TECNICA DELLE COSTRUZIONI
TEMA ESAME DEL 29 MARZO 2021

DOCENTI:
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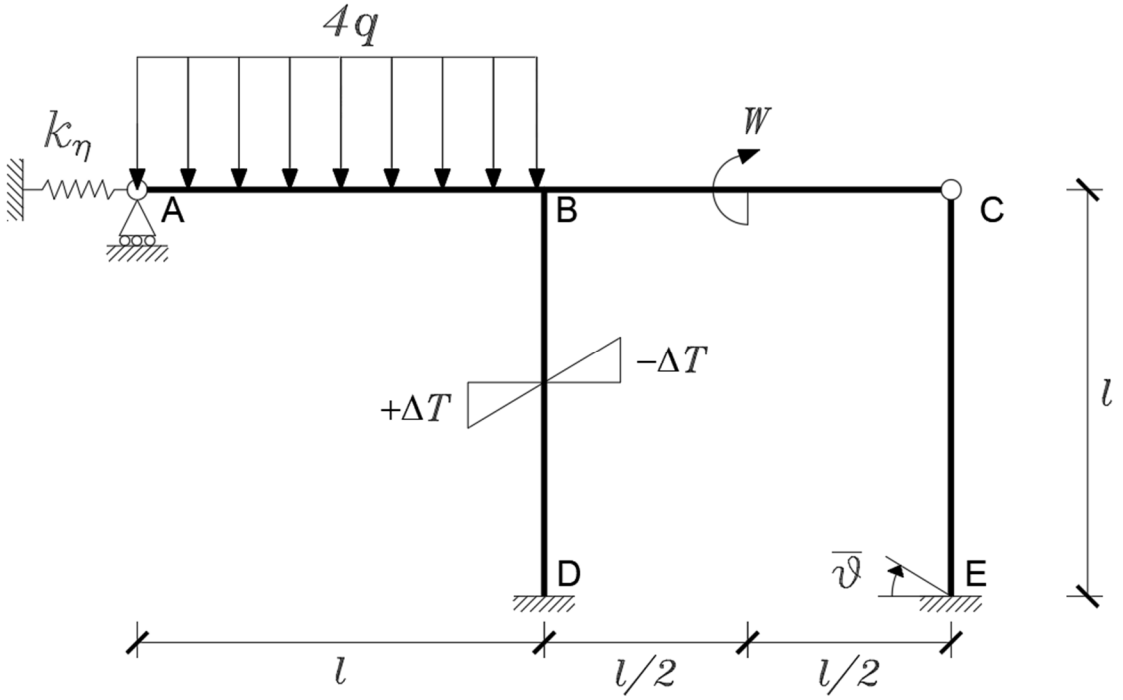
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**

Esercizio



$$k_\eta = 7\frac{EJ}{l^3} \qquad \frac{\alpha\Delta T}{h} = \frac{5}{3}\frac{ql^2}{EJ} \qquad \bar{\vartheta} = \frac{1}{4}\frac{ql^3}{EJ} \qquad W = 2 \cdot q \cdot l^2$$

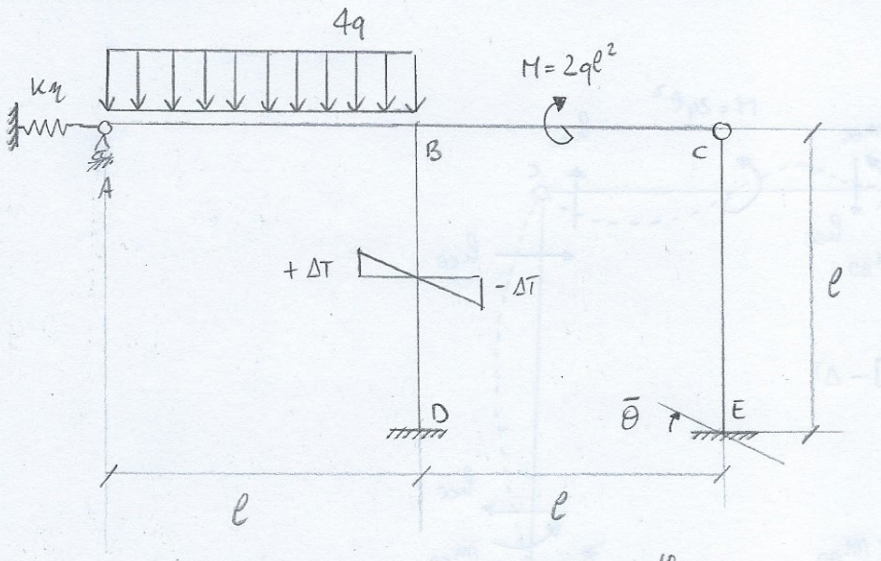
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TELAIO 29/03/2021

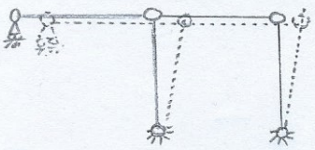


DATI

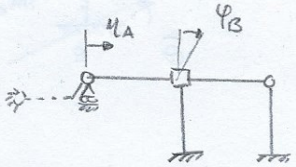
- $k_M = 7EJ/l^3$
- $\frac{2\Delta T}{h} = \frac{5}{3} ql^2/EJ$
- $\bar{\theta} = \frac{1}{4} ql^3/EJ$

ANALISI CINEMATICA

$\begin{cases} 6\ g.d.l \\ 9\ g.d.v. \end{cases}$ → 3 VOLTE IPERVINCOLATA



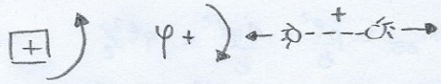
STRUTTURA LABILE



SISTEMA

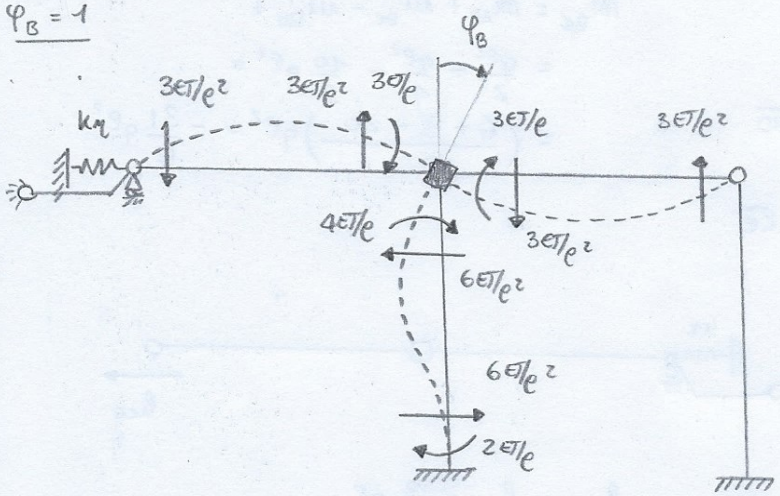
$$\begin{cases} m_{BB}\varphi_B + m_{BM}u_A + m_{B\phi} = 0 \\ h_{MB}\varphi_B + h_{MM}u_A + h_{M\phi} = 0 \end{cases}$$

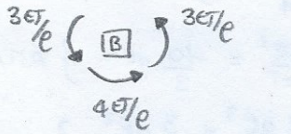
CONVENZIONI



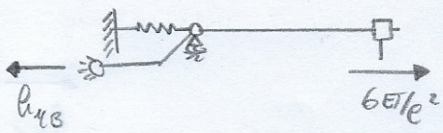
RISOLUZIONE

- $\varphi_B = 1$



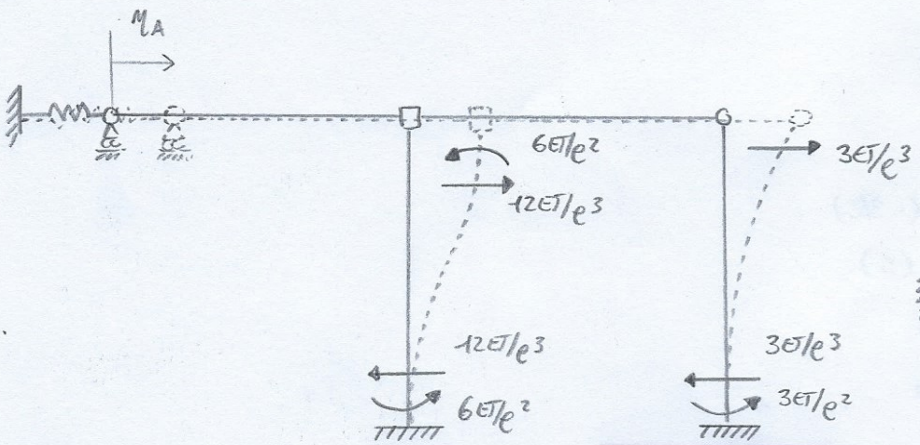


$m_{BB} = (3+3+4)EJ/l = 10\ EJ/l$

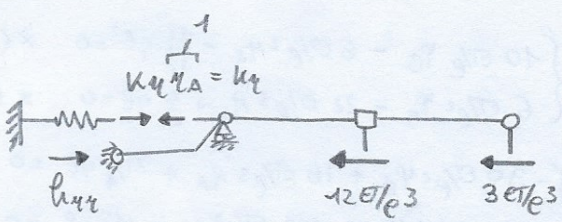


$h_{MB} = 6\ EJ/l^2$

- $u_A = 1$

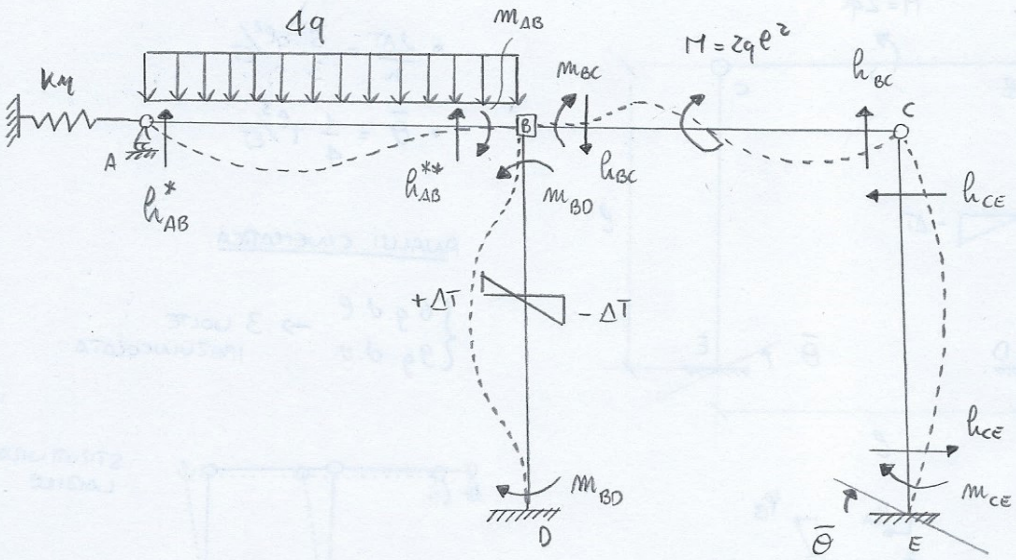


$m_{BM} = -6EJ/l^2$



$h_{MM} = -(15 + k_M) = -22\ EJ/l^3$

- $q \neq 0;\ \Delta T \neq 0;\ \bar{\theta} \neq 0$



$$h^{*}_{AB} = \frac{3}{8}\rho e^2 = \frac{3}{8}\cdot 4qe = \frac{3}{2}qe$$
$$h^{**}_{AB} = \frac{5}{8}\rho e^2 = \frac{5}{8}\,4qe = \frac{5}{2}qe$$
$$m_{AB} = \frac{\rho e^2}{8} = \frac{4qe^2}{8} = qe^2/2$$

} ASTA $\bar{AB}$

$$m_{BC} = \frac{M}{8} = \frac{2qe^2}{8} = qe^2/4$$
$$h_{BC} = \frac{9}{8}\,M/e = \frac{9}{8}\cdot\frac{2qe^2}{e} = \frac{9}{4}qe$$

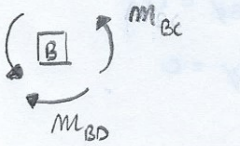
} ASTA $\bar{BC}$

$$m_{BD} = \frac{2EJ\,\Delta T}{h} = 2EJ\,\frac{5}{3}\frac{qe^2}{EJ} = \frac{10}{3}qe^2$$

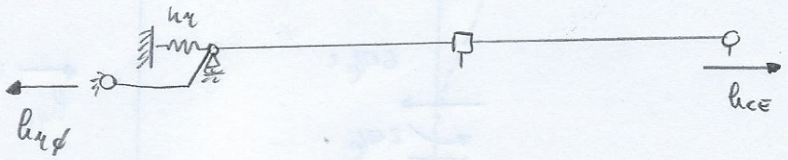
} ASTA $\bar{BD}$

$$m_{CE} = 3EJ/e\;\bar{\theta} = 3EJ/e\cdot\frac{1}{4}\frac{qe^3}{EJ} = \frac{3}{4}qe^2$$
$$h_{CE} = 3EJ/e^2\,\bar{\theta} = 3EJ/e^2\cdot\frac{1}{4}\frac{qe^3}{EJ} = \frac{3}{4}qe$$

} ASTA $\bar{CE}$



$$m_{B\phi} = m_{AB} + m_{BC} - m_{BD} =$$
$$= \frac{qe^2}{2} + \frac{qe^2}{4} - \frac{10}{3}qe^2 =$$
$$= \left(\frac{6+3-40}{12}\right)qe^2 = -\frac{31}{12}qe^2$$



$$h_{M\phi} = h_{CE} = \frac{3}{4}qe$$

SISTEMA RISOLVENTE

$$\begin{cases} m_{BB}\varphi_B + m_{BM}M_A + m_{B\phi} = 0 \\ h_{MB}\varphi_B + h_{MM}M_A + h_{M\phi} = 0 \end{cases}$$

$$\begin{cases} 10\,EJ/e\,\varphi_B - 6\,EJ/e^2 M_A - \frac{31}{12}qe^2 = 0 \quad \times(-3/e) \\ 6\,EJ/e^2\,\varphi_B - 22\,EJ/e^3 M_A + \frac{3}{4}qe = 0 \quad \times(5) \end{cases}$$

$$\begin{cases} -30\,EJ/e^2\varphi_B + 18\,EJ/e^3 M_A + 31/4\,qe = 0 \\ 30\,EJ/e^2\varphi_B - 110\,EJ/e^3 M_A + \frac{15}{4}qe = 0 \end{cases}$$

$$// \quad -92\,EJ/e^3 M_A + \frac{46}{4}qe = 0 \rightarrow \boxed{M_A = \frac{1}{8}\frac{qe^4}{EJ}}$$

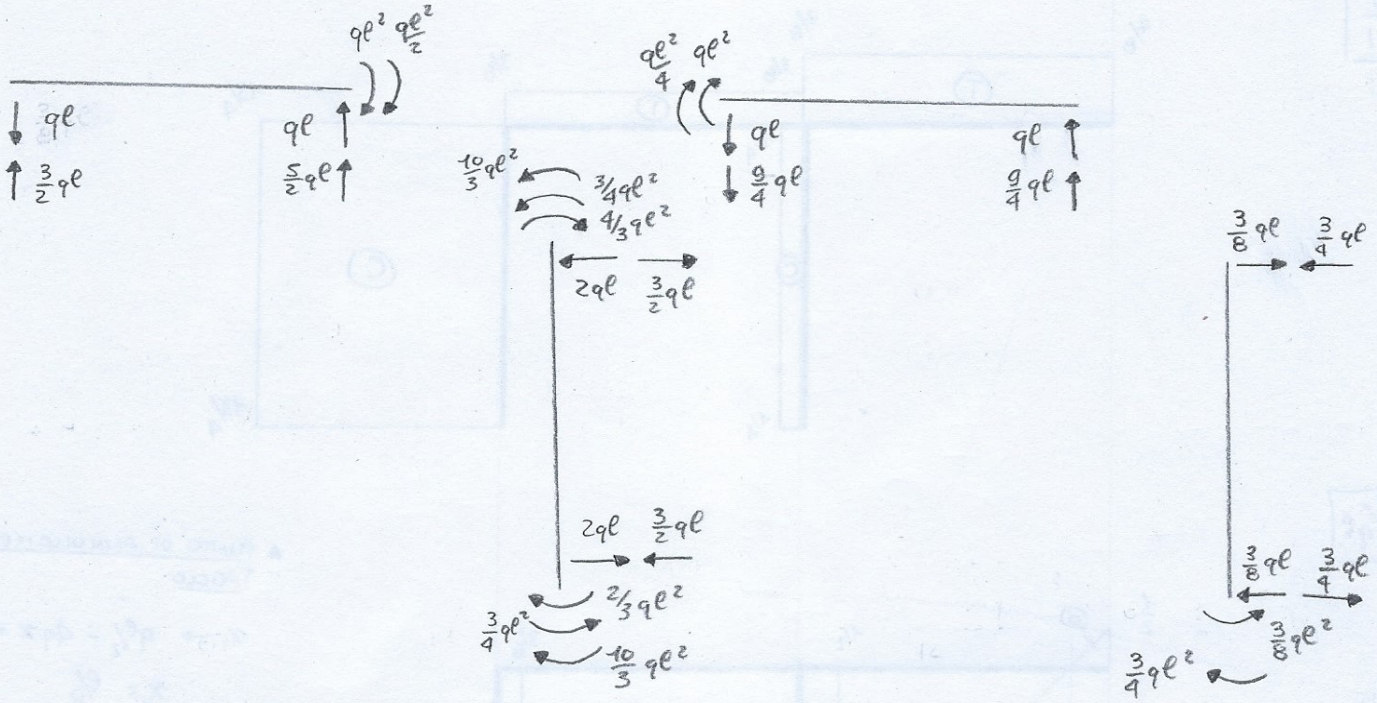
$$6\,EJ/\ell^2\,\varphi_B - 22\,EJ/\ell^3\,M_A + \frac{3}{4}q\ell = 0$$

$$6\,EJ/\ell^2\,\varphi_B - 22\,EJ/\ell^3 \cdot \frac{1}{8}\frac{q\ell^4}{EJ} + \frac{3}{4}q\ell = 0$$

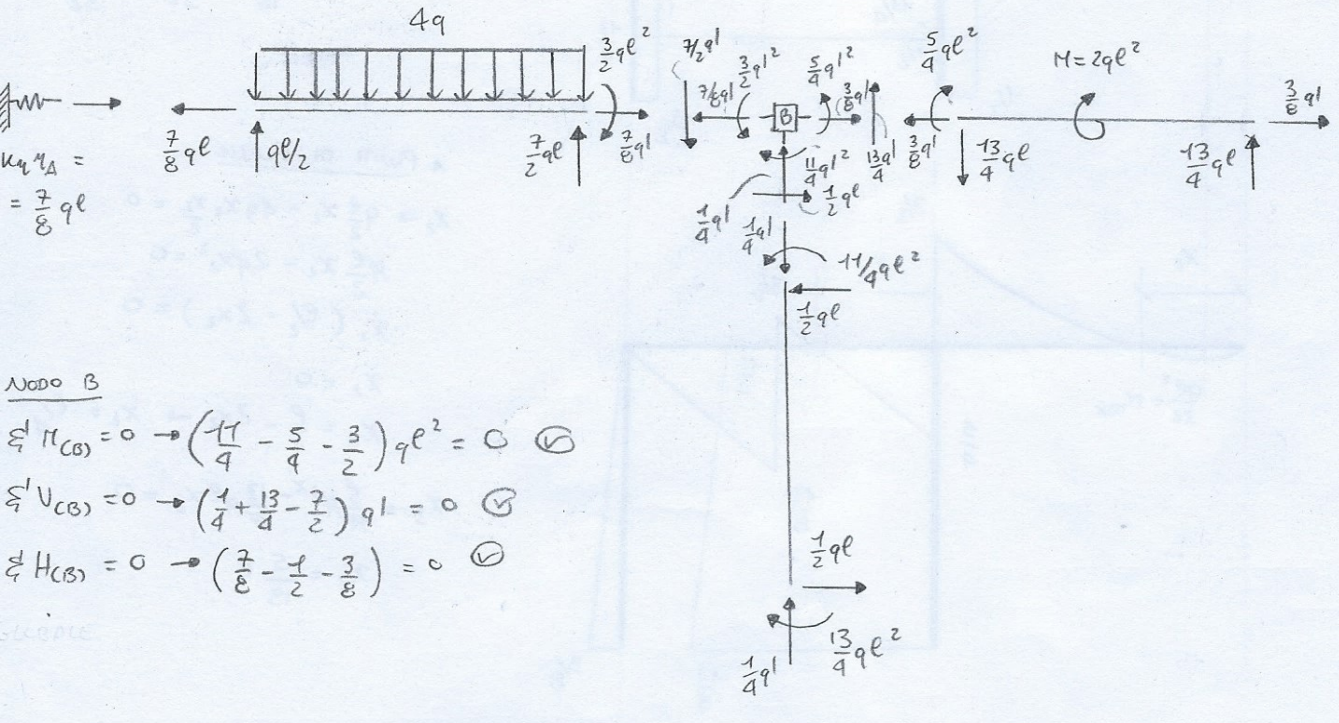
$$6\,EJ/\ell^2\,\varphi_B - \frac{11}{4}q\ell + \frac{3}{4}q\ell = 0$$

$$6\,EJ/\ell^2\,\varphi_B - \frac{8}{4}q\ell = 0 \rightarrow \boxed{\varphi_B = \frac{1}{3}\frac{q\ell^3}{EJ}}$$

AZIONI INTERNE



AZIONI INTERNE TOT.



$k_M\,M_A = \frac{7}{8}q\ell$

NODO B

$\sum_i M_{(B)} = 0 \rightarrow \left(\frac{11}{4} - \frac{5}{4} - \frac{3}{2}\right)q\ell^2 = 0$ ✓

$\sum_i V_{(B)} = 0 \rightarrow \left(\frac{1}{4} + \frac{13}{4} - \frac{7}{2}\right)q\ell = 0$ ✓

$\sum_i H_{(B)} = 0 \rightarrow \left(\frac{7}{8} - \frac{1}{2} - \frac{3}{8}\right) = 0$ ✓

VERIFICA GLOBALE

$\Sigma H^{GLOB} = 0 \rightarrow \left(\frac{3}{8} + \frac{1}{2} - \frac{7}{8}\right) q\ell = 0$ ✓

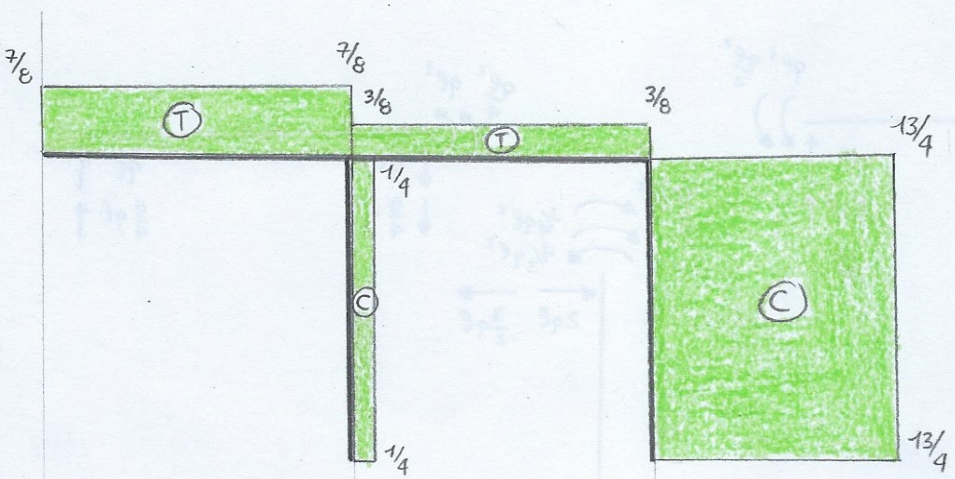
$\Sigma V^{GLOB} = 0 \rightarrow \frac{1}{4}q\ell + \frac{13}{4}q\ell + \frac{1}{2}q\ell - 4q\ell = 0$ ✓

$\Sigma M^{GLOB}_{(B)} = 0 \rightarrow q\frac{\ell}{2}\cdot\ell - 4q\ell\frac{\ell}{2} - q\frac{\ell}{2}\ell + \frac{13}{4}q\ell^2 + 2q\ell^2 - \frac{3}{8}q\ell\ell + \frac{3}{8}q\ell^2 - \frac{13}{4}q\ell\ell = 0$

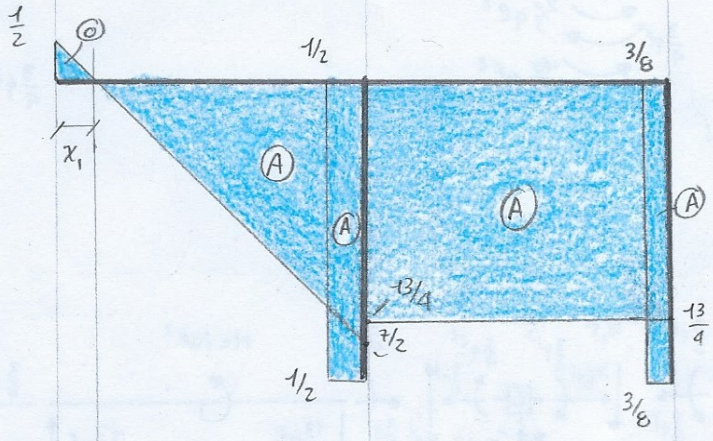
$\frac{q\ell^2}{2} - 2q\ell^2 - \frac{q\ell^2}{2} + \frac{13}{4}q\ell^2 + 2q\ell^2 - \frac{3}{8}q\ell^2 + \frac{3}{8}q\ell^2 = \frac{13}{4}q\ell^2 = 0$ ✓

DIAGRAMMI

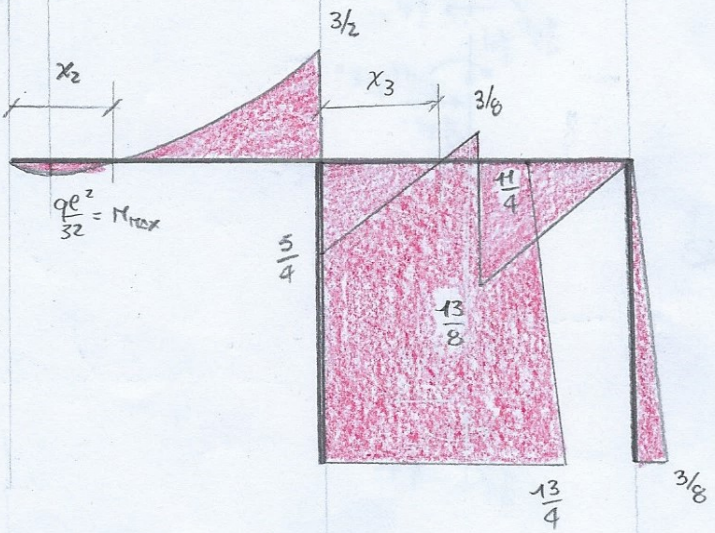
N / ql



V / qℓ



M / qℓ²



▲ PUNTO DI ANNULLAMENTO DEL TAGLIO

$x_1 \rightarrow q\ell/2 - 4qx = 0$

$x_1 = \ell/8$

$M_{MAX} \rightarrow q\frac{\ell}{2}\frac{\ell}{8} - 4q\frac{\ell}{8}\frac{\ell}{16} =$

$= \frac{q\ell^2}{16} - \frac{q\ell^2}{32} = \frac{q\ell^2}{32}$

▲ PUNTI DI FLESSO

$x_2 \rightarrow q\frac{\ell}{2}x_2 - 4qx_2\frac{x_2}{2} = 0$

$q\frac{\ell}{2}x_2 - 2qx_2^2 = 0$

$x_2(\ell/2 - 2x_2) = 0$

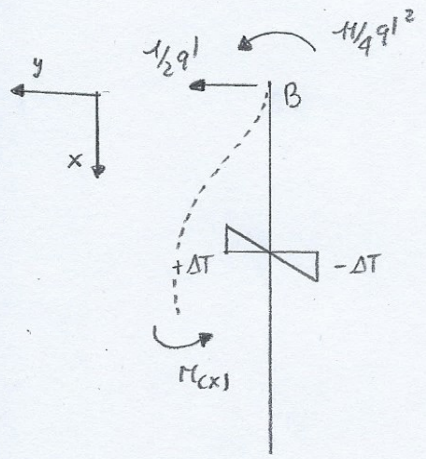
$x_2 = 0$

$x_2 = \frac{\ell}{2} - 2x_2 \rightarrow x_2 = \ell/4$

$x_3 \rightarrow \frac{5}{4}q\ell^2 - \frac{13}{4}q\ell x_3 = 0$

$x_3 = \frac{5}{13}\ell$

STUDIO DELLA DEFORMATA TERMICA



$$M(x) + \frac{11}{4}ql^2 + \frac{ql}{2}x = 0$$

$$M(x) = -\frac{11}{4}ql^2 - \frac{ql}{2}x$$

$$y''(x) = -\frac{M(x)}{EJ} - \frac{2\alpha\Delta T}{h} = -\frac{1}{EJ}\left(-\frac{11}{4}ql^2 - \frac{ql}{2}x\right) - 2\cdot\frac{5}{3}\frac{ql^2}{EJ} =$$

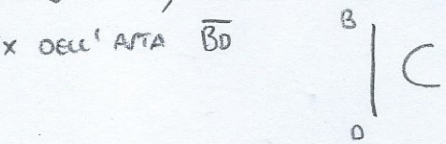
$$= \frac{11}{4}\frac{ql^2}{EJ} + \frac{qlx}{2EJ} - \frac{10}{3}\frac{ql^2}{EJ} =$$

$$= \frac{qlx}{2EJ} - \frac{7}{12}\frac{ql^2}{EJ}$$

$$y''(x) \geq 0 \rightarrow \frac{qlx}{2EJ} - \frac{7}{12}\frac{ql^2}{EJ} \geq 0 \rightarrow \frac{qlx}{2EJ} \geq \frac{7}{12}\frac{ql^2}{EJ} \rightarrow$$

$$\rightarrow x \geq \frac{14}{12}l$$

DA $x \geq l$, QUINDI LA CONCAVITÀ È VERSO SINISTRA $\forall x$ DELL'ASTA $\overline{BD}$



DEFORMATA QUALITATIVA