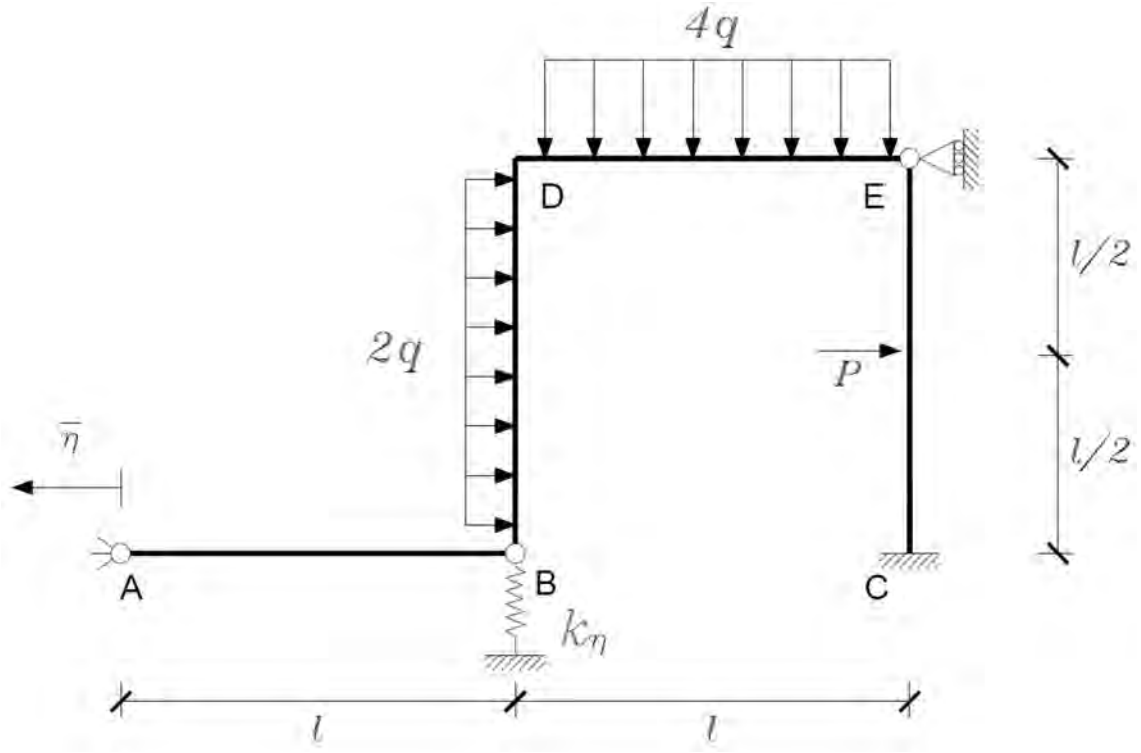
TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 13 GIUGNO 2022

DOCENTI:
PROF. GIOVANNI A. PLIZZARI
PROF. FAUSTO MINELLI

DURATA: **2 ORE.**

Esercizio



$$k_\eta = 6\frac{EJ}{l^3} \qquad \bar{\eta} = \frac{3}{4}\frac{ql^4}{EJ} \qquad P = 2\,q \cdot l$$

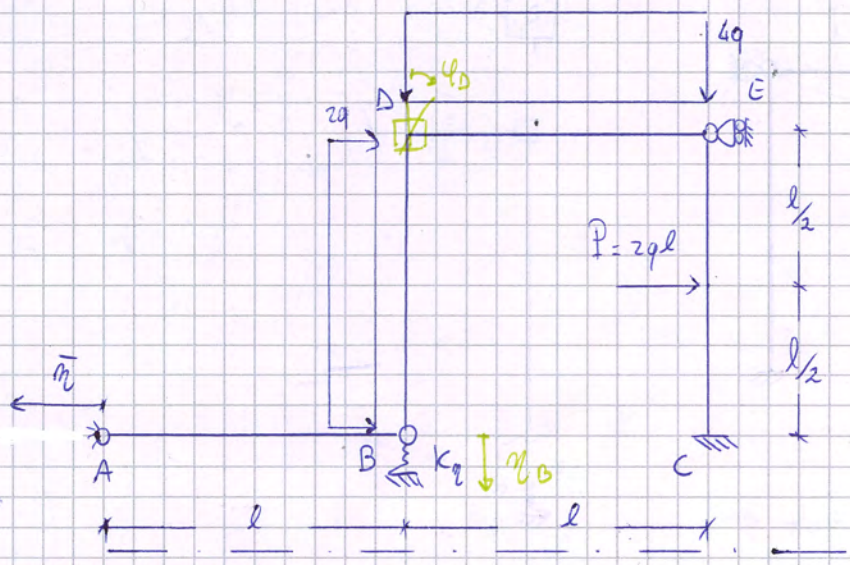
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TEMA ESAME - TELAIO 13/06/22



- DATI:

$K_\eta = 6\,EJ/\ell^3$

$\bar{\eta} = \frac{3}{4}\,q\ell^4/EJ$

GRADI DI LIBERTÀ → 9

GRADI DI VINCOLO → 2+1+3+1+2+2 = 11

STRUTTURA 2 VOLTE IPERVINCOLATA

- ANALISI CINEMATICA

SPOSTAMENTO VERTICALE CONSENTITO

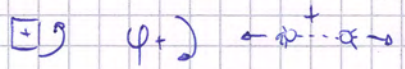
⇒ STRUTTURA A NODI SPOSTABILI

INCOGNITE: φ_D, η_B

- SISTEMA RISOLVENTE:

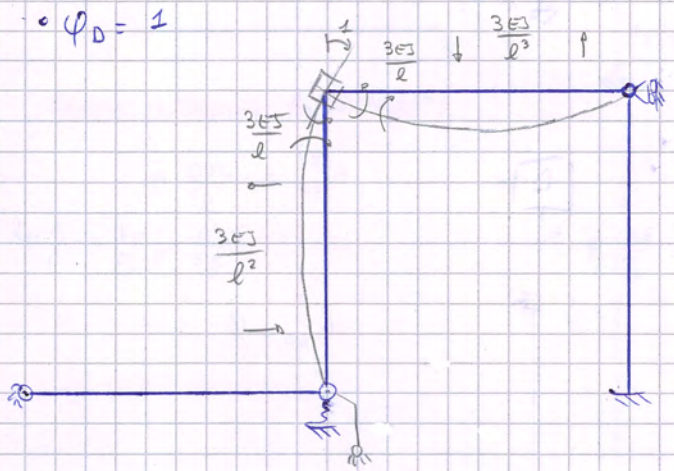
$$\begin{cases} m_{DD}\,\varphi_D + m_{DB}\,\eta_B + m_{D\phi} = \phi \\ h_{BD}\,\varphi_D + h_{BB}\,\eta_B + h_{B\phi} = \phi \end{cases}$$

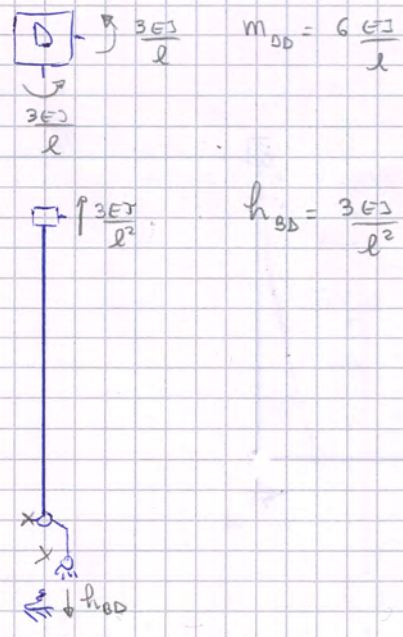
- CONVENZIONI:



- RISOLUZIONE

- $\varphi_D = 1$

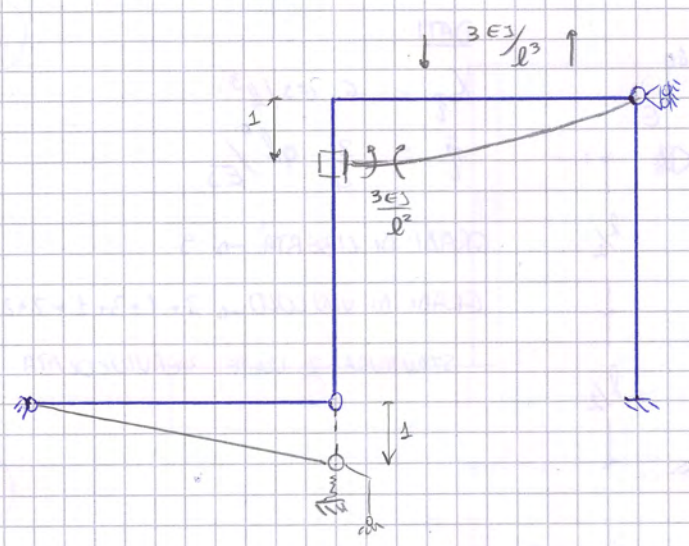


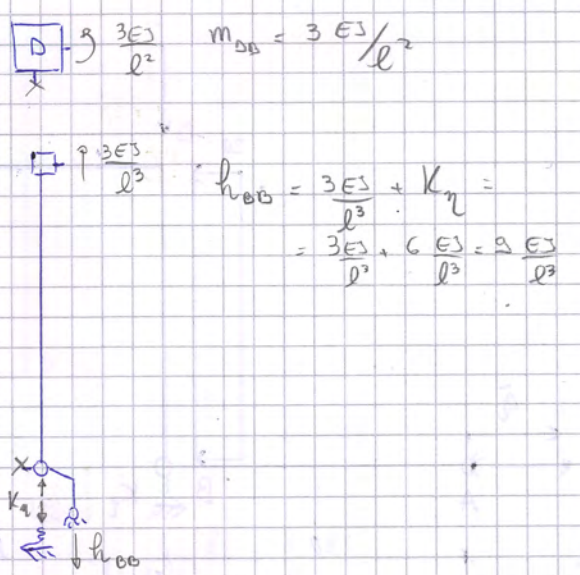


$m_{DD} = 6\frac{EJ}{\ell}$

$h_{BD} = \frac{3EJ}{\ell^2}$

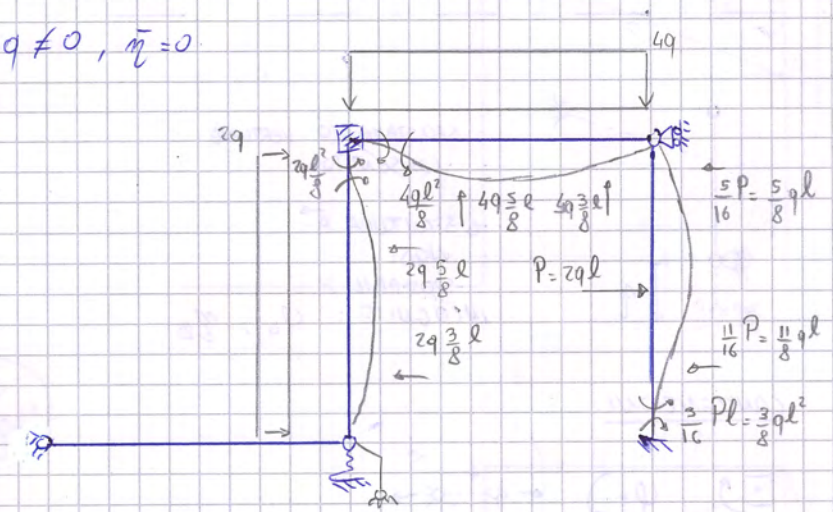
• $\eta_B = 1$

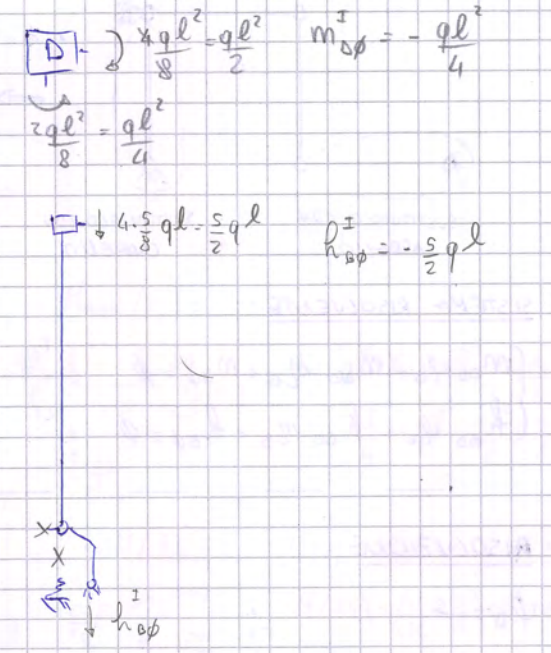




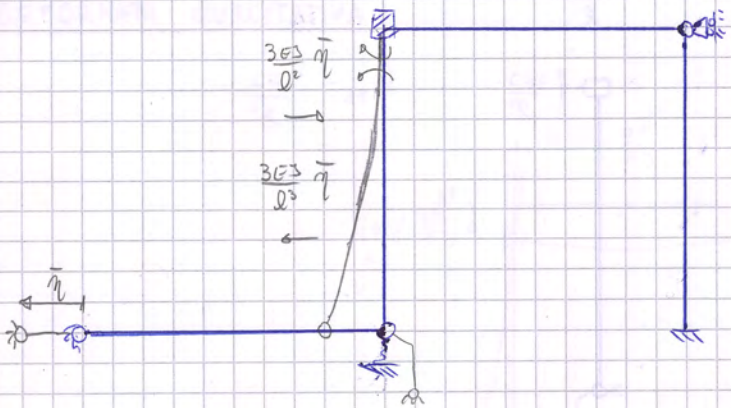
$$h_{BB} = \frac{3EJ}{l^3} + K_\eta = \frac{3EJ}{l^3} + \frac{6EJ}{l^3} = \frac{9EJ}{l^3}$$

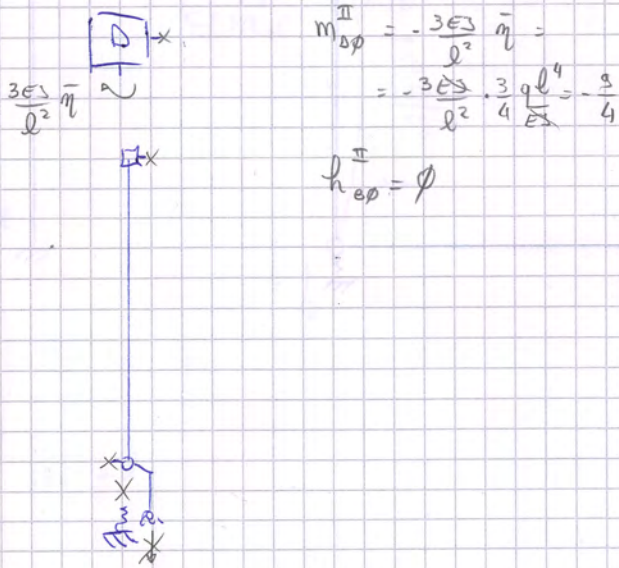
• $q \neq 0, \ \bar{\eta} = 0$





• $\bar{\eta} \neq 0, \ q = 0$





- SISTEMA RISOLVENTE

$$\begin{cases} m_{DD}\varphi_D + m_{D\Theta}\eta_\Theta + m_{D\phi} = \phi \\ h_{\Theta D}\varphi_D + h_{\Theta\Theta}\eta_\Theta + h_{\Theta\phi} = \phi \end{cases} \Rightarrow \begin{cases} 1) \; 6\frac{EJ}{\ell}\varphi_D + 3\frac{EJ}{\ell^2}\eta_\Theta + \left(-\frac{q\ell^2}{4} - \frac{9}{4}q\ell^2\right) = \phi \\ 2) \; 3\frac{EJ}{\ell^2}\varphi_D + 9\frac{EJ}{\ell^3}\eta_\Theta + \left(-\frac{5}{2}q\ell\right) = \phi \end{cases}$$

$$\begin{cases} 1) \; 6\frac{EJ}{\ell}\varphi_D + 3\frac{EJ}{\ell^2}\eta_\Theta - \frac{5}{2}q\ell^2 = \phi \\ 2) \; 3\frac{EJ}{\ell^2}\varphi_D + 9\frac{EJ}{\ell^3}\eta_\Theta - \frac{5}{2}q\ell = \phi \end{cases}$$

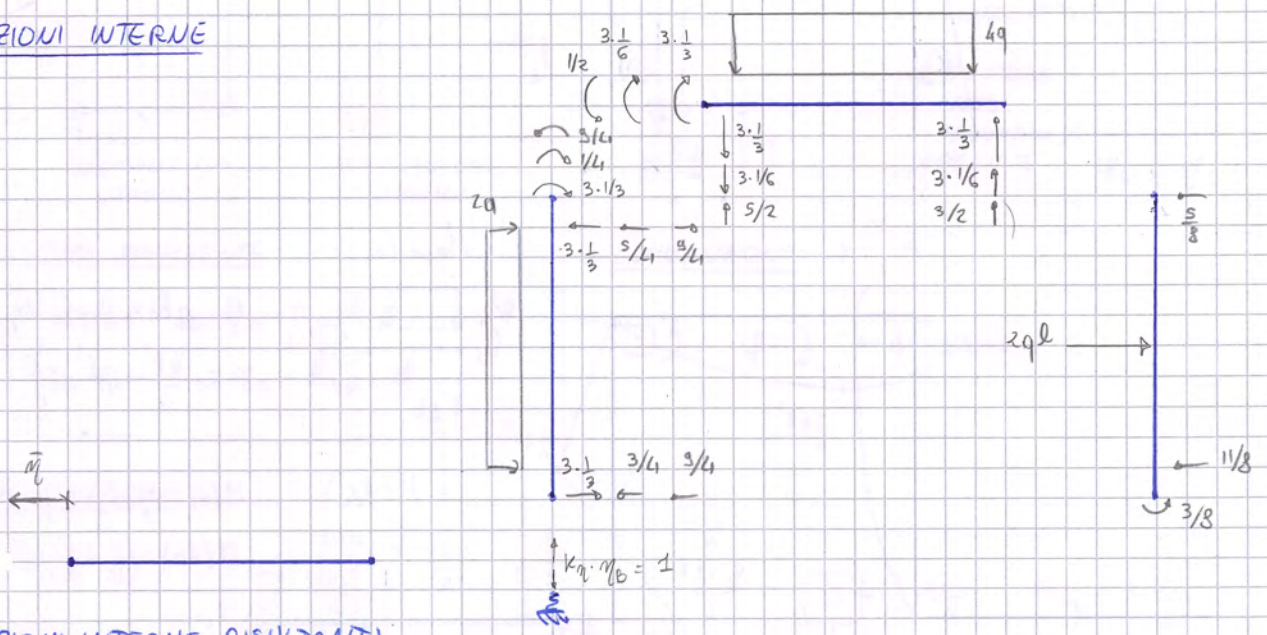
$1) - 2)\cdot 2\ell \Rightarrow (3-18)\frac{EJ}{\ell^2}\eta_\Theta + \left(-\frac{5}{2}+5\right)q\ell^2 = \phi \Rightarrow \eta_\Theta = \frac{5}{2}\cdot\frac{1}{15_3}\frac{q\ell^4}{EJ} = \frac{1}{6}\frac{q\ell^4}{EJ}$

$1) \Rightarrow 6\frac{EJ}{\ell}\varphi_D + 3\frac{EJ}{\ell^2}\cdot\frac{1}{8_2}\frac{q\ell^2}{EJ} - \frac{5}{2}q\ell^2 = \phi$

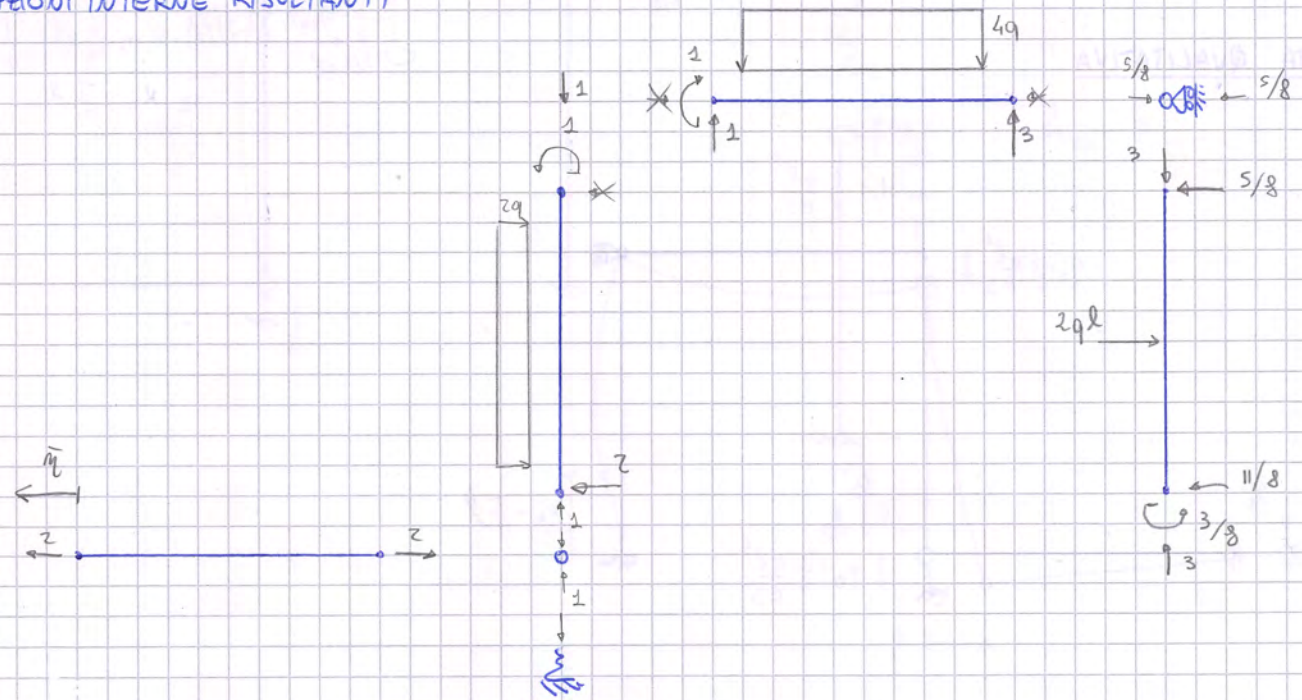
$$6\frac{EJ}{\ell}\varphi_D = 2q\ell^2 \Rightarrow \varphi_D = \frac{1}{3}\frac{q\ell^3}{EJ}$$

$$\boxed{\eta_\Theta = \frac{1}{6}\frac{q\ell^4}{EJ}, \quad \varphi_D = \frac{1}{3}\frac{q\ell^3}{EJ}}$$

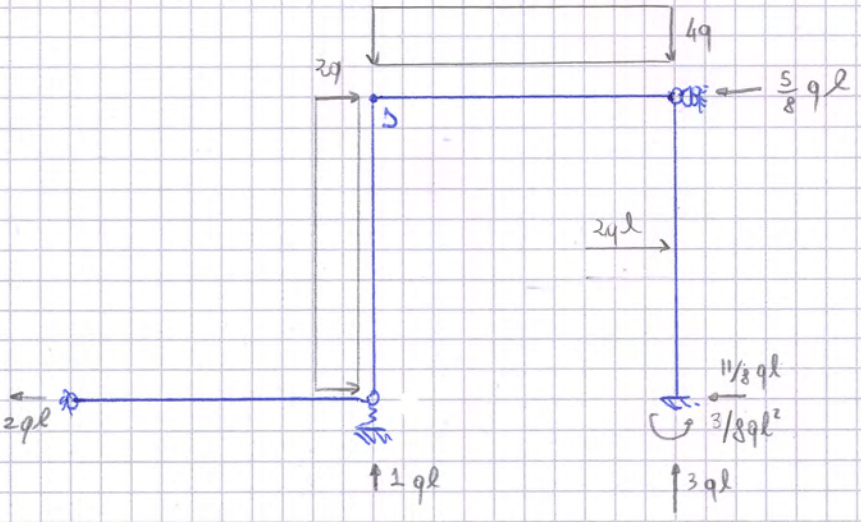
- AZIONI INTERNE



- AZIONI INTERNE RISULTANTI



• EQUILIBRI GLOBALI



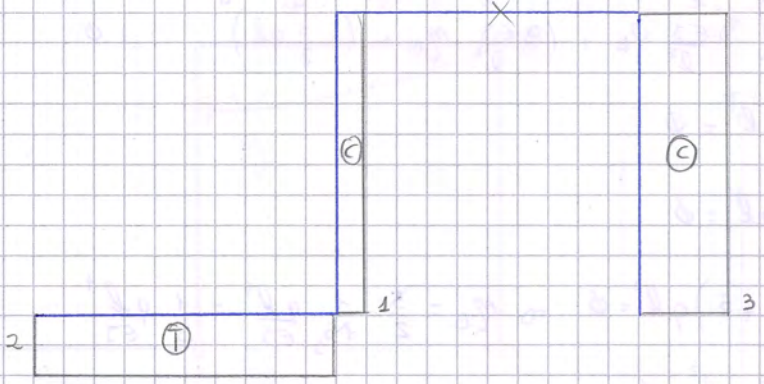
$$\Sigma F_H = -2ql + 2ql + 2ql - \frac{5}{8}ql - \frac{11}{8}ql = 0$$

$$\Sigma F_V = +1ql - 4ql + 3ql = 0$$

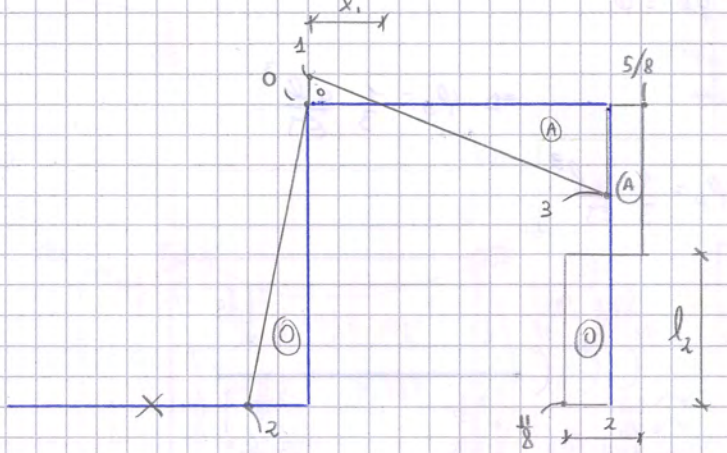
$$+\circlearrowleft \Sigma M(D) = -2ql\,l + 2ql\,\frac{l}{2} + 3ql\,l + \frac{3}{8}ql^2 - \frac{11}{8}ql\,l + 2ql\,\frac{l}{2} - 4ql\,\frac{l}{2} = 0$$

• DIAGRAMMI AZIONI INTERNE

N/ql

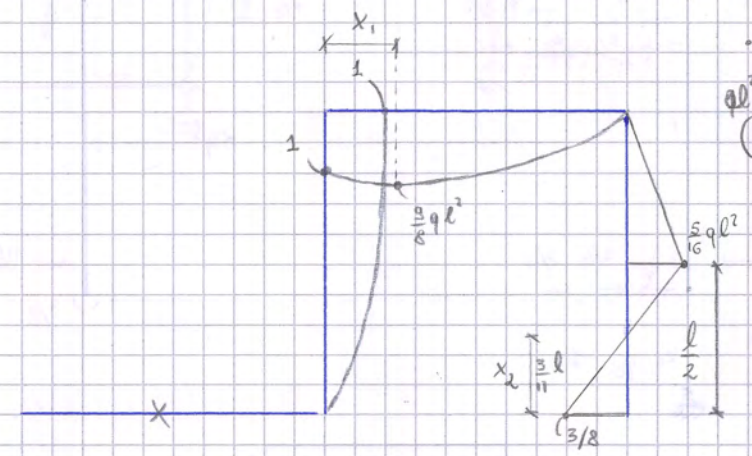


V/ql

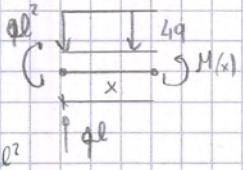


$$X_1 = \frac{1}{1+3}\,l = \frac{1}{4}\,l$$

M/ql²

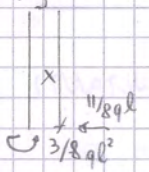


• $M_{max}(x_1)$:



$$M = ql\cdot x_1 + ql^2 - 4q\frac{x_1^2}{2} = \frac{ql^2}{4} + ql^2 - \frac{ql^2}{8} = \frac{9}{8}ql^2$$

• $M(l/2)$:



$$M(x) = \frac{11}{8}ql\,x - \frac{3}{8}ql^2$$

$$M(l/2) = \left(\frac{11}{16} - \frac{3}{8}\right)ql^2 = \frac{5}{16}ql^2$$

X_2:

$$M(x) = 0 \rightarrow \frac{11}{8}ql\,x_2 = \frac{3}{8}ql^2$$

$$\rightarrow X_2 = \frac{3}{11}\,l$$

• DEFORMATA QUALITATIVA