

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 22 GIUGNO 2021

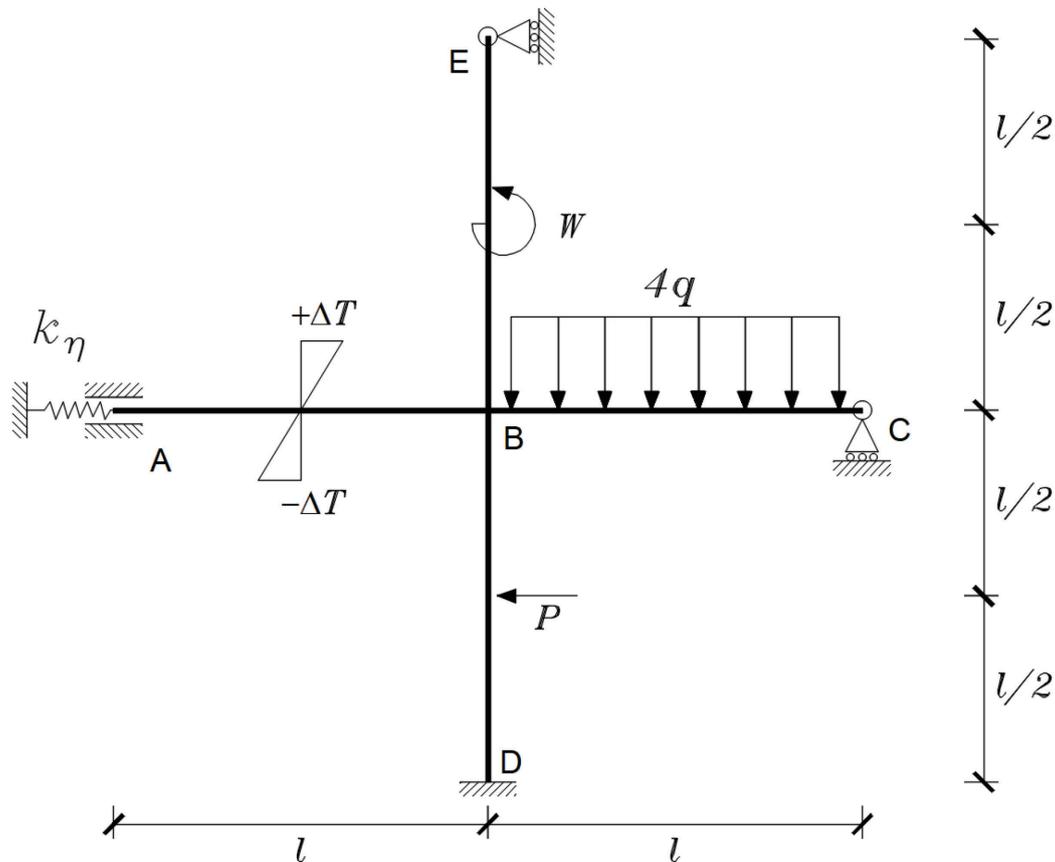
DOCENTI:

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PROF. FAUSTO MINELLI

DURATA: 2 ORE.

Esercizio



$$k_{\eta} = 9 \frac{EJ}{l^3}$$

$$\frac{\alpha \Delta T}{h} = \frac{7}{16} \frac{q l^2}{EJ}$$

$$W = q \cdot l^2$$

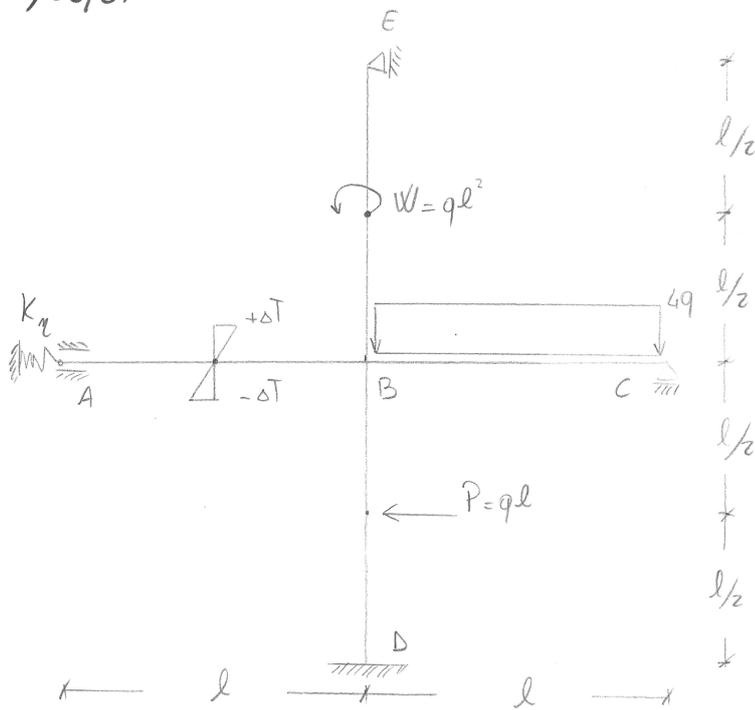
$$P = q \cdot l$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \rightarrow \infty$, $EJ = \text{costante}$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.



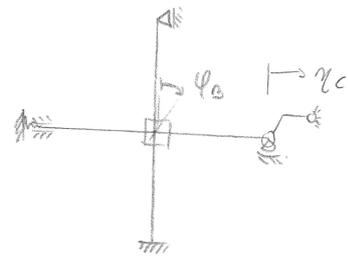
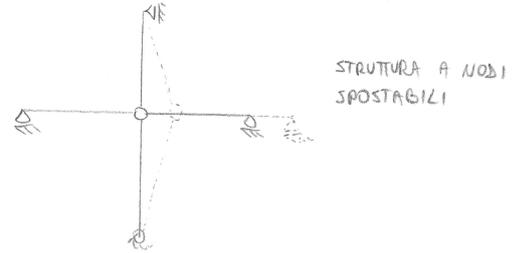
• DATI

$$K_q = 9 \frac{EI}{l^3}$$

$$\frac{\alpha \Delta T}{h} = \frac{7}{16} \frac{ql^2}{EI}$$

• ANALISI CINEMATICA

$$\begin{cases} 3 \text{ g.d.l.} \\ 7 \text{ g.d.v.} \end{cases} \rightarrow 4 \text{ VOLTE IPERVINCOLATA}$$



• SISTEMA RISOLVENTE

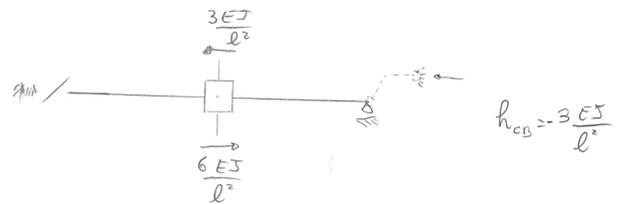
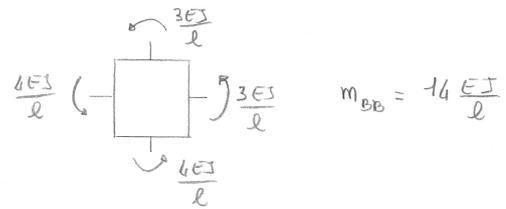
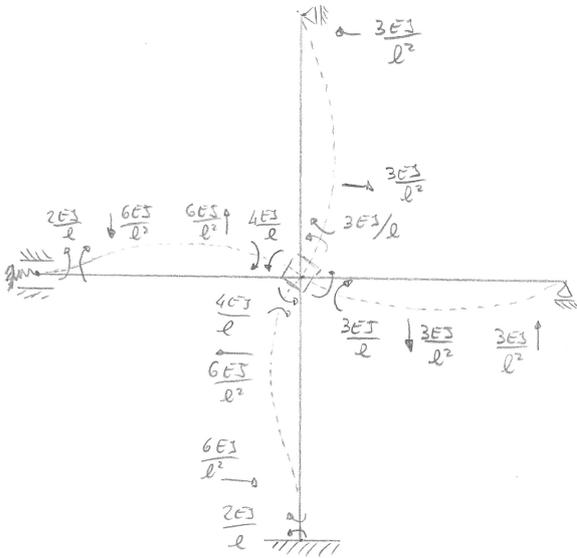
$$\begin{cases} m_{BB} \phi_B + m_{BC} \eta_C + m_{B\phi} = 0 \\ h_{CB} \phi_B + h_{CC} \eta_C + h_{C\phi} = 0 \end{cases}$$

• CONVENZIONI

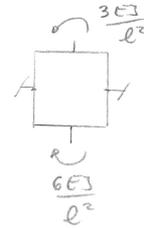
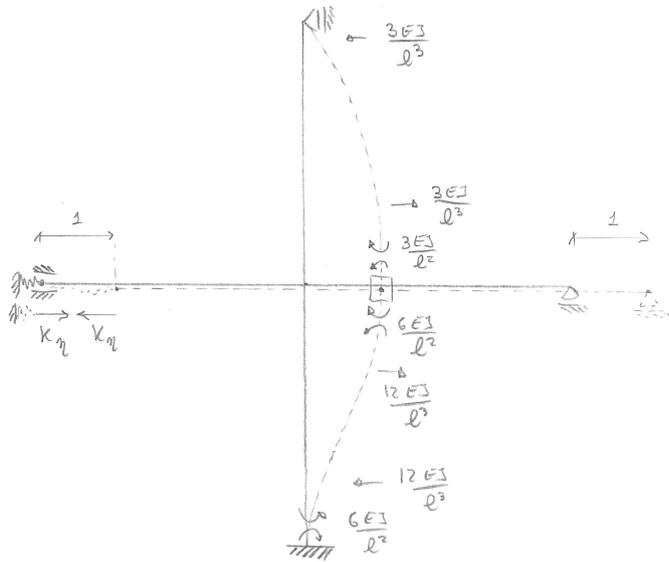


• RISOLUZIONE

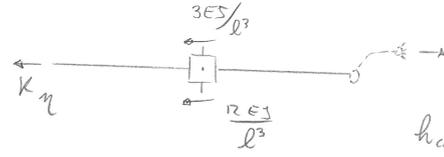
• $\phi_B = 1$



• $\eta = 1$

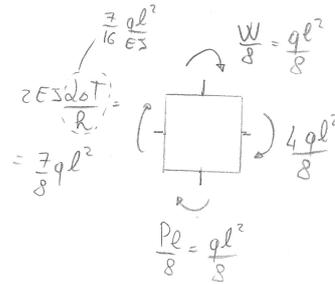
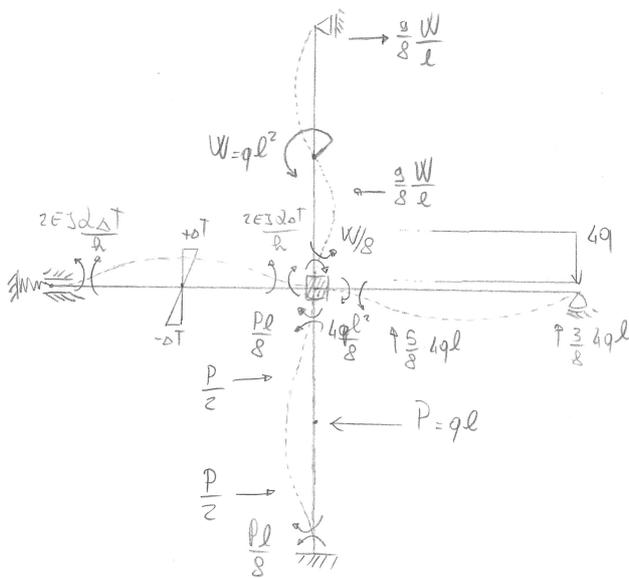


$m_{bc} = -\frac{3EI}{l^2}$

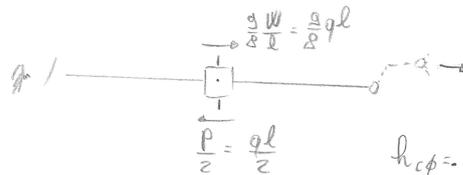


$h_{cc} = 15 \frac{EI}{l^3} + K_{\eta} = 24 \frac{EI}{l^3}$

• $q \neq 0; \Delta T \neq 0; M \neq 0; P \neq 0$



$m_{bd} = -\frac{W}{8} - \frac{4ql^2}{8} - \frac{Pl}{8} - \frac{2EI\alpha\Delta T}{h}$
 $= -\left(\frac{ql^2}{8} + \frac{ql^2}{2} + \frac{ql^2}{8} + \frac{7}{8}ql^2\right) = -\frac{13}{8}ql^2$



$h_{cd} = -\frac{3}{8}ql + \frac{ql}{2} = -\frac{5}{8}ql$

• SISTEMA

$\begin{cases} m_{BB}\phi_B + m_{BC}\eta_C + m_{Bd}\phi = \phi \\ h_{cB}\phi_B + h_{cC}\eta_C + h_{cd}\phi = \phi \end{cases}$

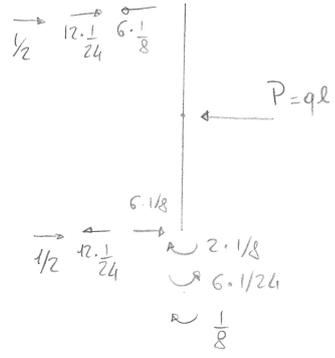
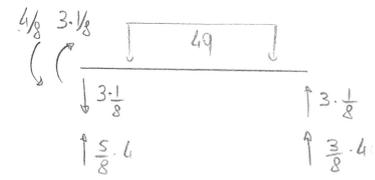
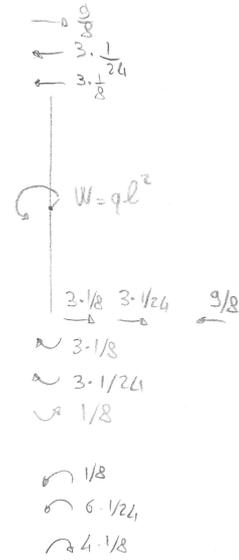
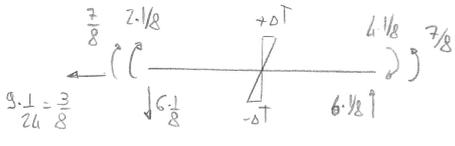
$\begin{cases} 14 \frac{EI}{l^2} \phi_B - 3 \frac{EI}{l^2} \eta_C - \frac{13}{8} ql^2 = \phi & (1) \\ -3 \frac{EI}{l^2} \phi_B + 24 \frac{EI}{l^3} \eta_C - \frac{5}{8} ql = \phi & (2) \end{cases}$

$(2) + \frac{8}{l} \times (1) \Rightarrow +103 \frac{EI}{l^2} \phi_B - \frac{103}{8} ql = \phi \Rightarrow \phi_B = \frac{1}{8} \frac{ql^3}{EI}$

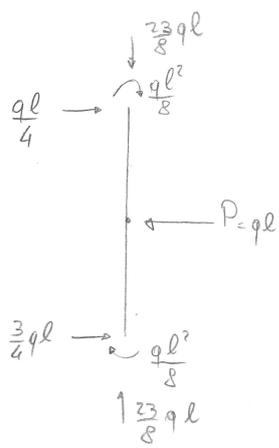
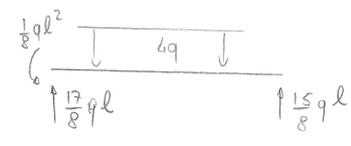
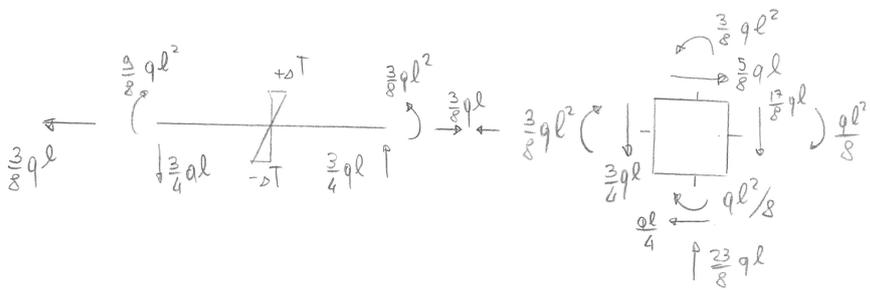
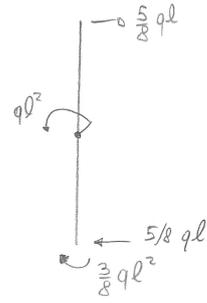
SOSTITUISCO ϕ_B IN (2) $-\frac{3}{8} ql + 24 \frac{EI}{l^3} \eta_C - \frac{5}{8} ql = \phi \Rightarrow \eta_C = \frac{1}{24} \frac{ql^4}{EI}$

$\phi_B = \frac{1}{8} \frac{ql^3}{EI}; \eta_C = \frac{1}{24} \frac{ql^4}{EI}$

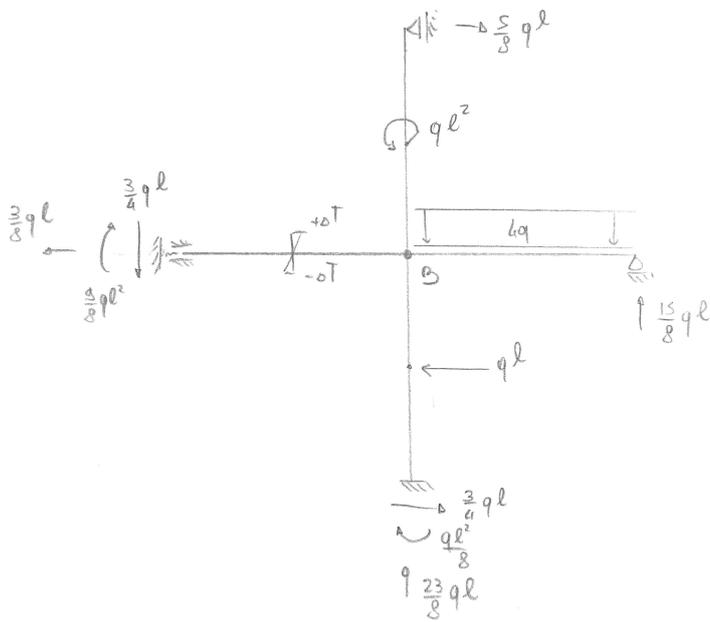
• AZIONI INTERNE



• AZIONI INTERNE TOT



• EQUILIBRIO GLOBALE



$$\uparrow \sum F_V = 0 \quad 4ql + \frac{3}{4}ql - \frac{15}{8}ql - \frac{23}{8}ql = 0 \quad \checkmark$$

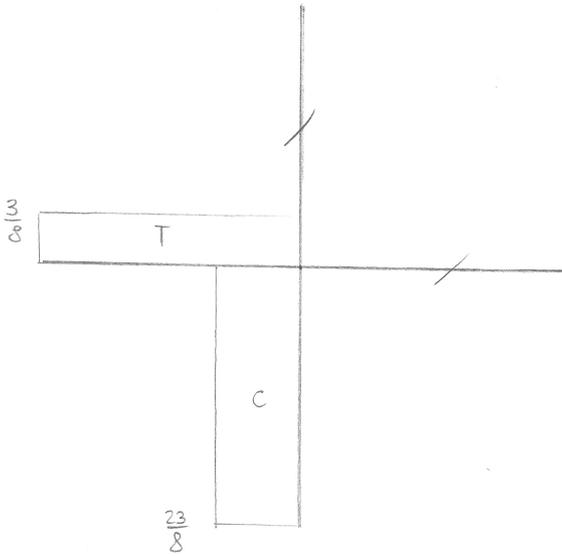
$$\rightarrow \sum F_H = 0 \quad \frac{5}{8}ql - \frac{3}{8}ql - ql + \frac{3}{4}ql = 0 \quad \checkmark$$

$$\circlearrowleft \sum M_B = 0$$

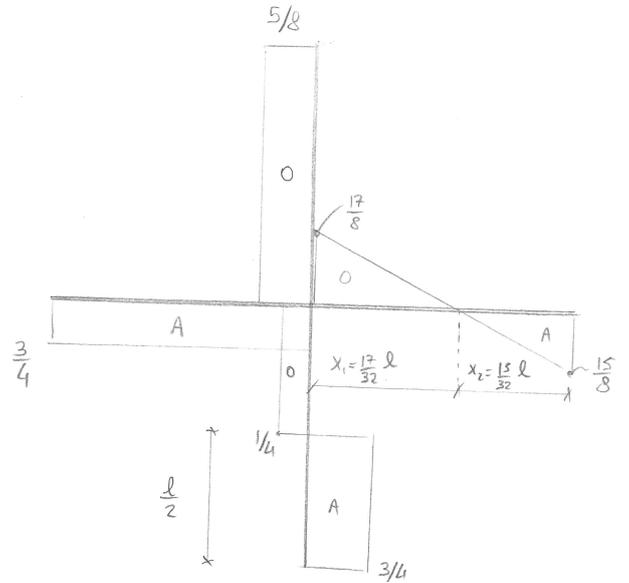
$$\frac{5}{8}ql^2 - ql^2 - \frac{3}{4}ql^2 + \frac{9}{8}ql^2 + 4ql \cdot \frac{l}{2} - \frac{15}{8}ql^2 + ql^2 - \frac{3}{4}ql^2 + \frac{ql^2}{8} = 0 \quad \checkmark$$

• DIAGRAMMI AZIONI INTERNE

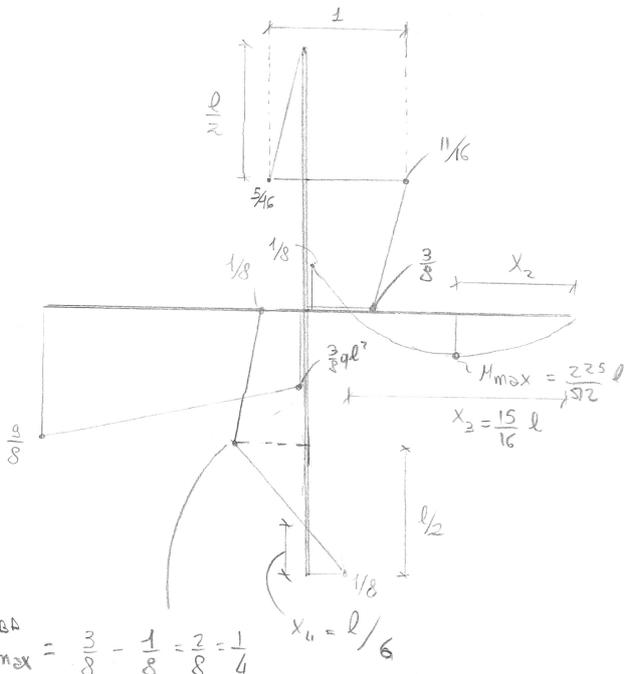
$\frac{N}{ql}$



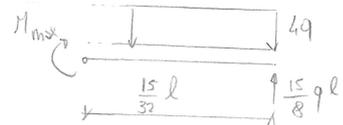
$\frac{V}{ql}$



$\frac{M}{ql^2}$

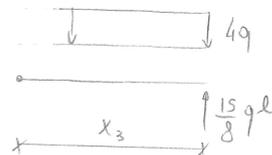


M_{max}^{BC}



$$M_{max} = \frac{15}{8}ql \cdot \frac{15}{32}l - 4q \left(\frac{15}{32}l \right)^2 \cdot \frac{1}{2} = \frac{225}{512}ql^2$$

x_3 :



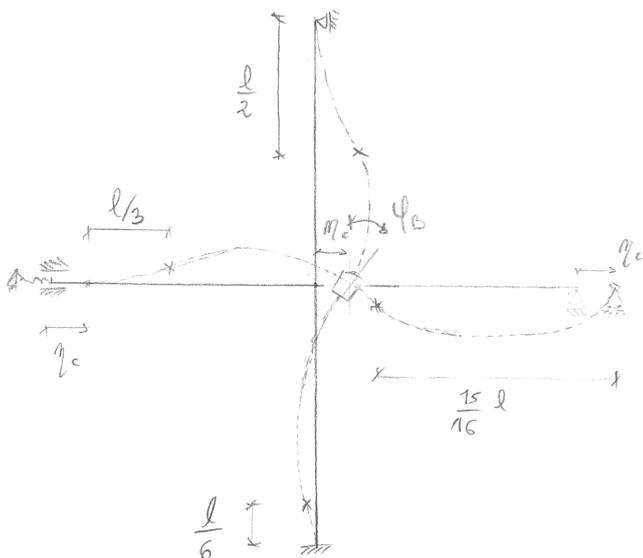
$$\frac{15}{8}ql x_3 = 4q \frac{x_3^2}{2} \Rightarrow x_3 = \frac{15}{16}l$$

x_4 :

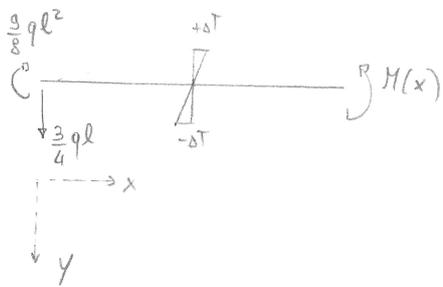
$$M(x) = 0 : \frac{3}{8}ql x = \frac{1}{8}ql^2 \Rightarrow x = \frac{l}{6}$$

$$M_{max}^{CA} = \frac{3}{8}ql^2 - \frac{1}{8}ql^2 = \frac{2}{8}ql^2 = \frac{1}{4}ql^2$$

• DEFORMATA QUALITATIVA



• DEFORMATA TERMICA



$$M(x) = \frac{3}{8} ql^2 - \frac{3}{4} qlx$$

$$y'' = -\frac{M(x)}{EI} + 2\alpha\Delta T = \frac{3}{4} \frac{qlx}{EI} - \frac{3}{8} \frac{ql^2}{EI} + 2 \cdot \frac{7}{16} \frac{ql^2}{EI} = \frac{3}{4} \frac{qlx}{EI} - \frac{1}{4} \frac{ql^2}{EI}$$

$$y'' > 0 \quad \frac{3}{4} \frac{qlx}{EI} > \frac{1}{4} \frac{ql^2}{EI}; \quad x > \frac{l}{3}$$