

# TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 17 FEBBRAIO 2021

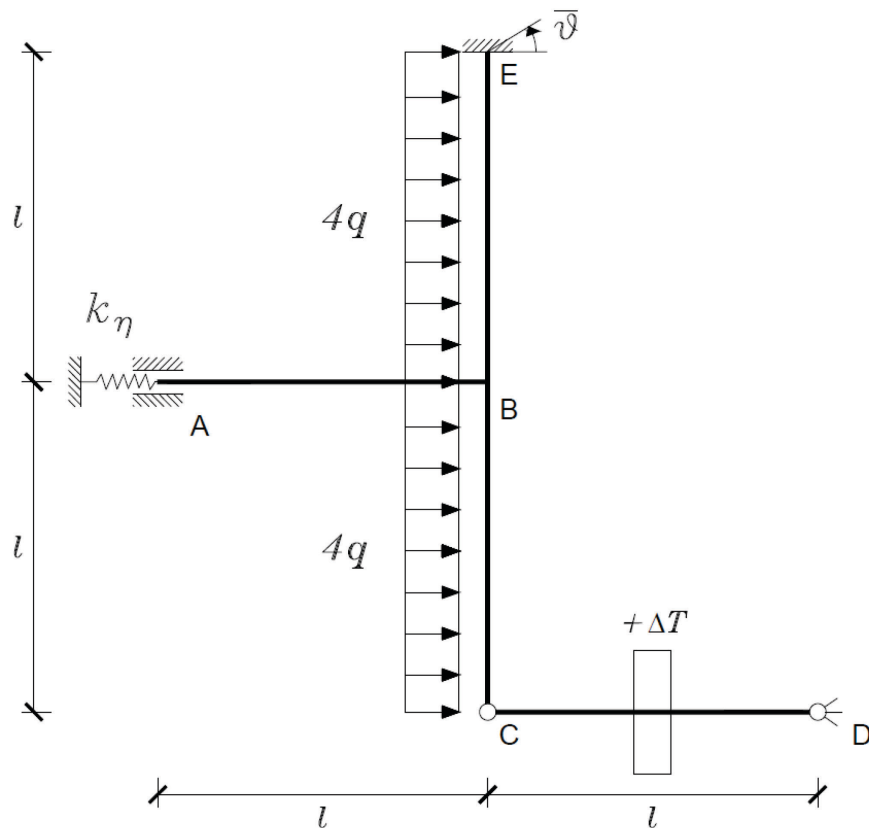
DOCENTI:

PROF. GIOVANNI A. PLIZZARI

PROF. FAUSTO MINELLI

DURATA: 2 ORE.

## Esercizio



$$k_\eta = 3 \frac{EJ}{l^3} \quad \alpha \Delta T = \frac{13}{24} \frac{ql^3}{EJ} \quad \bar{\vartheta} = \frac{1}{3} \frac{ql^3}{EJ}$$

Dato il telaio in figura, **si richiedono i grafici di:**

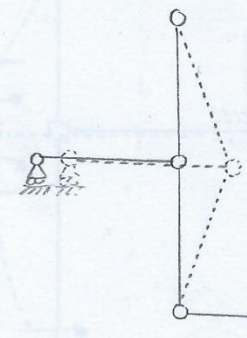
1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma  $EA \rightarrow \infty$ ,  $EJ = \text{costante}$ .

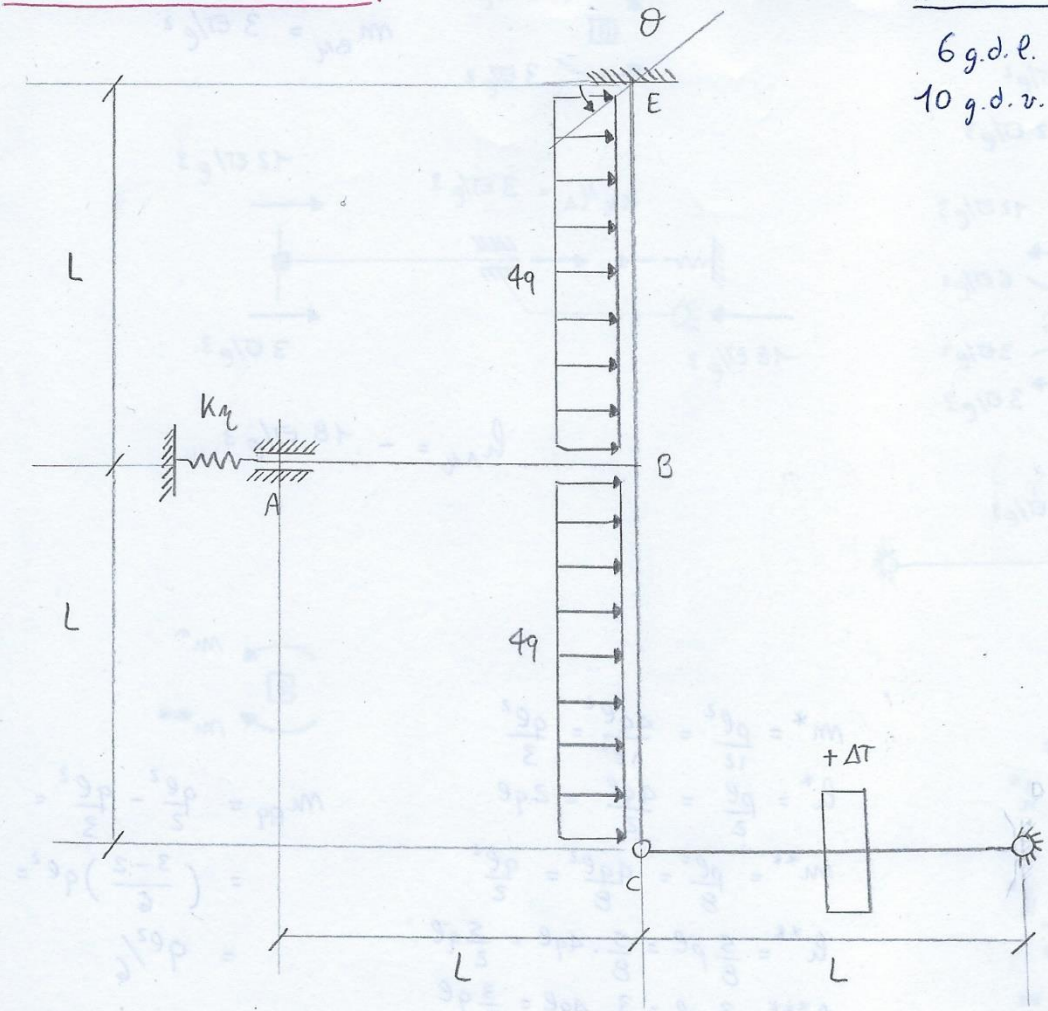
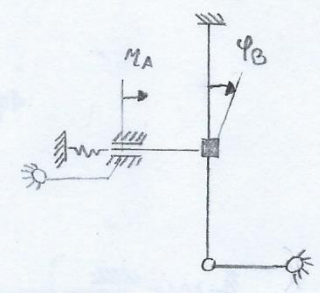
I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

6 g.d.l. } 4 UOGLIE IPERVINCOLATA  
10 g.d.v.

[TELAIO A  
NODI  
SPOSTABILI]



[SCELTA DELLE INCOGNITE]



DATI

$$k_A = 3 \text{ ET}/e^3$$

$$\Delta T = \frac{13}{24} \frac{q e^3}{\text{ET}}$$

$$\theta = \frac{1}{3} \frac{q e^3}{\text{ET}}$$

EQUAZIONI DI EQUILIBRIO

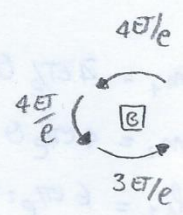
$$\begin{cases} \sum M_{CB} = 0 \rightarrow m_{B\varphi} \varphi_B + m_{BA} M_A + m_{B\theta} \theta + m_{Bq} + m_{B\Delta T} = 0 \\ \sum h_{CA} = 0 \rightarrow h_{A\varphi} \varphi_B + h_{AA} M_A + h_{A\theta} \theta + h_{Aq} + h_{A\Delta T} = 0 \end{cases}$$

CONVENZIONI

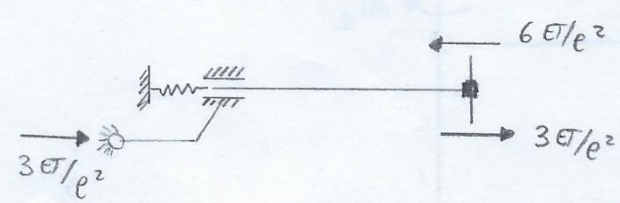
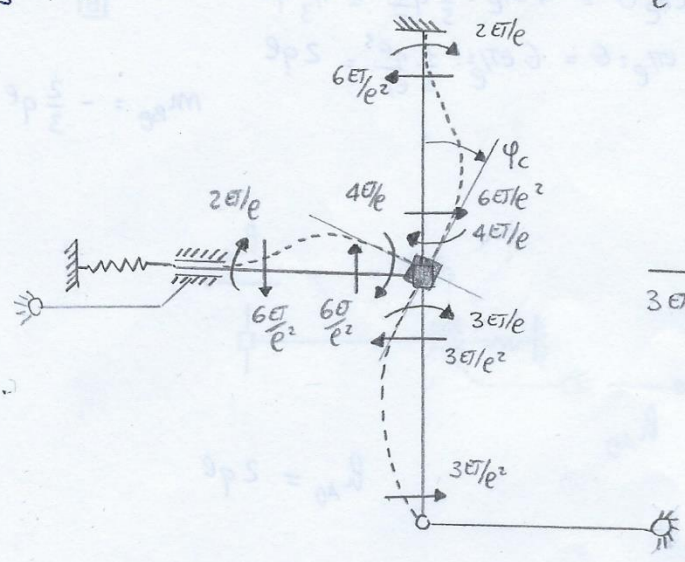


RISOLUZIONE

$$[\varphi_B = 1]$$



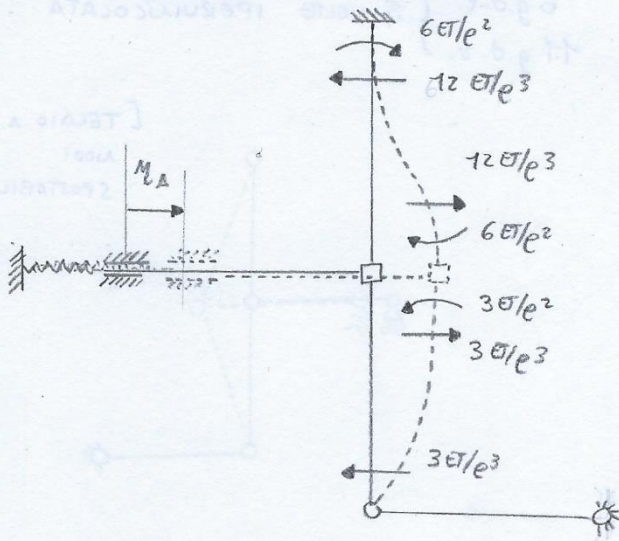
$$m_{B\varphi} = 11 \text{ ET}/e$$



$$h_{A\varphi} = -3 \text{ ET}/e^2$$



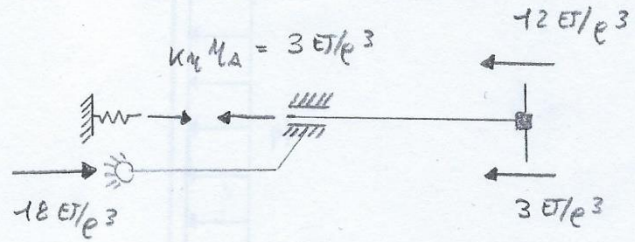
$$[M_A = 1]$$



$$\frac{6EI}{l^2}$$

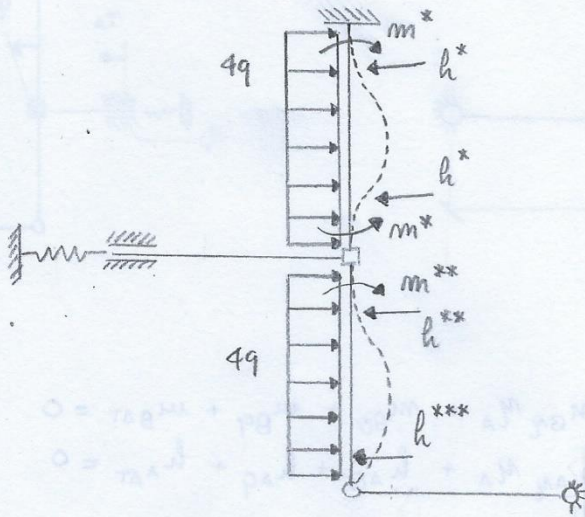
$$\frac{3EI}{l^2}$$

$$M_{B4} = 3EI/l^2$$



$$h_{A4} = -18EI/l^3$$

$$[q \neq 0]$$



$$m^* = \frac{pl^2}{12} = \frac{4ql^2}{12} = \frac{ql^2}{3}$$

$$h^* = \frac{pl}{2} = \frac{4ql}{2} = 2ql$$

$$m^{**} = \frac{pl^2}{8} = \frac{4ql^2}{8} = \frac{ql^2}{2}$$

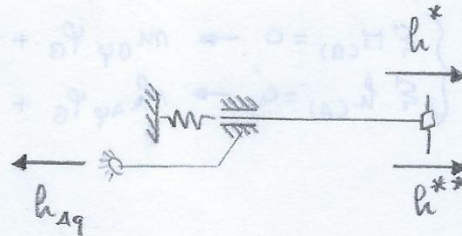
$$h^{**} = \frac{5}{8}pl = \frac{5}{8} \cdot 4ql = \frac{5}{2}ql$$

$$h^{***} = \frac{3}{8}pl = \frac{3}{8} \cdot 4ql = \frac{3}{2}ql$$

$$m^*$$

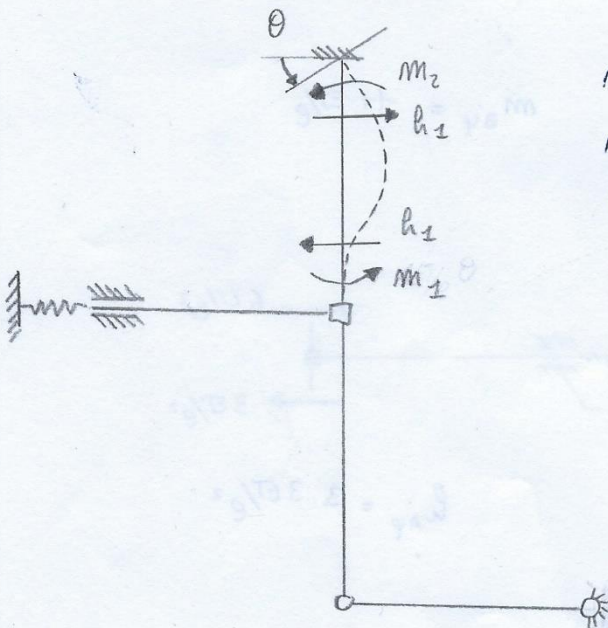
$$m^{**}$$

$$M_{Bq} = \frac{ql^2}{2} - \frac{ql^2}{3} = \left(\frac{3-2}{6}\right)ql^2 = \frac{ql^2}{6}$$



$$h_{Aq} = 2ql + \frac{5}{2}ql = \left(\frac{4+5}{2}\right)ql = \frac{9}{2}ql$$

$$[\theta \neq 0]$$



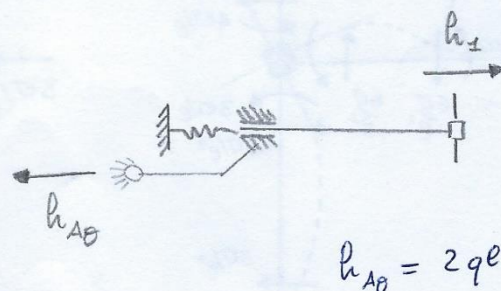
$$m_1 = 2EI/l \theta = 2EI/l \cdot \frac{1}{3} \frac{ql^3}{EI} = \frac{2}{3}ql^2$$

$$m_2 = 4EI/l \theta = 4EI/l \cdot \frac{1}{3} \frac{ql^3}{EI} = \frac{4}{3}ql^2$$

$$h_1 = 6EI/l^2 \theta = 6EI/l^2 \cdot \frac{1}{3} \frac{ql^3}{EI} = 2ql$$

$$m_1$$

$$M_{B\theta} = -\frac{2}{3}ql^2$$



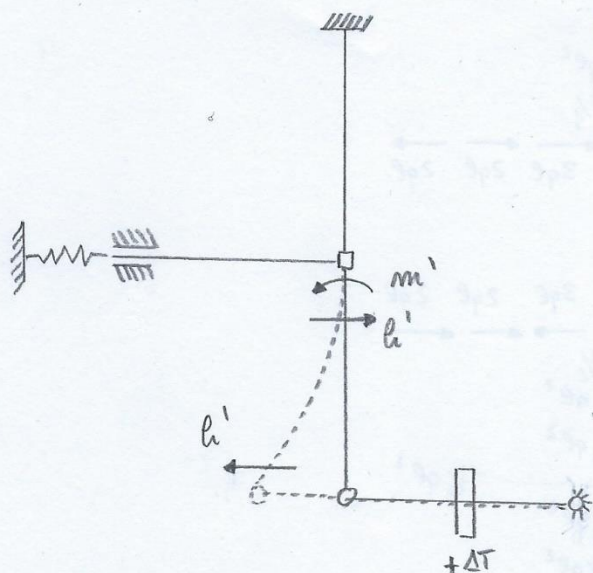
$$h_{A\theta} = 2ql$$



$[\Delta T \neq 0]$

$$m' = 3EI/e^2 \cdot 2\Delta T e = 3EI/e^2 \cdot \frac{13}{24} \frac{qe^3}{EI} \cdot e = \frac{13}{8} qe^2$$

$$h' = 3EI/e^3 \cdot 2\Delta T e = 3EI/e^3 \cdot \frac{13}{24} \frac{qe^3}{EI} \cdot e = \frac{13}{8} qe$$



$$m_{B\Delta T} = -\frac{13}{8} qe^2$$

$$h_{A\Delta T} = -\frac{13}{8} qe$$

SISTEMA RESOLVENTE

$$\begin{cases} \sum M_{(B)} = 0 \rightarrow m_{Bq} \cdot \varphi_B + m_{BA} \cdot \chi_A + m_{B\theta} + m_{Bq} + m_{B\Delta T} = 0 \\ \sum h_{(A)} = 0 \rightarrow h_{Aq} \cdot \varphi_B + h_{AA} \cdot \chi_A + h_{A\theta} + h_{Aq} + h_{A\Delta T} = 0 \end{cases}$$

$$\begin{cases} 11 EI/e \varphi_B + 3 EI/e^2 \chi_A - \frac{2}{3} qe^2 + \frac{qe^2}{6} - \frac{13}{8} qe^2 = 0 \quad (\times 6/e) \\ -3 EI/e^2 \varphi_B - 18 EI/e^3 \chi_A + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0 \end{cases}$$

$$\begin{cases} 66 EI/e^2 \varphi_B + 18 EI/e^3 \chi_A - 4qe + qe - \frac{39}{4} qe = 0 \\ -3 EI/e^2 \varphi_B - 18 EI/e^3 \chi_A + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0 \end{cases}$$

$$\begin{cases} 66 EI/e^2 \varphi_B + 18 EI/e^3 \chi_A - 4qe + qe - \frac{39}{4} qe = 0 \\ -3 EI/e^2 \varphi_B - 18 EI/e^3 \chi_A + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0 \end{cases}$$

$$\begin{cases} 66 EI/e^2 \varphi_B + 18 EI/e^3 \chi_A - 4qe + qe - \frac{39}{4} qe = 0 \\ -3 EI/e^2 \varphi_B - 18 EI/e^3 \chi_A + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0 \end{cases}$$

$$63 EI/e^2 \varphi_B // + qe - 4qe - \frac{39}{4} qe + \frac{9}{2} qe + 2qe - \frac{13}{8} qe = 0$$

$$63 EI/e^2 \varphi_B // + \left( \frac{+8 - 32 - 78 + 36 + 16 - 13}{8} \right) qe = 0$$

$$63 EI/e^2 \varphi_B // - \frac{63}{8} qe = 0$$

$$63 EI/e^2 \varphi_B = \frac{63}{8} qe \rightarrow \boxed{\varphi_B = \frac{1}{8} \frac{qe^3}{EI}}$$

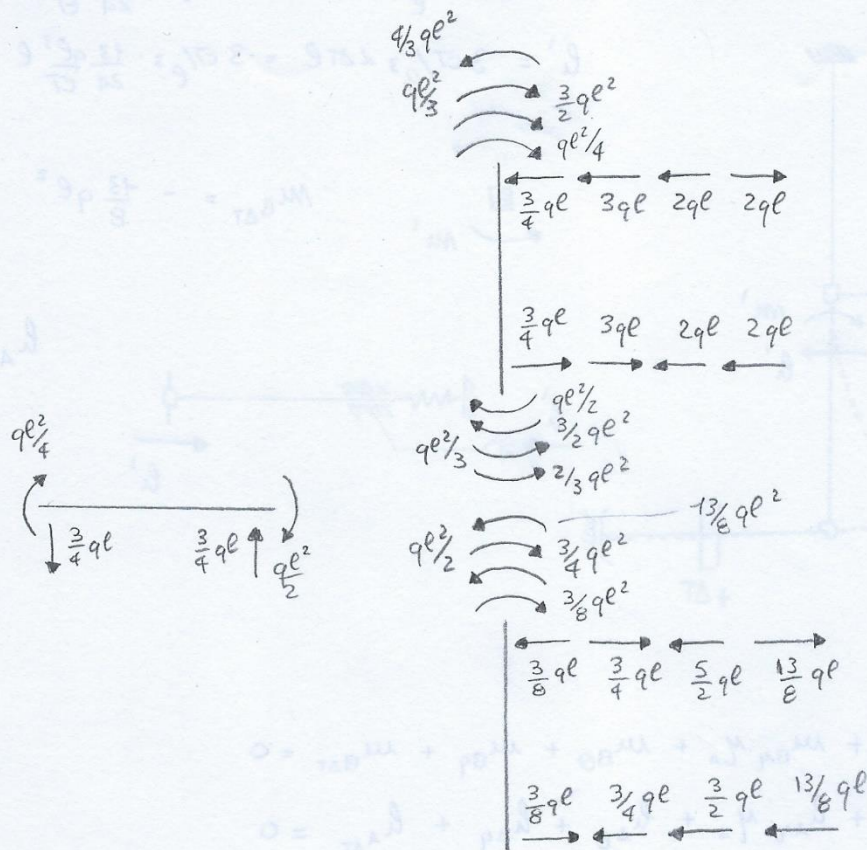
$$-3 EI/e^2 \cdot \frac{1}{8} \frac{qe^3}{EI} - 18 EI/e^3 \chi_A + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0$$

$$-18 EI/e^3 \chi_A - \frac{3}{8} qe + 2qe + \frac{9}{2} qe - \frac{13}{8} qe = 0$$

$$-18 EI/e^3 \chi_A + \frac{9}{2} qe = 0$$

$$-18 EI/e^3 \chi_A = -\frac{9}{2} qe \rightarrow \boxed{\chi_A = \frac{1}{4} \frac{qe^4}{EI}}$$

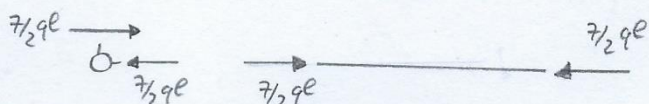
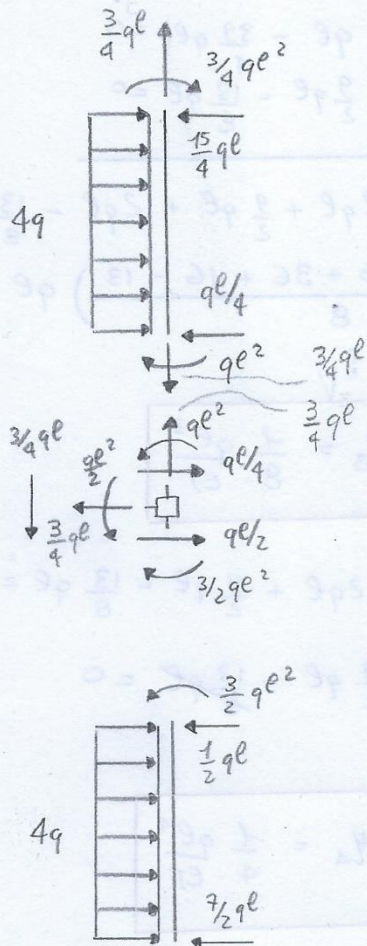
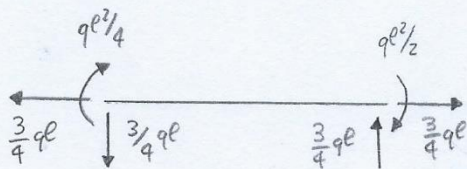
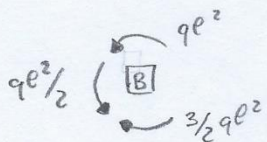




AZIONI SULLA STRUTTURA (TOTALI)

$$K_M \cdot \mu_A = 3 \frac{EI}{l^3} \cdot \frac{1}{4} \frac{q l^4}{EI} = \frac{3}{4} q l$$

$$\sum M_{(CB)} = q l^2 + \frac{q l^2}{2} - \frac{3}{2} q l^2 = 0 \quad \checkmark$$





$$\sum V_{GLOBALE} = 0 \rightarrow \frac{3}{4} q\ell - \frac{3}{4} q\ell = 0 \quad \textcircled{V}$$

$$\sum H_{GLOBALE} = 0 \rightarrow \frac{15}{4} q\ell + \frac{7}{2} q\ell + \frac{3}{4} q\ell - 4q\ell - 4q\ell = 0 \quad \textcircled{V}$$

$$\sum M_{GLOBALE}^{(D)} = 0 \rightarrow \frac{15}{4} q\ell \cdot 2\ell - \frac{3}{4} q\ell^2 - \frac{3}{4} q\ell \cdot \ell + \frac{3}{4} q\ell \cdot 2\ell + \frac{3}{4} q\ell \cdot \ell - \frac{q\ell^2}{4} - 4q\ell \cdot \frac{\ell}{2} - 4q\ell \cdot \frac{3}{2}\ell = 0$$

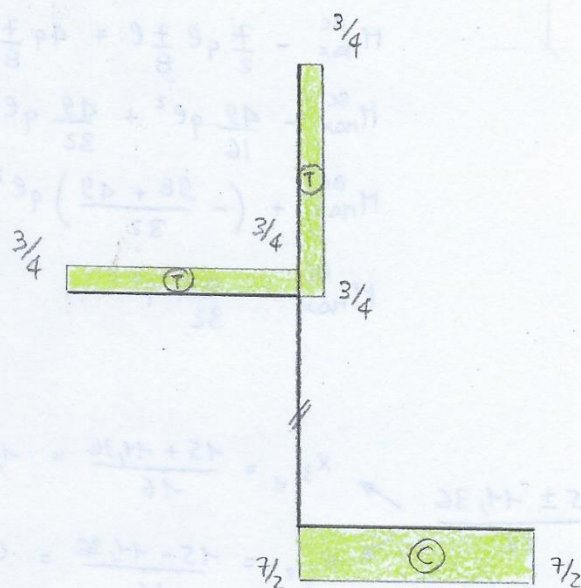
$$\frac{15}{2} q\ell^2 - \frac{3}{4} q\ell^2 - \frac{3}{4} q\ell^2 + \frac{3}{2} q\ell^2 + \frac{3}{4} q\ell^2 - \frac{q\ell^2}{4} - 2q\ell^2 - 6q\ell^2 = 0$$

$$9q\ell^2 - q\ell^2 - 2q\ell^2 - 6q\ell^2 = 0 \quad \textcircled{V}$$

DIAGRAMMI

[N/qℓ]

AZ. ASSIALE



TAGLIO

[V/qℓ]

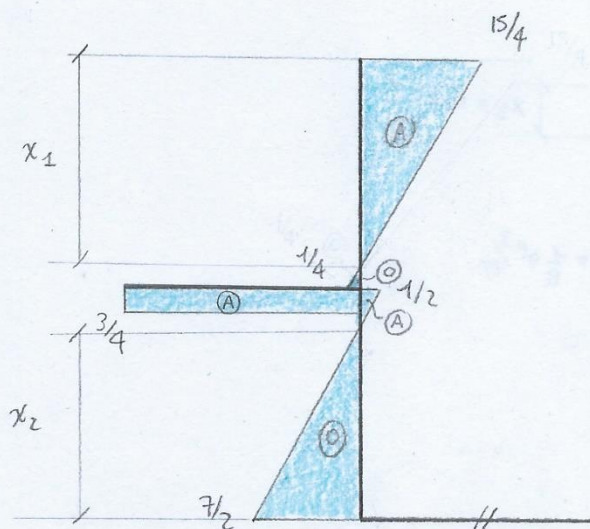
PUNTI DI ANNULLAMENTO DEL TAGLIO :

$$\bullet x_1 : \frac{15}{4} q\ell - 4qx_1 = 0$$

$$x_1 = \frac{15}{16} \ell$$

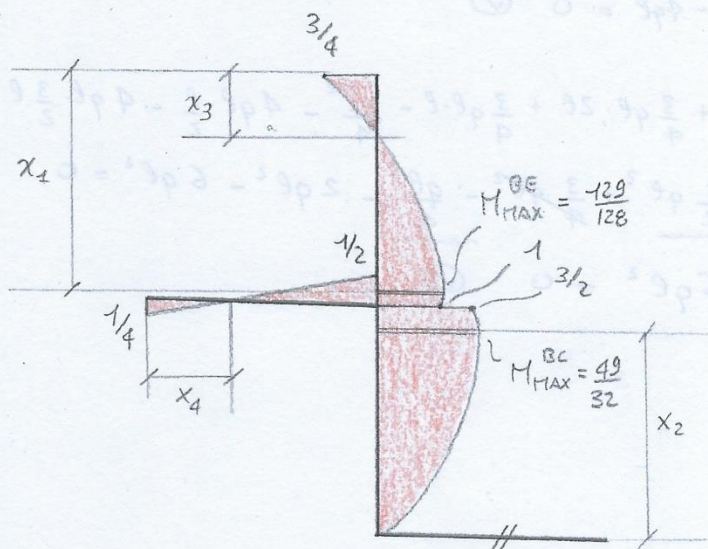
$$\bullet x_2 : \frac{7}{2} q\ell - 4qx_2 = 0$$

$$x_2 = \frac{7}{8} \ell$$





MOMENTO FLETTENTE  $[M/ql^2]$



•  $M_{MAX}^{BE}$ :

$$M_{MAX}^{BE} + \frac{3}{4} ql^2 + 4q x_1 \frac{x_1}{2} - \frac{15}{4} ql x_1 = 0$$

$$M_{MAX}^{BE} + \frac{3}{4} ql^2 + 4q \frac{15l}{16} \frac{15l}{32} - \frac{15}{4} ql \frac{15l}{16} = 0$$

$$M_{MAX}^{BE} + \frac{3}{4} ql^2 + \frac{225}{128} ql^2 - \frac{225}{64} ql^2 = 0$$

$$M_{MAX}^{BE} + \left( \frac{96 + 225 - 450}{128} \right) ql^2 = 0$$

$$M_{MAX}^{BE} = -\frac{129}{128} ql^2$$

•  $M_{MAX}^{BC}$ :

$$M_{MAX}^{BC} - \frac{7}{2} ql \frac{7l}{8} + 4q \frac{7l}{8} \frac{7l}{16} = 0$$

$$M_{MAX}^{BC} - \frac{49}{16} ql^2 + \frac{49}{32} ql^2 = 0$$

$$M_{MAX}^{BC} + \left( -\frac{98 + 49}{32} \right) ql^2 = 0$$

$$M_{MAX}^{BC} = \frac{49}{32} ql^2$$

PUNTI DI FLESSO:

•  $x_3$ :

$$\frac{3}{4} ql^2 + 4q x_3 \frac{x_3}{2} - \frac{15}{4} x_3 = 0$$

$$\frac{3}{4} ql^2 + 2q x_3^2 - \frac{15}{4} x_3 = 0$$

$$3ql^2 + 8q x_3^2 - 15x_3 = 0$$

$$8q x_3^2 - 15x_3 + 3ql^2 = 0$$

$$x_{1,2} = \frac{15 \pm \sqrt{225 - 96}}{16} = \frac{15 \pm \sqrt{129}}{16} = \frac{15 \pm 11,36}{16}$$

$$x_{3,1} = \frac{15 + 11,36}{16} = 1,65l \text{ NO}$$

$$x_{3,2} = \frac{15 - 11,36}{16} = 0,23l \text{ ✓}$$

•  $x_4$ :

$$\frac{ql^2}{4} - \frac{3}{4} ql x_4 = 0 \rightarrow x_4 = \frac{1}{3} l$$

DEFORMATA QUALITATIVA

