TECNICA DELLE COSTRUZIONI
TEMA ESAME DEL 17 GENNAIO 2022

DOCENTI:
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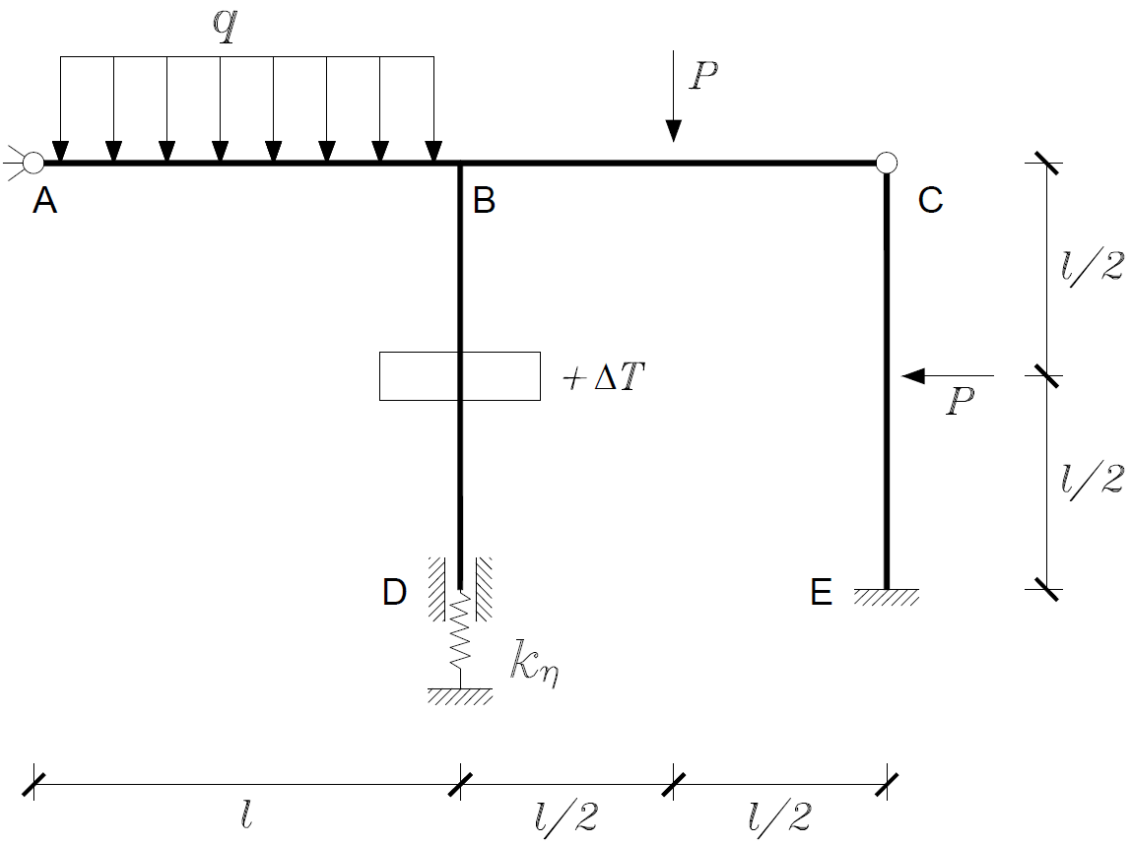
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**

Esercizio



$$k_\eta = 9\frac{EJ}{l^3} \qquad \alpha\Delta T = \frac{ql^3}{EJ} \qquad P = 4 \cdot q \cdot l$$

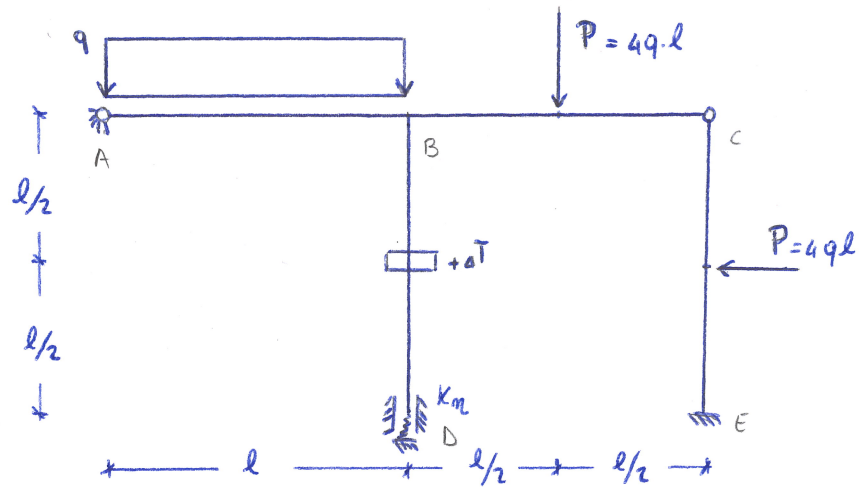
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TEMA ESAME - TELAIO 17/01/2022



DATI:

- $\alpha\Delta T = \frac{q\ell^3}{EJ}$
- $K_\eta = 9\,EJ/\ell^3$

GRADI DI LIBERTÀ → 3+3=6

GRADI DI VINCOLO → 2+2+3+2+1 = 10

STRUTTURA 4 VOLTE IPERSTATICA

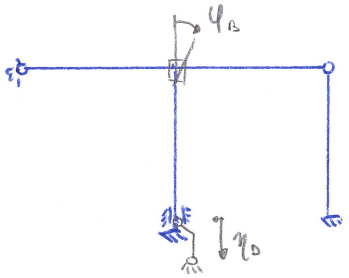
- LABILITÀ

SPOSTAMENTO VERTICALE CONSENTITO → STRUTTURA NODI SPOSTABILI

INCOGNITE: φ_B, η_D

- CONVENZIONI

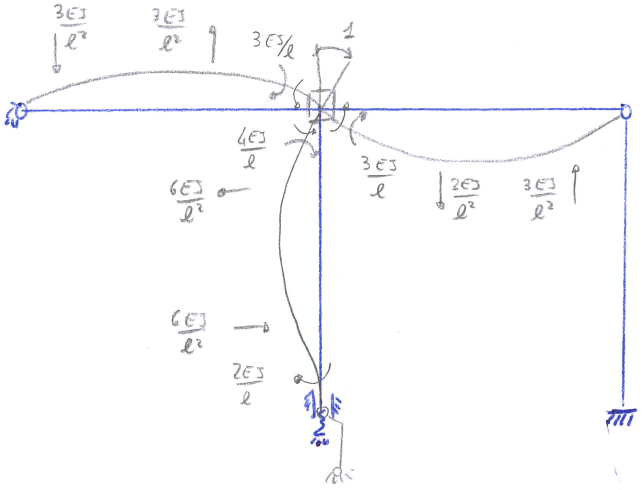
- SISTEMA RISOLVENTE

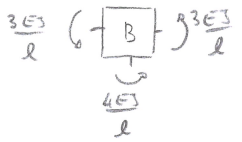


$$\begin{cases} m_{BB}\,\varphi_B + m_{B\eta}\,\eta_D + m_{Bq} = \phi \\ h_{\eta B}\,\varphi_B + h_{\eta\eta}\,\eta_D + h_{\eta q} = \phi \end{cases}$$

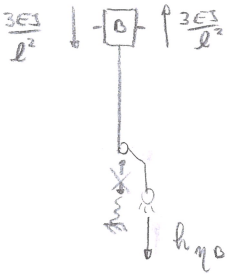
- RISOLUZIONE

- $\varphi_B = 1$



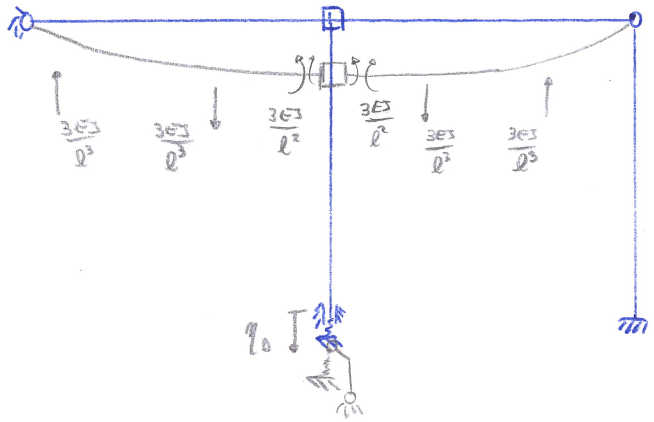


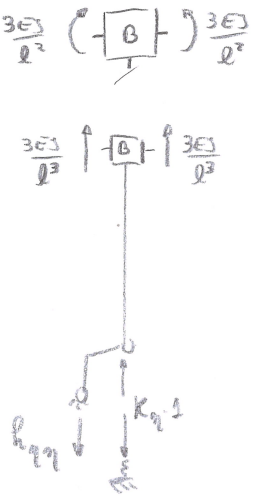
$m_{BB} = 10\,EJ/\ell$



$h_{\eta B} = \phi$

• $\eta_0 = 1$



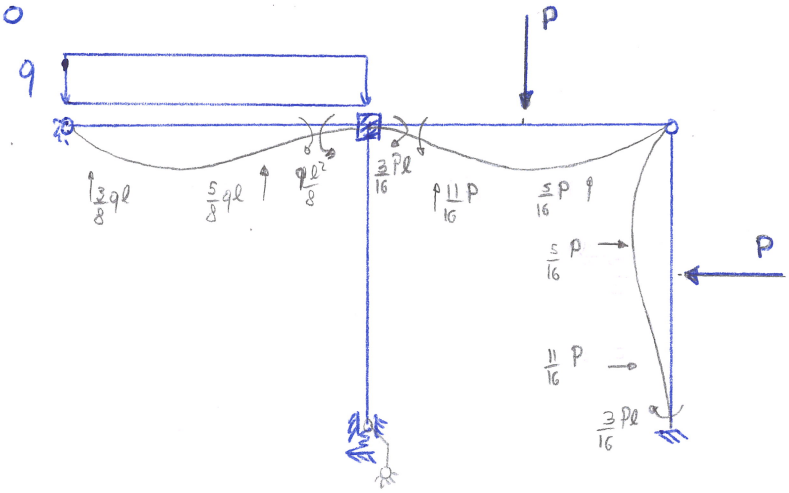


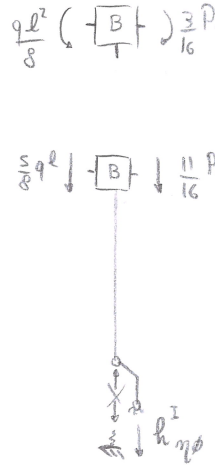
$m_{B\eta} = \emptyset$

$r_{\eta\eta} = 6\frac{EJ}{l^3} + K_\eta =$

$= 6\frac{EJ}{l^3} + 9\frac{EJ}{l^3} = 15\frac{EJ}{l^3}$

• $q \neq 0$





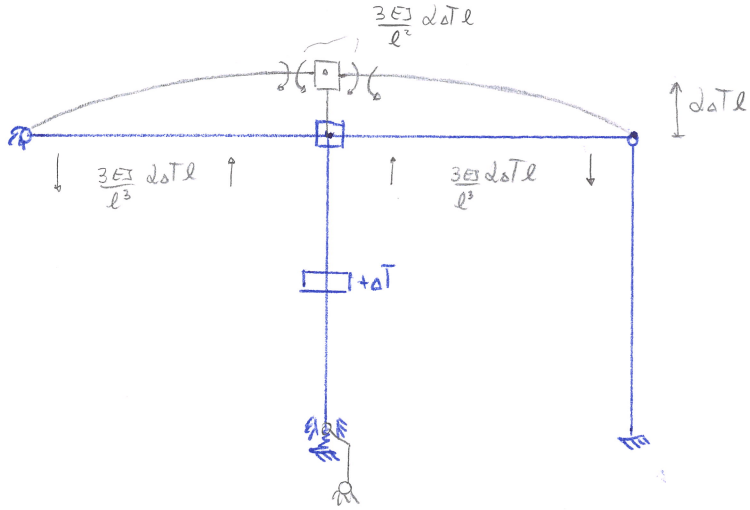
$m^I_{B\emptyset} = \frac{ql^2}{8} - \frac{3}{16}Pl =$

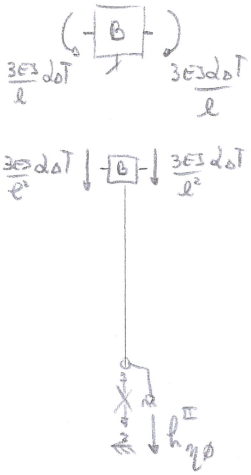
$= \frac{ql^2}{8} - \frac{3}{4}ql^2 = -\frac{5}{8}ql^2$

$r^I_{\eta\emptyset} = -\frac{5}{8}ql - \frac{11}{16}Pl =$

$= -\frac{5}{8}ql - \frac{11}{4}ql = -\frac{27}{8}ql$

• $\Delta T \neq 0$





$m^{II}_{B\emptyset} = \emptyset$

$r^{II}_{\eta\emptyset} = -\frac{6EJ}{l^2}\alpha\Delta T =$

$= -6ql$

- **SISTEMA RISOLVENTE**

$$\begin{cases} m_{BB}\,\varphi_B + m_{B\eta}\,\eta_D + m_{B\phi}^{I} + m_{B\phi}^{II} = \phi \\ h_{\eta B}\,\varphi_B + h_{\eta\eta}\,\eta_D + h_{\eta\phi}^{I} + h_{\eta\phi}^{II} = \phi \end{cases}$$

$$\begin{cases} 10\,\frac{EJ}{\ell}\,\varphi_B + \phi\cdot\eta_D - \frac{5}{8}\,q\ell^2 = \phi \\ \phi\cdot\varphi_B + 15\,\frac{EJ}{\ell^3}\,\eta_D - \frac{27}{8}\,q\ell - 6q\ell = \phi \end{cases}$$

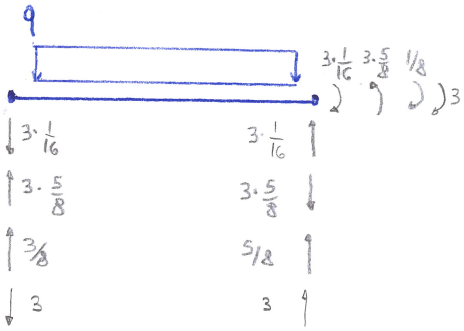
$$\begin{cases} 10\,\frac{EJ}{\ell}\,\varphi_B - \frac{5}{8}\,q\ell^2 = \phi \\ 15\,\frac{EJ}{\ell^3}\,\eta_D - \frac{75}{8}\,q\ell = \phi \end{cases} \Rightarrow \varphi_B = \frac{5}{8}\cdot\frac{1}{10}\,q\ell^3/EJ = \frac{1}{16}\,q\ell^3/EJ$$

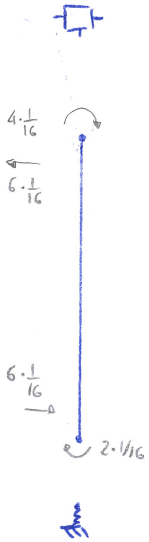
$$\Rightarrow \eta_D = \frac{75}{8}\cdot\frac{1}{15}\,q\ell^4/EJ = \frac{5}{8}\,q\ell^4/EJ$$

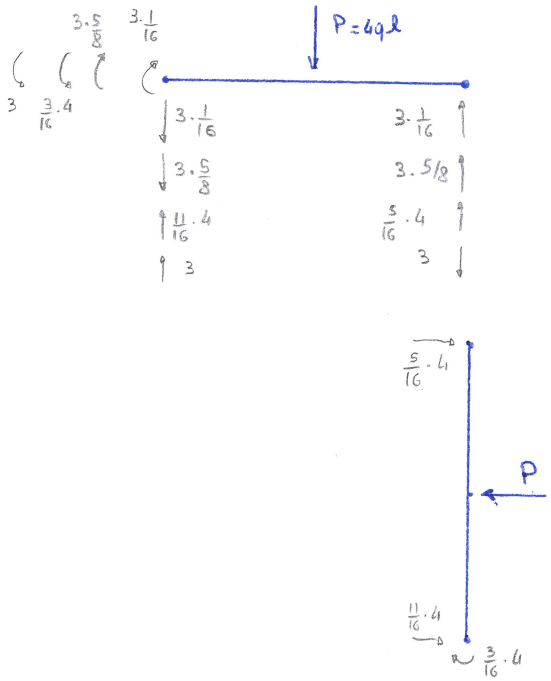
$$\varphi_B = \frac{1}{16}\,\frac{q\ell^3}{EJ}$$

$$\eta_D = \frac{5}{8}\,\frac{q\ell^4}{EJ}$$

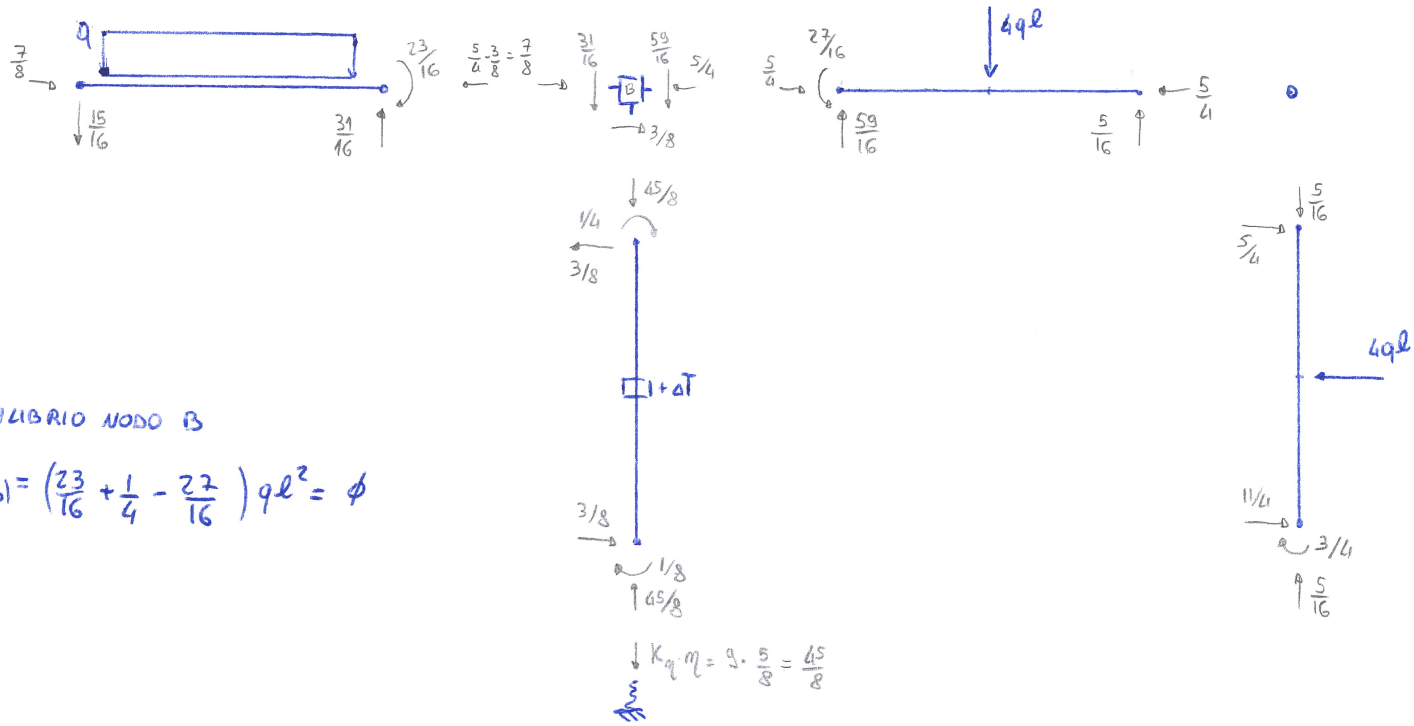
- **AZIONI INTERNE**







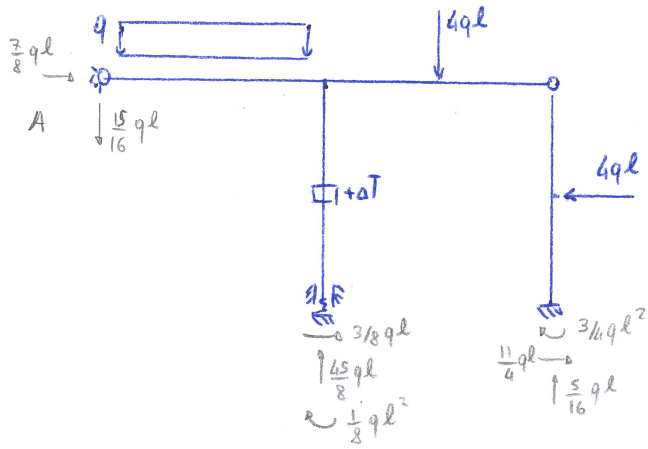
• AZIONI INTERNE RISULTANTI



• EQUILIBRIO NODO B

$$\Sigma M_{(B)} = \left(\frac{23}{16} + \frac{1}{4} - \frac{27}{16}\right) q\ell^2 = \phi$$

• EQUILIBRI GLOBALI



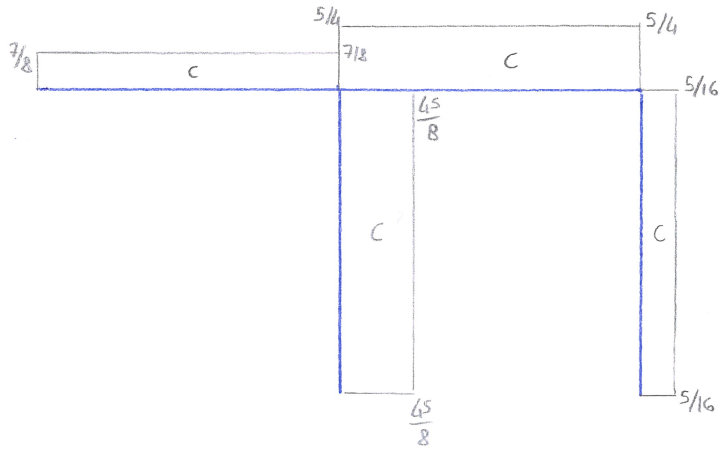
$$+\uparrow \Sigma F_v = \left(-\frac{15}{16} + \frac{45}{8} + \frac{5}{16} - 1 - 4\right) q\ell = \phi$$

$$+\rightarrow \Sigma F_H = \left(\frac{7}{8} + \frac{3}{8} + \frac{11}{4} - 4\right) q\ell = \phi$$

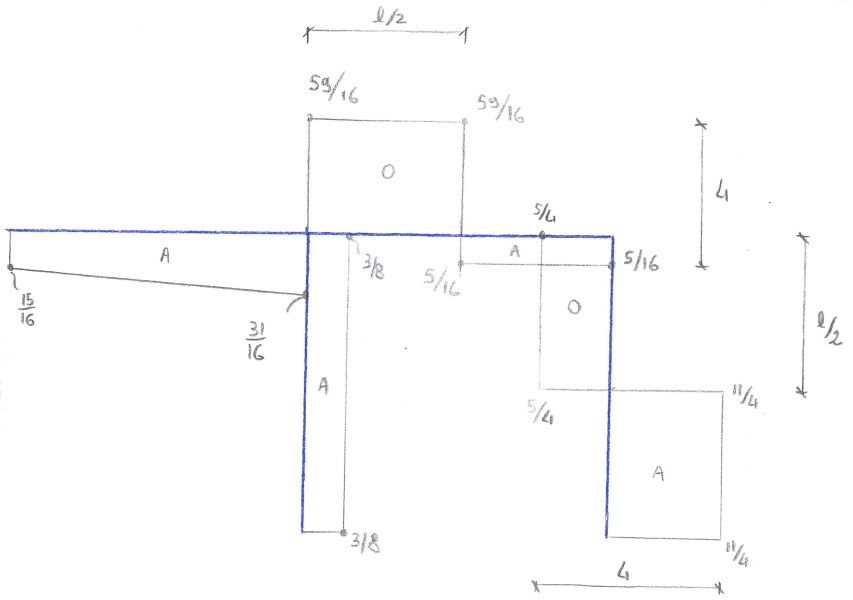
$$\left(+\circlearrowleft_A\right) \Sigma M_{(A)} = \frac{3}{8} q\ell \cdot \ell + \frac{45}{8} q\ell \cdot \ell - \frac{1}{8} q\ell^2 + \frac{11}{4} q\ell \cdot \ell + \frac{5}{16} q\ell 2\ell + - \frac{3}{4} q\ell^2 - q \cdot \ell \cdot \frac{\ell}{2} - 4q\ell \frac{3}{2}\ell - 4q\ell \cdot \frac{\ell}{2} = \phi$$

• DIAGRAMMI AZIONI INTERNE

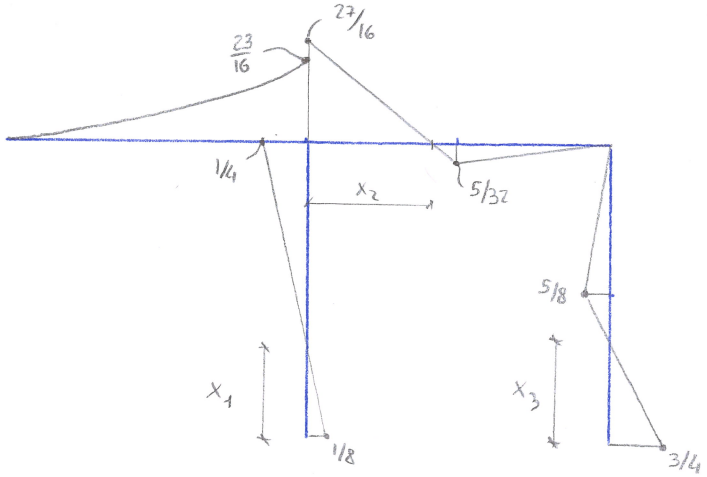
N/qℓ



V/ql



H/ql^2



- X_2: $X_2 : \frac{27}{16} = \frac{l}{2} : \left(\frac{27}{16} + \frac{5}{32}\right)$

  $\rightarrow X_2 = \frac{l}{2} \cdot \frac{27}{16} \cdot \frac{1}{\left(\frac{27}{16} + \frac{5}{32}\right)} = \frac{27}{59} l$

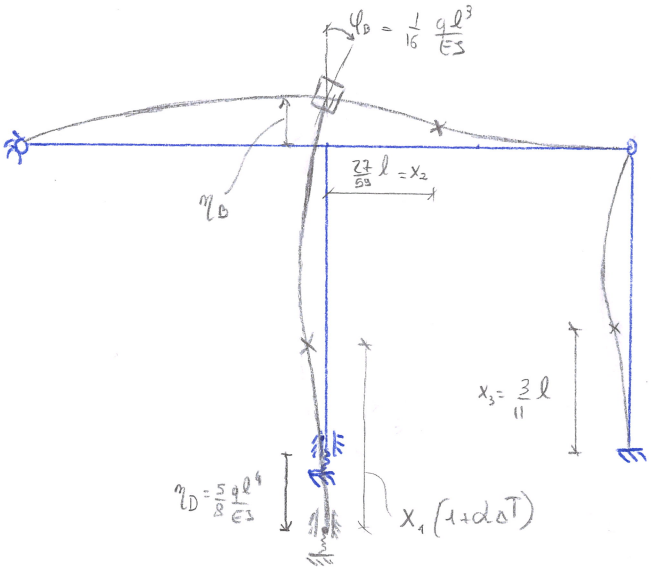
- X_1: $X_1 : \frac{1}{8} = l : \left(\frac{1}{8} + \frac{1}{4}\right)$

  $\rightarrow X_1 = l \cdot \frac{1}{8} \cdot \frac{1}{\left(\frac{1}{8} + \frac{1}{4}\right)} = \frac{l}{3}$

- X_3: $X_3 : \frac{3}{4} = \frac{l}{2} : \left(\frac{3}{4} + \frac{5}{8}\right)$

  $\rightarrow X_3 = \frac{l}{2} \cdot \frac{3}{4} \cdot \frac{1}{\left(\frac{3}{4} + \frac{5}{8}\right)} = \frac{3}{11} l$

- DEFORMATA QUALITATIVA



$\eta_D = \alpha \Delta T \cdot l \cdot \eta_D =$

$= \frac{ql^4}{EJ} \cdot \frac{5}{8} \frac{ql^4}{EJ} = \frac{3}{8} \frac{ql^4}{EJ}$