TECNICA DELLE COSTRUZIONI
TEMA ESAME DEL 7 LUGLIO 2021

DOCENTI:
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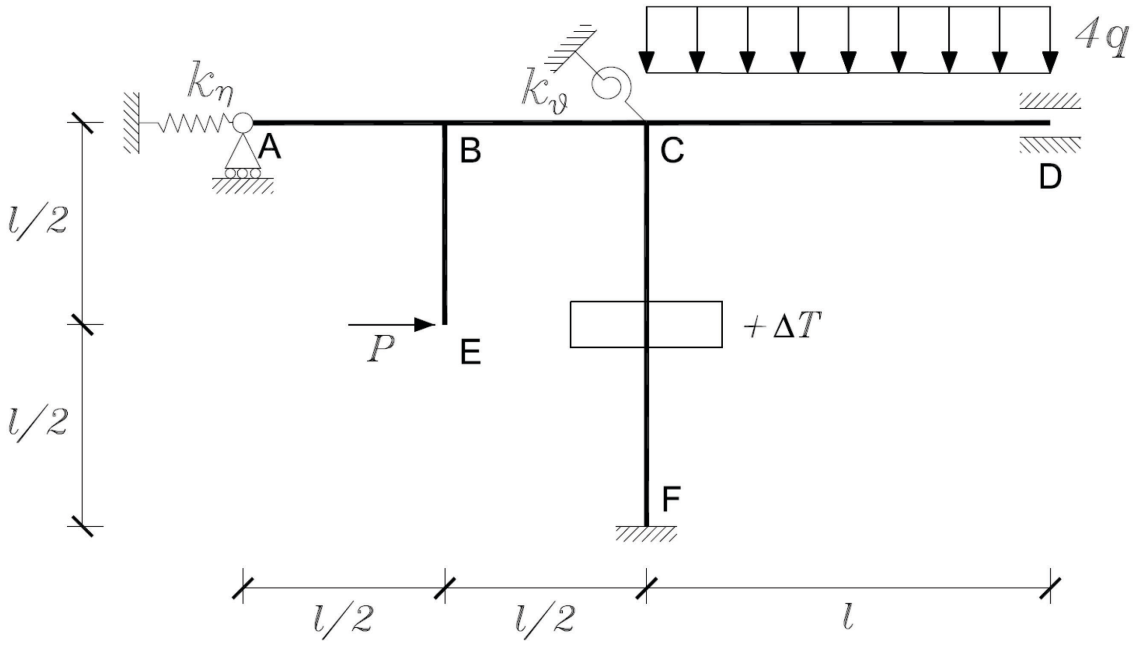
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**

Esercizio



$$k_\eta = 12\frac{EJ}{l^3} \qquad k_\theta = \frac{1}{4}\frac{EJ}{l} \qquad P = 2 \cdot q \cdot l \qquad \alpha\Delta T = \frac{2}{9}\frac{ql^3}{EJ}$$

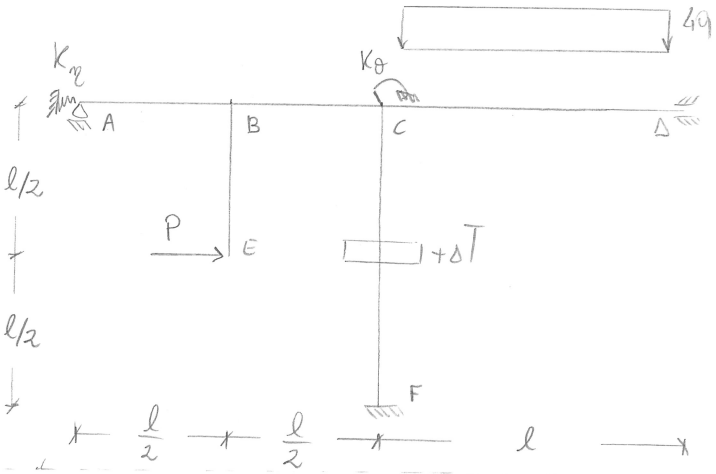
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TELAIO 07/07/2021



- DATI

$P = 2ql$

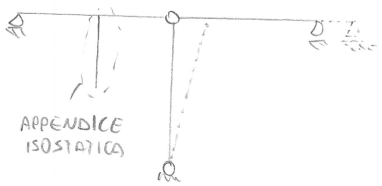
$K_\eta = 12\,EJ/l^3$

$K_\vartheta = \frac{1}{4}\,EJ/l$

$\alpha\Delta T = \frac{2}{3}\frac{ql^3}{EJ}$

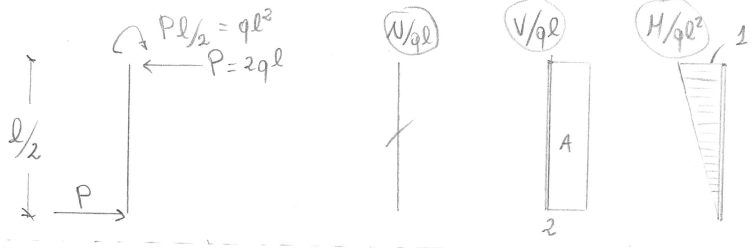
- ANALISI CINEMATICA

$\begin{cases} 3\ g.d.l \\ 8\ g.d.v. \end{cases} \Rightarrow$ 5 IPERVINCOLATA

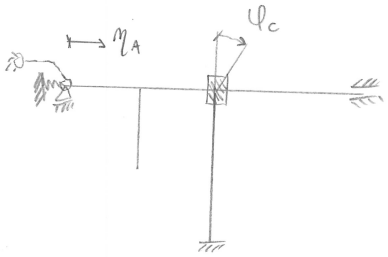


TELAIO A NODI SPOSTABILI

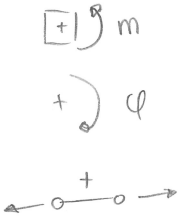
- RISOLVO APPENDICE ISOSTATICA BE



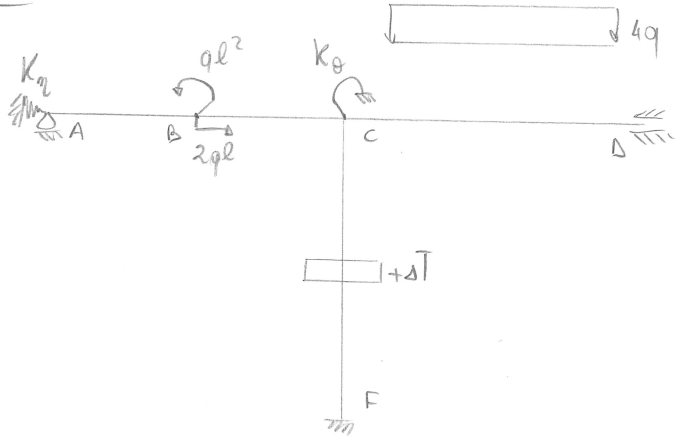
- SCELTA INCOGNITE



- CONVENZIONI



- TELAIO

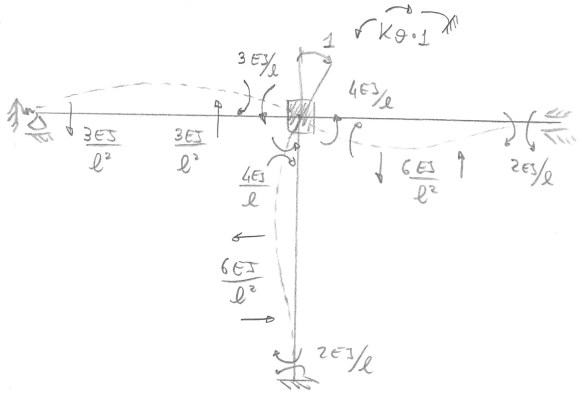


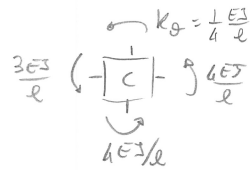
- SISTEMA RISOLVENTE

$$\begin{cases} m_{cc}\varphi_C + m_{CA}\eta_A + m_{C\phi} = \phi \\ h_{AC}\varphi_C + h_{AA}\eta_A + h_{A\phi} = \phi \end{cases}$$

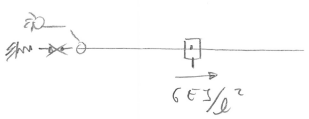
- RISOLUZIONE

- $\varphi_C = 1$



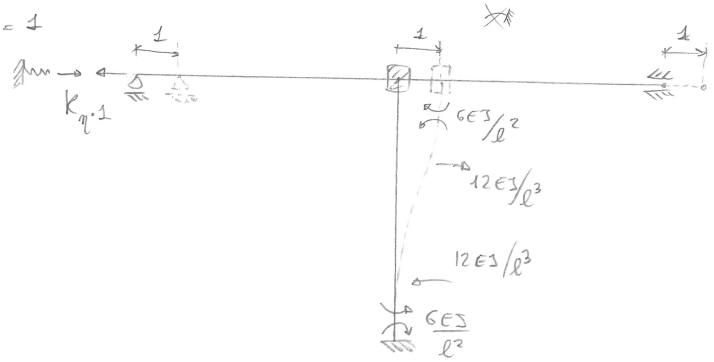


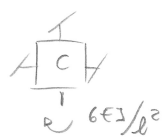
$m_{cc} = 11\frac{EJ}{l} + K_\vartheta$



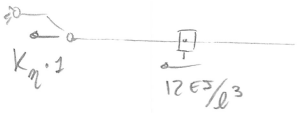
$h_{AC} = 6EJ/l^2$

- $\eta_A = 1$



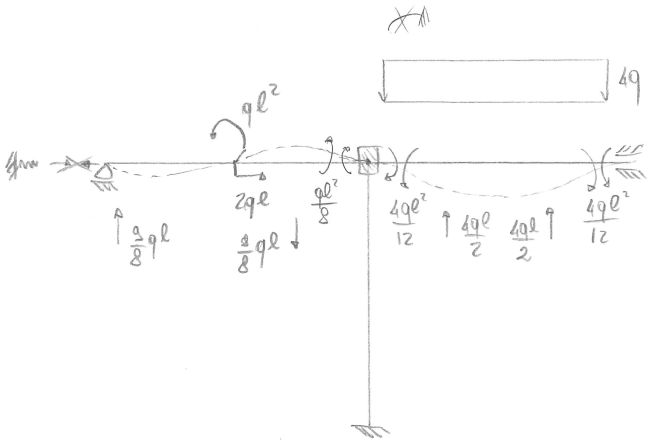


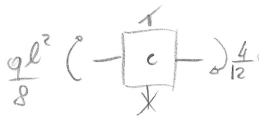
$m_{CA} = -6EJ/l^2$



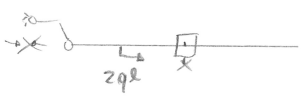
$h_{AA} = -\left(12\frac{EJ}{l^3} + K_\eta\right)$

• $q \neq 0$



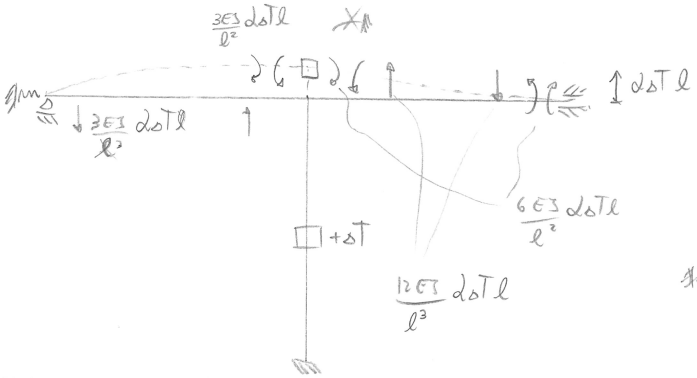


$$m^q_{C\phi} = -\left(\frac{4}{12}+\frac{1}{8}\right)q\ell^2 = -\frac{11}{24}q\ell^2$$



$$h^q_{A\phi} = 2q\ell$$

• $\delta T \neq 0$



$$m^{\delta T}_{C\phi} = -\frac{3EJ}{\ell^2}\alpha\delta T\ell$$

$$h^{\delta T}_{A\phi} = \phi$$

• SISTEMA RISOLVENTE

$$\begin{cases} \left(\frac{11EJ}{\ell}+K_\theta\right)\varphi_C - \frac{6EJ}{\ell^2}\eta_A - \left(\frac{11}{24}q\ell^2 + 3\frac{EJ}{\ell}\alpha\delta T\right) = \phi \\ \frac{6EJ}{\ell^2}\varphi_C - \left(\frac{12EJ}{\ell^3}+K_\eta\right)\eta_A + 2q\ell = \phi \end{cases}$$

$$\begin{cases} \left(\frac{11EJ}{\ell}+\frac{1}{4}\frac{EJ}{\ell}\right)\varphi_C - \frac{6EJ}{\ell^2}\eta_A - \left(\frac{11}{24}q\ell^2 + 3\cdot\frac{2}{9}q\ell^2\right) = \phi \\ \frac{6EJ}{\ell^2}\varphi_C - \left(\frac{12EJ}{\ell^3}+\frac{12EJ}{\ell^3}\right)\eta_A + 2q\ell = \phi \end{cases}$$

$$\begin{cases} \frac{45}{4}\frac{EJ}{\ell}\varphi_C - 6\frac{EJ}{\ell^2}\eta_A - \frac{9}{8}q\ell^2 = \phi \quad (1) \\ 6\frac{EJ}{\ell^2}\varphi_C - 24\frac{EJ}{\ell^3}\eta_A + 2q\ell = \phi \quad (2) \end{cases}$$

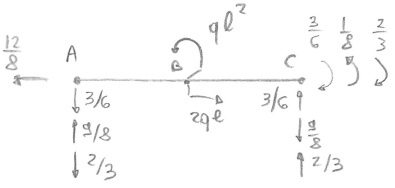
(1) $-\frac{\ell}{4}$ (2) $\Rightarrow \frac{39}{4}\frac{EJ}{\ell}\varphi_C = \frac{13}{8}q\ell^2 \Rightarrow \varphi_C = \frac{13}{8}\cdot\frac{4}{39}\frac{q\ell^3}{EJ} \Rightarrow$

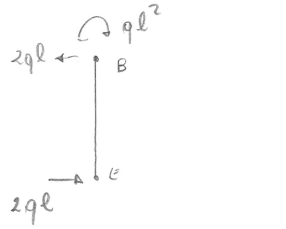
$$\varphi_C = \frac{1}{6}\frac{q\ell^3}{EJ}$$

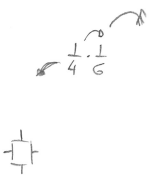
(2) $6\frac{EJ}{\ell}\cdot\frac{1}{6}\frac{q\ell^3}{EJ} - 24\frac{EJ}{\ell^3}\eta_A + 2q\ell = \phi \qquad 24\frac{EJ}{\ell^3}\eta_A = 3q\ell \Rightarrow$

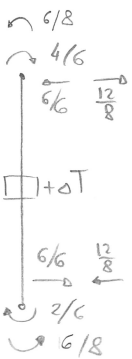
$$\eta_A = \frac{1}{8}\frac{q\ell^4}{EJ}$$

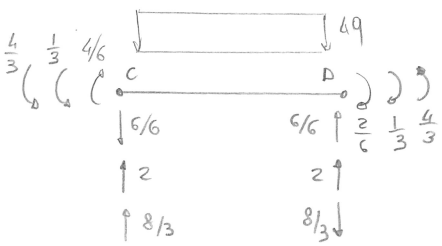
• AZIONI



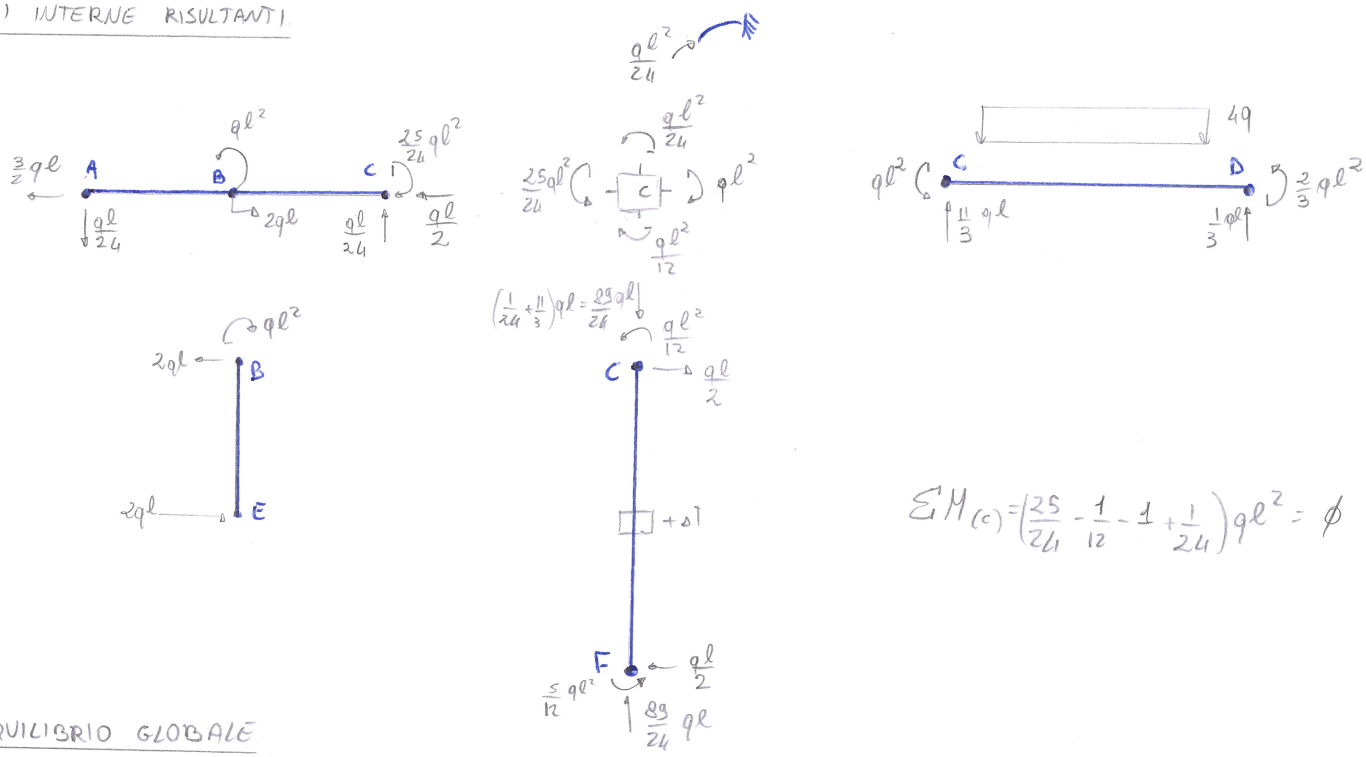






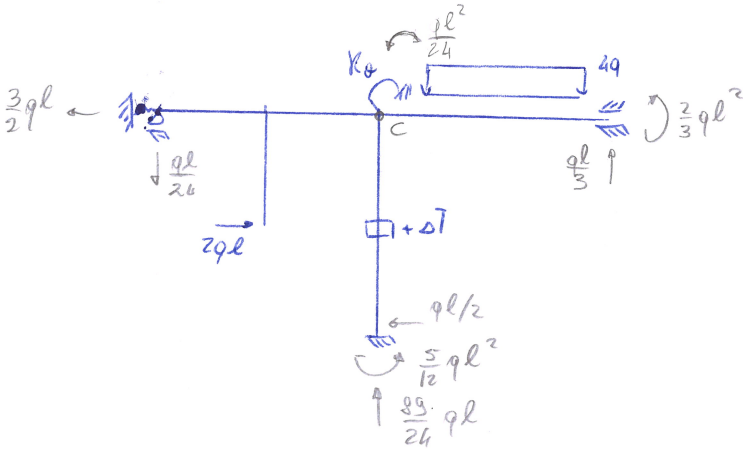


• AZIONI INTERNE RISULTANTI



$$\Sigma M_{(C)} = \left(\frac{25}{24} - \frac{1}{12} - 1 + \frac{1}{24}\right) ql^2 = \phi \; \checkmark$$

• EQUILIBRIO GLOBALE



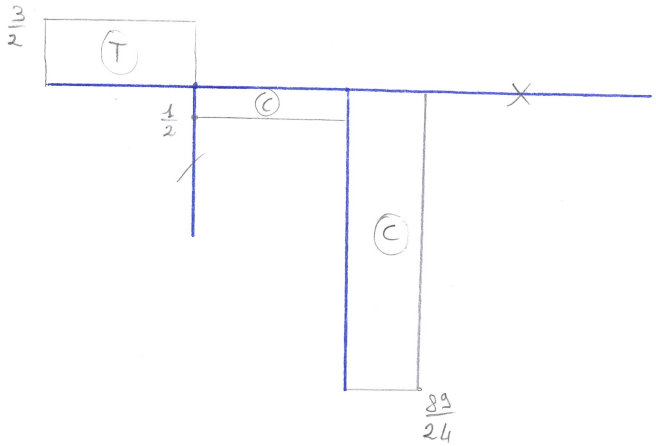
$$+\uparrow \Sigma F_V = \left(\frac{1}{24} + \frac{89}{24} - 4 + \frac{1}{3}\right) ql = \phi$$

$$+\rightarrow \Sigma F_H = \left(-\frac{3}{2} - \frac{1}{2} + 2\right) ql = \phi$$

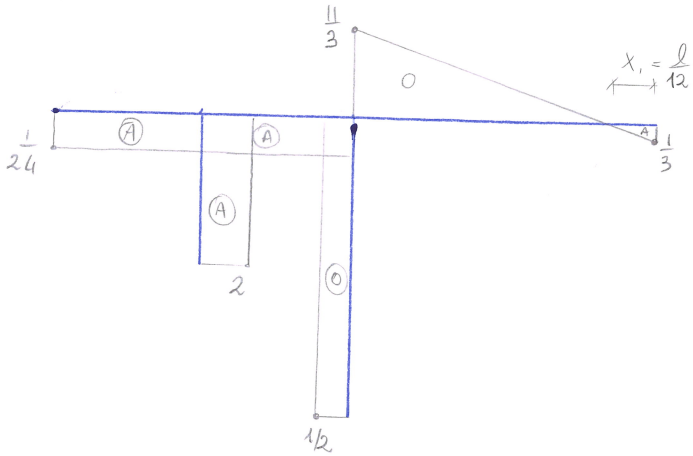
$$+\circlearrowleft \Sigma M_C = \left(-\frac{1}{24} - 2\cdot\frac{1}{2} - \frac{1}{24} + \frac{1}{2} - \frac{5}{12} + \frac{4}{2} - \frac{1}{3} - \frac{2}{3}\right) ql^2 = \phi$$

• DIAGRAMMI AZIONI INTERNE

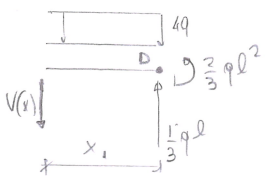
N / ql



V / ql



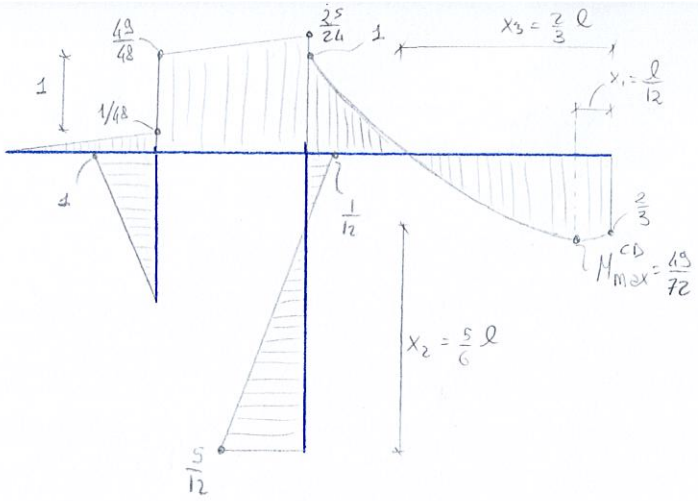
X_1:



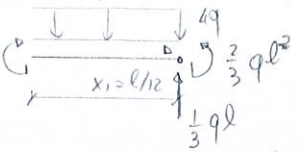
$$V(x) = \frac{1}{3} ql - 4q X_1 = 0$$

$$4 X_1 = \frac{1}{3} l \Rightarrow X_1 = \frac{l}{12}$$

$\frac{M}{qb^2}$



M^{CD}_{max}:



$$M^{CD}_{max} = \frac{1}{3}q\ell\cdot\frac{\ell}{12} + \frac{2}{3}q\ell^2 - \frac{49}{2}\left(\frac{\ell}{12}\right)^2 = \frac{49}{72}q\ell^2$$

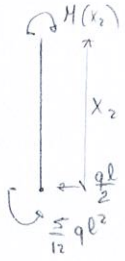
X_3: $M(x) = 0 \quad \frac{1}{3}q\ell x + \frac{2}{3}q\ell^2 - \frac{49}{2}x^2 = 0$

$$6qx^2 - q\ell x - 2q\ell^2 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 6\cdot4\cdot2}}{12}\ell = \frac{1\pm\sqrt{49}}{12}\ell$$

$$x_3 = \frac{1+7}{12}\ell = \frac{2}{3}\ell$$

X_2:



$$M(x_2) = \frac{5}{12}q\ell^2 - \frac{q\ell}{2}x_2 = 0 \qquad x_2 = \frac{5}{6}\ell$$

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- DEFORMATA