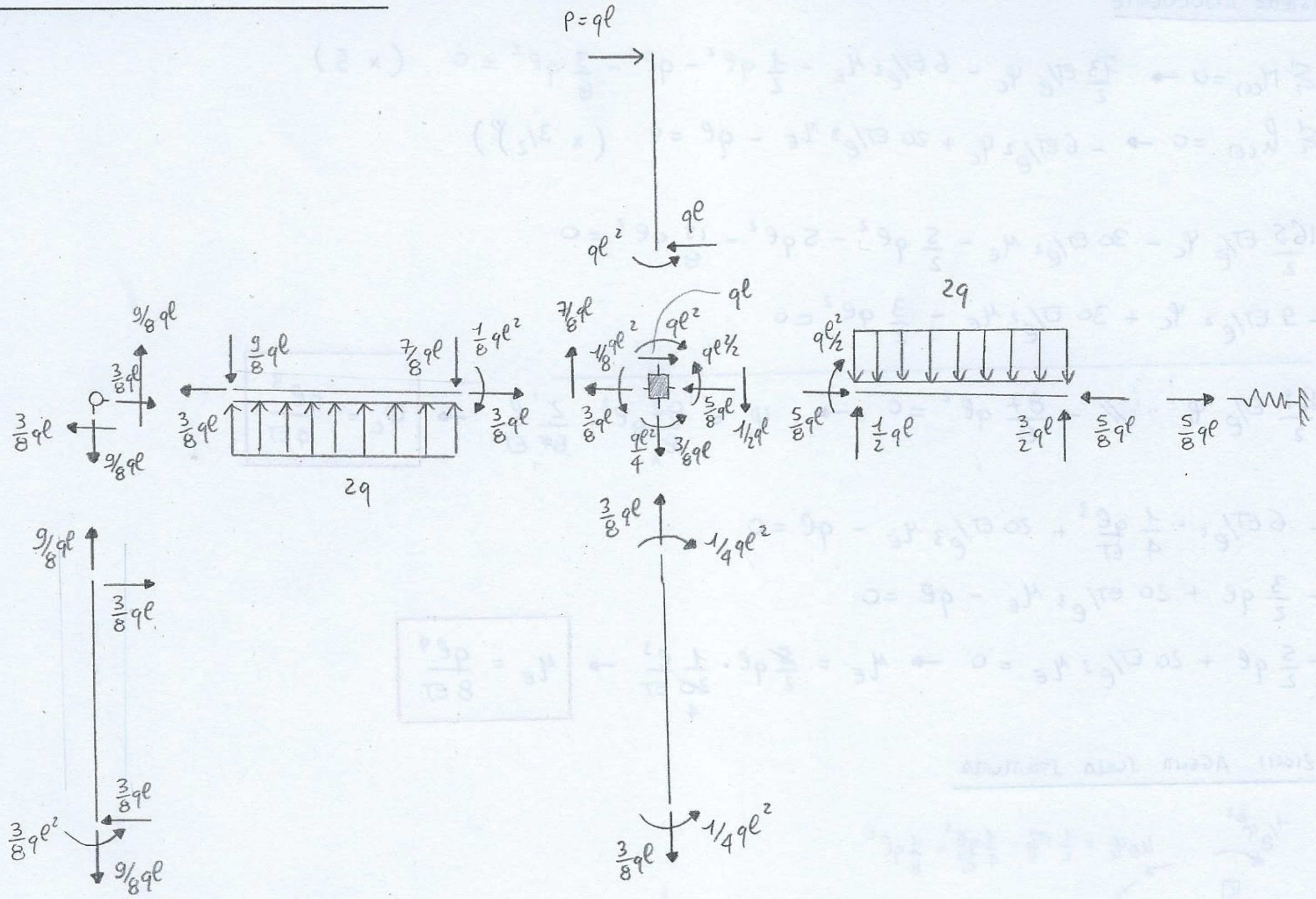
$$- \frac{5}{2} ql + 20 EI/l^3\, \eta_E = 0 \rightarrow \eta_E = \frac{\cancel{5}}{2} ql \cdot \frac{1}{\cancel{20}_4} \frac{l^3}{EI} \rightarrow \boxed{\eta_E = \frac{ql^4}{8EI}}$$

AZIONI AGENTI SULLA STRUTTURA

$k_\theta \varphi_C = \frac{1}{2} EI/l \cdot \frac{1}{4} \frac{ql^3}{EI} = \frac{1}{8} ql^2$



AZIONI TOTALI AGENTI SULLA STRUTTURA



$$\Sigma M_{(C)} = 0 \rightarrow \left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8} - 1 + \underbrace{\frac{1}{8}}_{\text{CONTRIBUTO MOLLA TORSIONALE}}\right) q\ell^2 = 0$$ [EQUILIBRIO AL NODO]

$$\Sigma h = 0 \rightarrow \left(\frac{3}{8} - 1 + \frac{5}{8}\right) q\ell = 0$$ [EQUILIBRIO ORIZZONTALE GLOBALE]

$$\Sigma V = 0 \rightarrow \left(9/8 + \frac{3}{8} - \frac{3}{2}\right) q\ell = 0$$ [EQUILIBRIO VERTICALE GLOBALE]

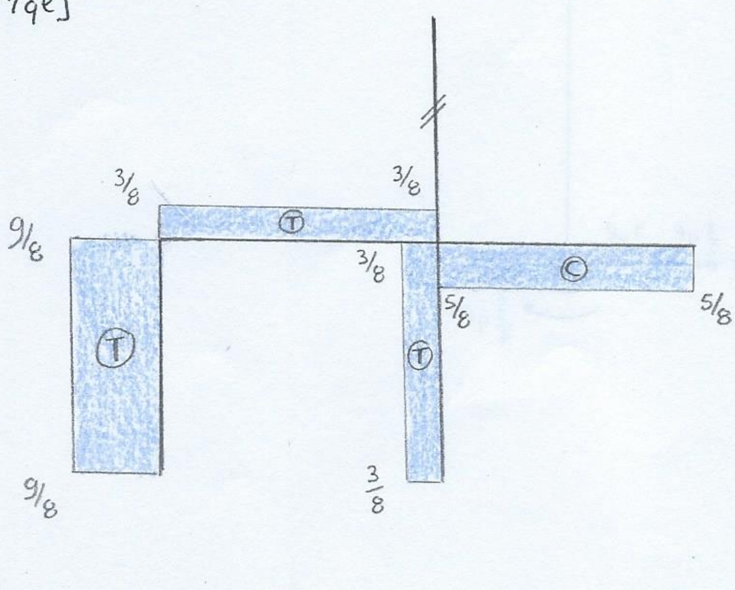
$$\Sigma M_{(C)} = 0 \rightarrow 9/8\, q\ell^2 + \frac{3}{8} q\ell^2 + \frac{1}{4} q\ell^2 - 2q\ell\ell/2 - 2q\ell\ell/2 - q\ell - \frac{3}{8} q\ell^2 + \frac{3}{2} q\ell^2 + \underbrace{\frac{1}{8}}_{\text{CONTRIBUTO MOLLA TORSIONALE}} q\ell^2 = 0$$

(GLOBALE)

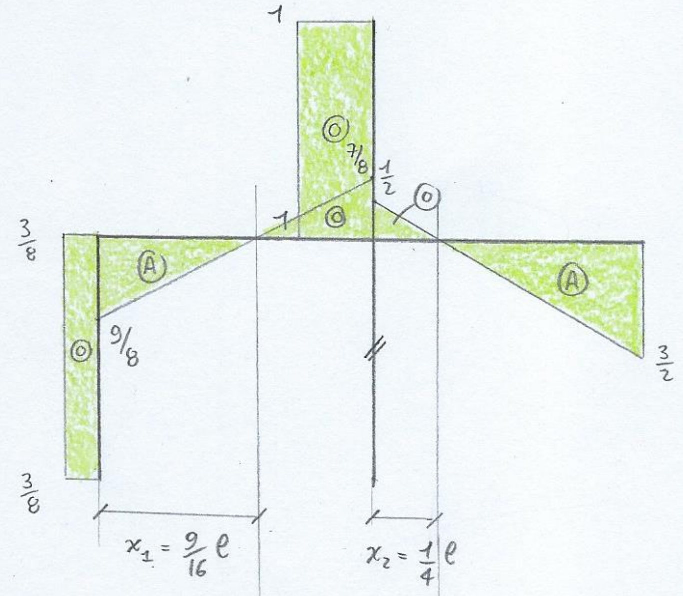
[EQUILIBRIO GLOBALE DEI MOMENTI]

DIAGRAMMI

AZIONE ASSIALE $[N/q\ell]$



TAGLIO $[V/ql]$



ANNULLAMENTO DEL TAGLIO:

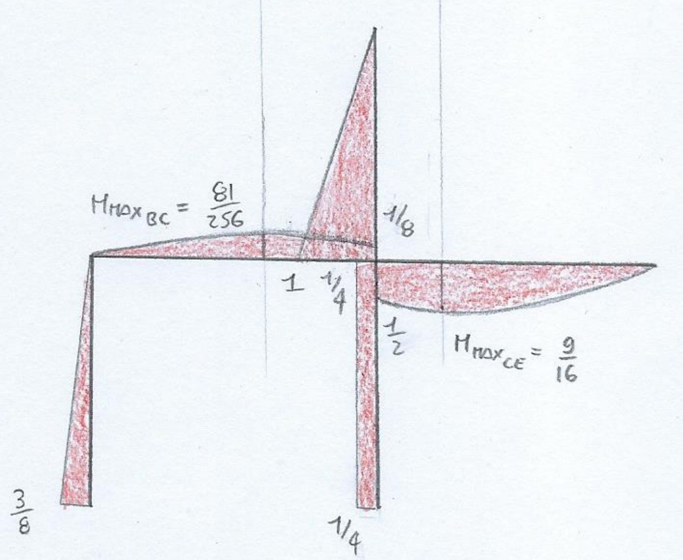
- ASTA $\overline{BC}$

$$\frac{9}{8} ql - 2qx = 0 \rightarrow x_1 = 9/16\, l$$

- ASTA $\overline{CE}$

$$\frac{1}{2} ql - 2qx = 0 \rightarrow x_2 = \frac{1}{4} l$$

MOMENTO $[M/ql^2]$



MOMENTI MASSIMI:

- ASTA $\overline{BC}$

$$\frac{9}{8} ql \cdot x_1 - 2q x_1 \frac{x_1}{2} =$$
$$= \frac{9}{8} ql \cdot \frac{9}{16} l - 2q \cdot \frac{9}{16} l \cdot \frac{9}{32} l =$$
$$= \frac{81}{128} ql^2 - \frac{81}{256} ql^2 = \frac{81}{256} ql^2$$

- ASTA $\overline{CE}$

$$ql^2/2 + \frac{1}{2} ql x_2 - 2q x_2 \frac{x_2}{2} =$$
$$= \frac{ql^2}{2} + \frac{1}{2} ql \frac{1}{4} l - 2q \frac{1}{4} l \frac{1}{8} l =$$
$$= \frac{ql^2}{2} + \frac{1}{8} ql^2 - \frac{1}{16} ql^2 = \frac{9}{16} ql^2$$

[NO PUNTI DI FLESSO]

DEFORMATA QUALITATIVA

$$\alpha \Delta T l = \frac{1}{8} ql^4/EJ$$