

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 1 SETTEMBRE 2021

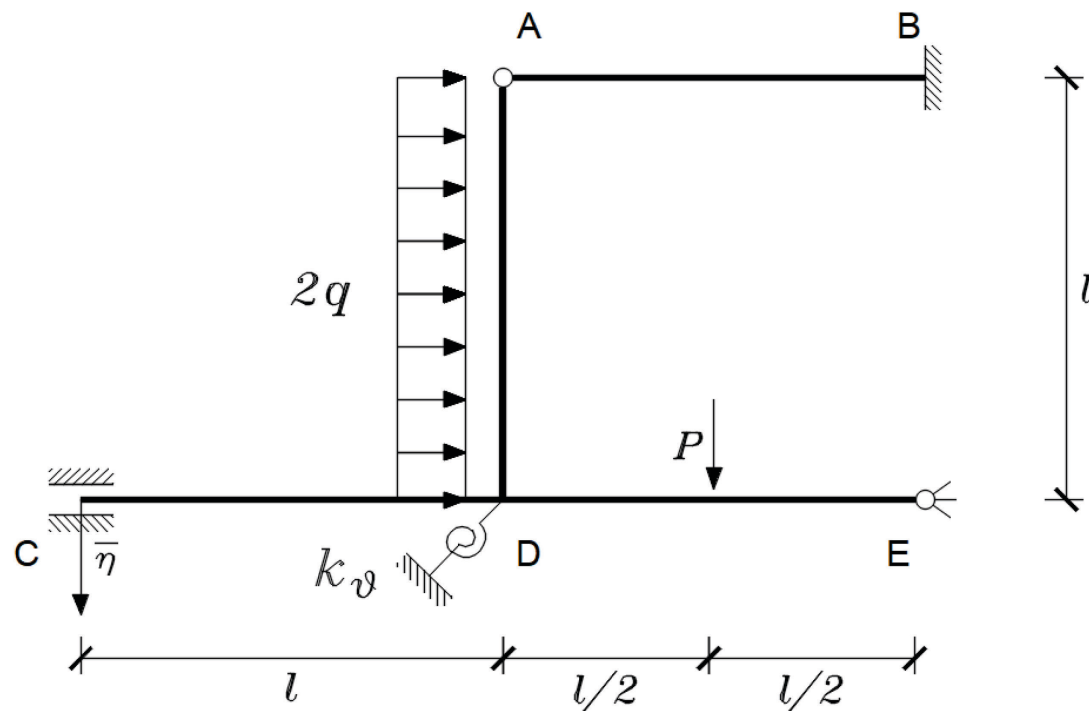
DOCENTI:

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PROF. FAUSTO MINELLI

DURATA: 2 ORE.

Esercizio



$$k_{\theta} = 2 \frac{EJ}{l}$$

$$\bar{\eta} = \frac{1}{8} \frac{ql^4}{EJ}$$

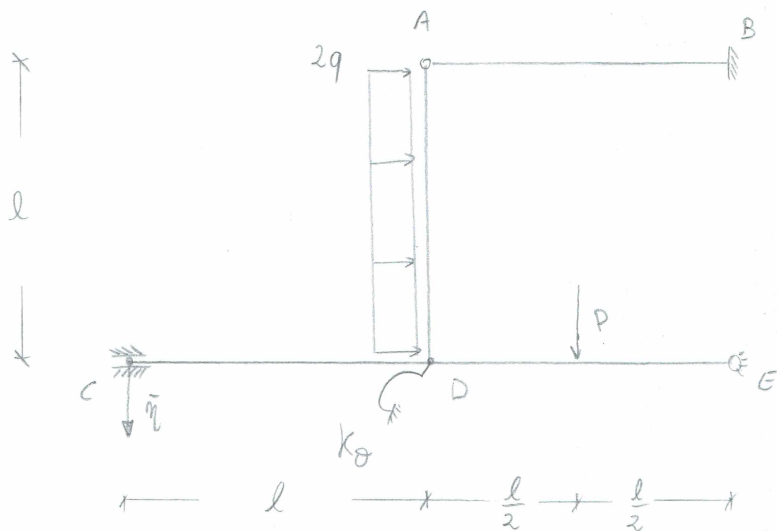
$$P = 4 \cdot q \cdot l$$

Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \rightarrow \infty$, $EJ = \text{costante}$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.



• DATI

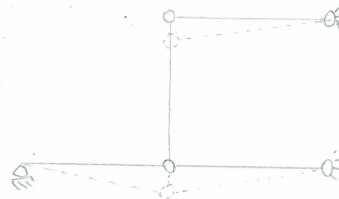
$$k_\theta = 2 \frac{EI}{l}$$

$$\bar{\eta} = \frac{1}{8} \frac{ql^4}{EI^3}$$

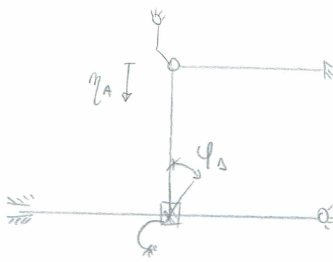
$$P = 4ql$$

• ANALISI CINEMATICA

$$\begin{cases} 6 q \cdot dl \\ 10 q \cdot dl \end{cases} \Rightarrow \Delta \text{ 4 IPERVINCOLATA}$$

TELAIO A NODI
SPOSTABILI

• SCELTA INCOGNITE



• SISTEMA RISOLVENTE

$$\begin{cases} m_{\Delta\Delta} \varphi_\Delta + m_{\Delta\eta} \eta_A + m_{\Delta\phi} = \phi \\ h_{\eta\Delta} \varphi_\Delta + h_{\eta\eta} \eta_A + h_{\eta\phi} = \phi \end{cases}$$

• CONVENZIONI

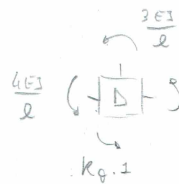
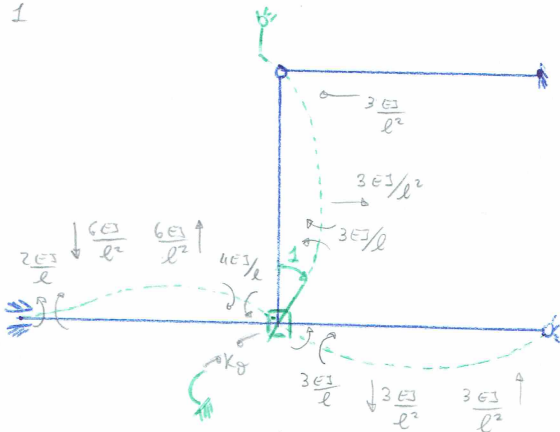
$$\boxed{+} \Rightarrow m$$

$$\boxed{+} \Rightarrow \varphi$$

$$\rightarrow \boxed{+} \Rightarrow h$$

• RISOLUZIONE

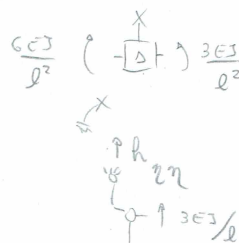
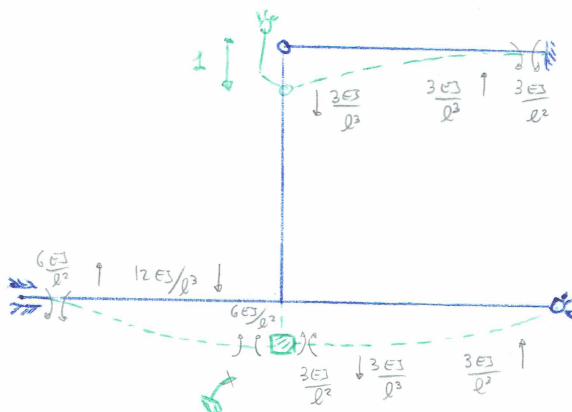
$$\varphi_\Delta = 1$$



$$\frac{4EI}{l} \left(\boxed{-\Delta} \right) \frac{3EI}{l} \quad m_{\Delta\Delta} = \frac{10EI}{l} + k_\theta = \frac{10EI}{l} + \frac{2EI}{l} = \frac{12EI}{l}$$

$$h_{\eta\Delta} = \frac{3EI}{l^2}$$

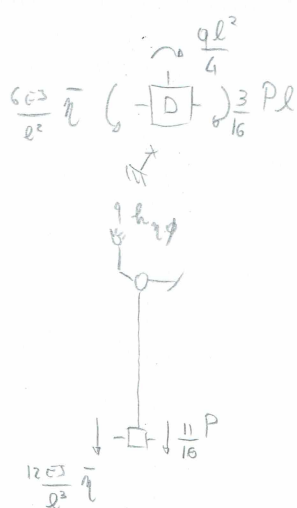
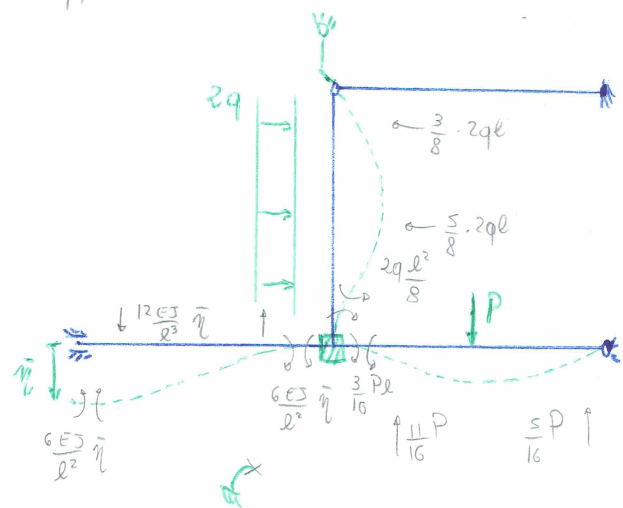
$$\eta_A = 1$$



$$m_{\Delta\eta} = -\frac{3EI}{l^2}$$

$$h_{\eta\eta} = -18EI/l^3$$

• $q \neq 0$



$$m_{\Delta\phi} = -\frac{ql^2}{4} - \frac{3}{16} Pl + \frac{6ES}{l^2} \bar{\eta} =$$

$$= -\frac{ql^2}{4} - \frac{3}{4} ql^2 + \frac{3}{4} ql^2 = -\frac{ql^2}{4}$$

$$h_{\eta\phi} = \frac{11}{16} Pl + \frac{12}{l^3} ES \bar{\eta} = \frac{11}{4} ql + \frac{3}{2} ql = \frac{17}{4} ql$$

SISTEMA RISOLVENTE

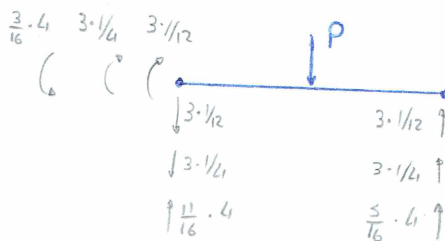
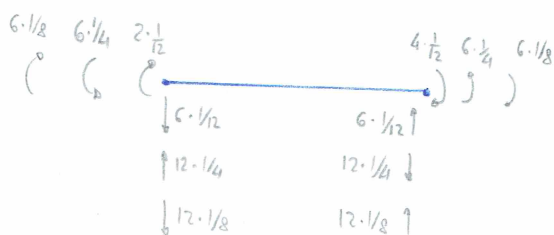
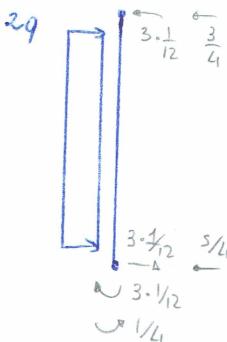
$$\begin{cases} m_{\Delta\Delta} \varphi_{\Delta} + m_{\Delta\eta} \eta_A + m_{\Delta\phi} = \phi \\ h_{\eta\Delta} \varphi_{\Delta} + h_{\eta\eta} \eta_A + h_{\eta\phi} = \phi \end{cases}$$

$$\begin{cases} 12 \frac{ES}{l} \varphi_{\Delta} - 3 \frac{ES}{l^2} \eta_A - \frac{ql^2}{4} = \phi & (1) \\ 3 \frac{ES}{l^2} \varphi_{\Delta} - 18 \frac{ES}{l^3} \eta_A + \frac{17}{4} ql = \phi & (2) \end{cases}$$

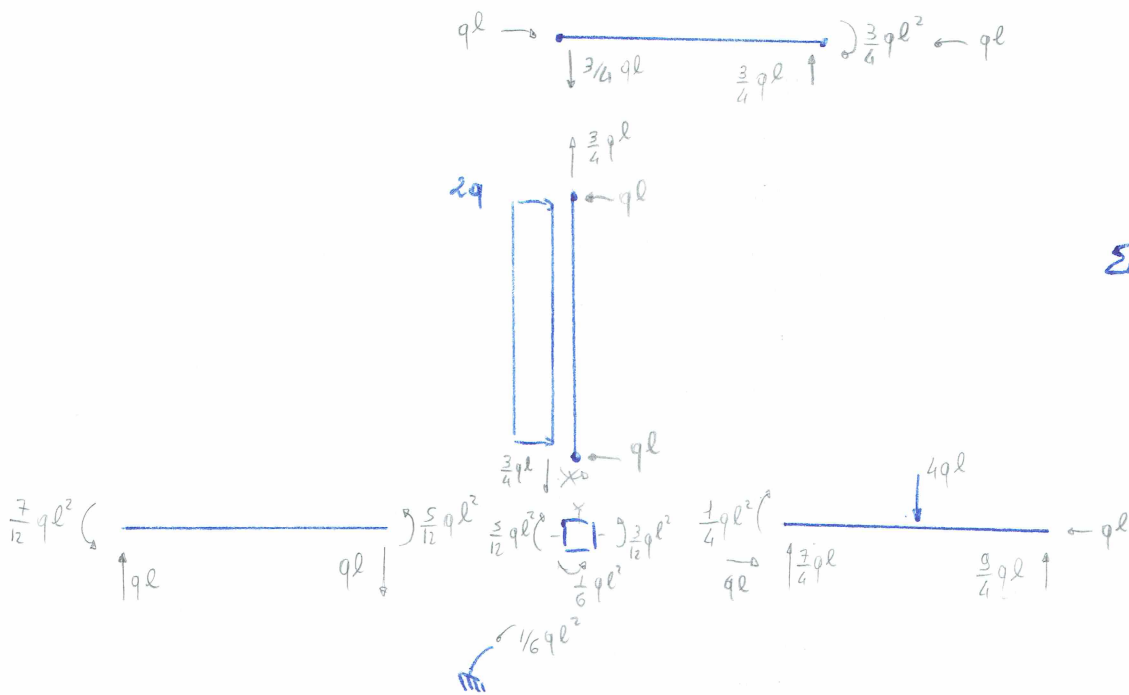
$$(1) \cdot 4l \cdot (2) \Rightarrow 89 \frac{ES}{l^2} \eta_A - \frac{89}{4} ql^2 = \phi \Rightarrow \eta_A = \frac{1}{4} \frac{ql^4}{ES}$$

$$\eta_A \Rightarrow (2) \Rightarrow 12 \frac{ES}{l} \varphi_{\Delta} - \frac{3}{4} ql^2 - \frac{1}{4} ql^2 = \phi \Rightarrow \varphi_{\Delta} = \frac{1}{12} \frac{ql^3}{ES}$$

AZIONI INTERNE

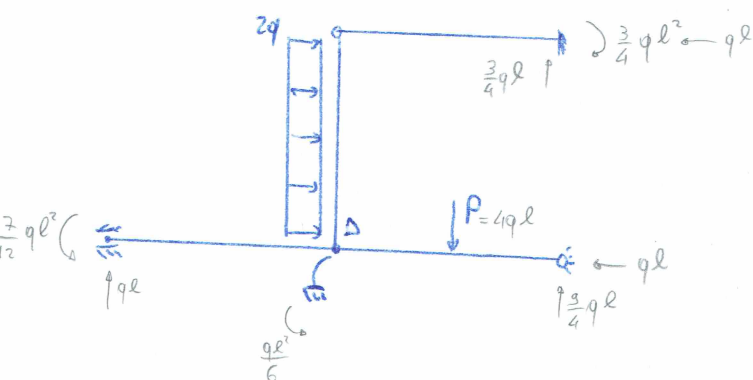


AZIONI INTERNE RISULTANTI



$$\sum H_{(2)} = \left(\frac{3}{12} + \frac{1}{6} - \frac{5}{12} \right) ql^2 = 0$$

EQUILIBRIO GLOBALE

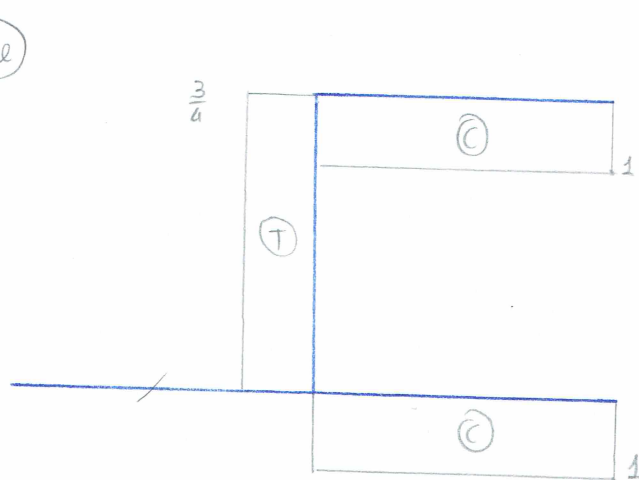


$$\uparrow \sum F_v = ql + \frac{9}{4} ql + \frac{3}{4} ql - 4ql = 0$$

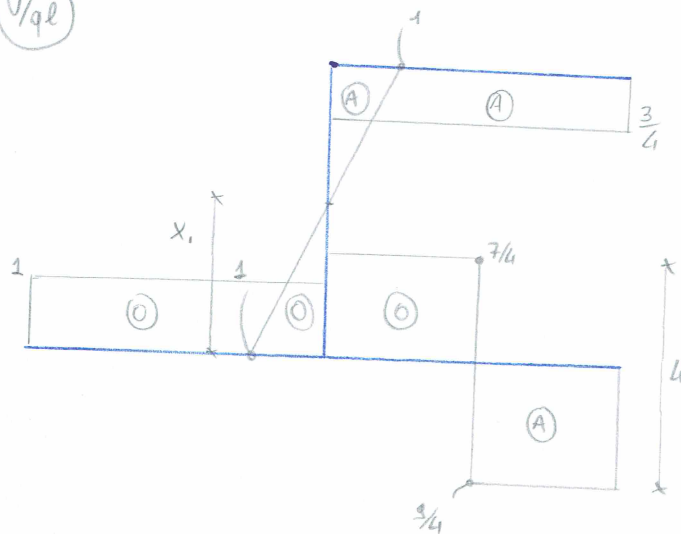
$$\rightarrow \sum F_h = 2ql - ql - ql = 0$$

$$\begin{aligned} \sum H_0 &= \frac{7}{12} ql^2 - ql^2 + \frac{ql^2}{6} - \frac{2ql^2}{2} - \frac{4ql^2}{2} + \frac{3}{4} ql^2 - \frac{3}{4} ql^2 + \\ &+ ql^2 + \frac{9}{4} ql^2 = 0 \end{aligned}$$

AZIONI INTERNE

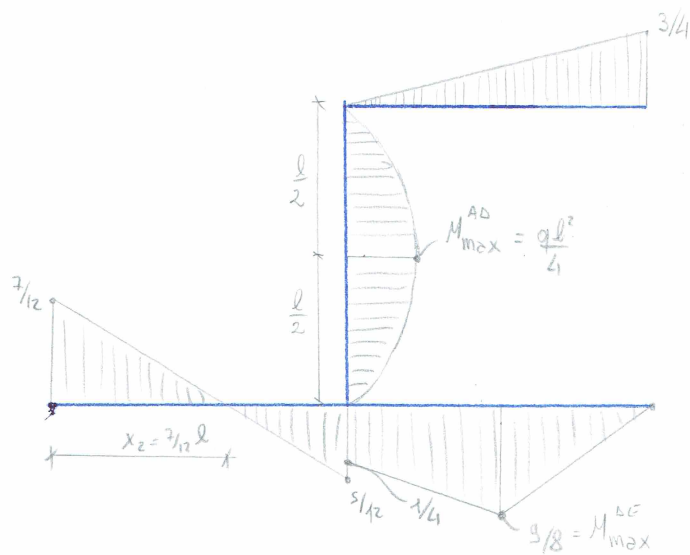


(V/ql)



$$x_1 = l/2$$

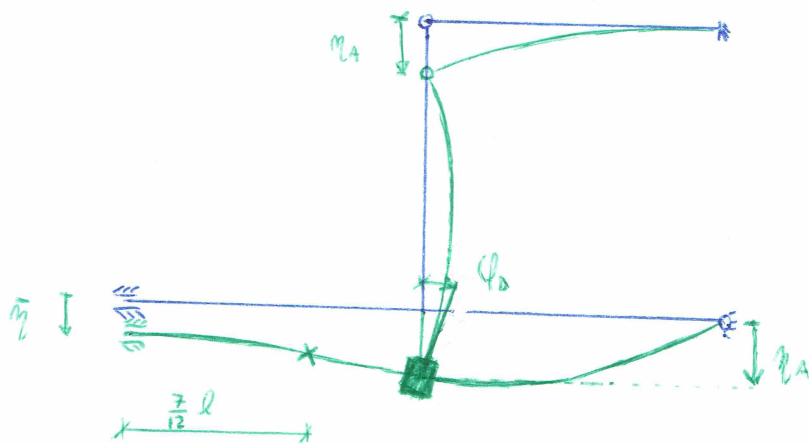
$$\frac{M}{ql^2}$$



$$M_{max}^{AD} = 2q \frac{l^2}{8} = \frac{ql^2}{4}$$

$$x_2 = \frac{\frac{7}{12}}{\frac{7}{12} + \frac{5}{12}} = \frac{7}{12} l$$

• DEFORMATA QUALITATIVA



$$\bar{\eta} = \frac{1}{8} \frac{ql^4}{EI}$$

$$\eta_A = \frac{1}{4} \frac{ql^4}{EI}$$

$$\phi_D = \frac{1}{12} \frac{ql^3}{EI}$$