

TECNICA DELLE COSTRUZIONI

TEMA ESAME DEL 04 LUGLIO 2022

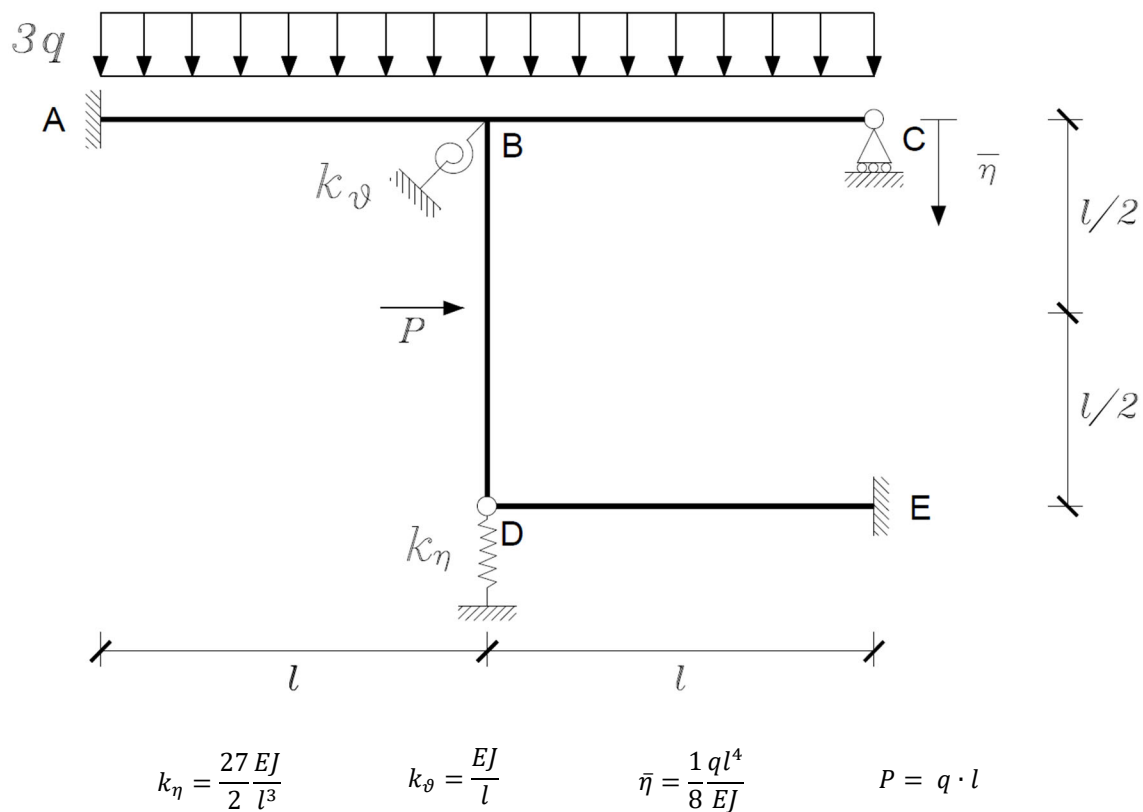
DOCENTI:

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PROF. FAUSTO MINELLI

DURATA: 2 ORE.

Esercizio



Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \rightarrow \infty$, $EJ = \text{costante}$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

• DATI:

$$K_\theta = 1 \frac{EJ}{l}$$

$$K_\eta = \frac{27}{2} \frac{EJ}{l^3}$$

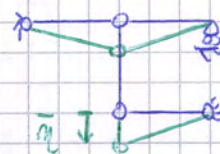
$$\bar{\eta} = \frac{1}{8} q l^4 / EJ$$

GRADI DI LIBERTÀ $\rightarrow 6$ GRADI DI VINCOLO $\rightarrow 3+1+3+2+1+1=11$

STRUTTURA 5 VOLTE IPERVINCOLATA

• ANALISI CINEMATICA

SPOSTAMENTO VERT. CONSENTITO

 $\Rightarrow$ STRUTTURA A NODI SPOSTABILIINCOGNITE φ_B, η_D 

• SISTEMA RISOLVENTE:

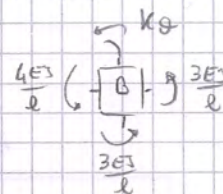
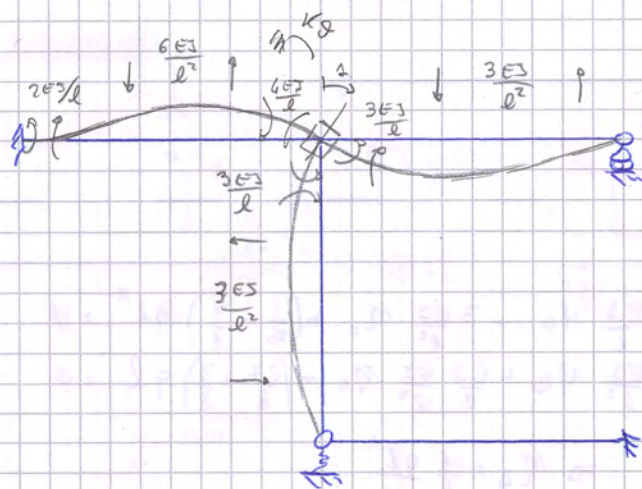
$$\begin{cases} m_{BB} \varphi_B + m_{BD} \eta_D + m_{B\phi} = 0 \\ h_{BB} \varphi_B + h_{BD} \eta_D + h_{B\phi} = 0 \end{cases}$$

• CONVENZIONI

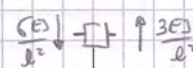
$$[\oplus] \quad \varphi \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$$

• RISOLUZIONE

$$\varphi_B = 1$$

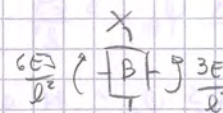
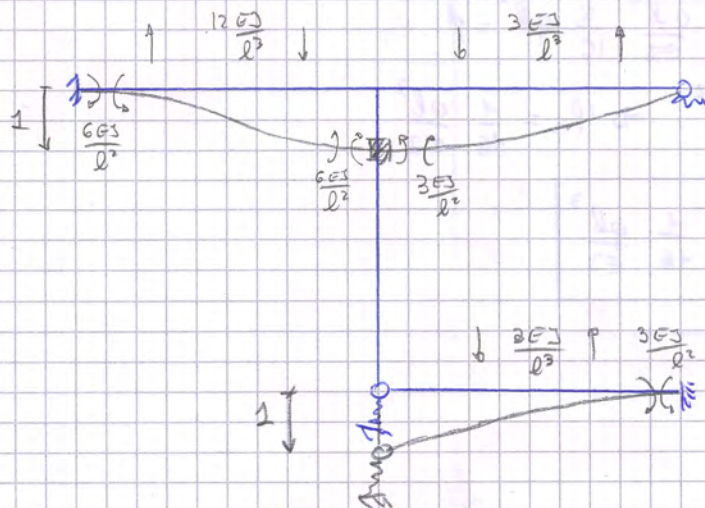


$$m_{BB} = \frac{10 EJ}{l} + K_\theta = \frac{11 EJ}{l}$$



$$h_{BB} = -\frac{3 EJ}{l^2}$$

$$\eta_D = 1$$

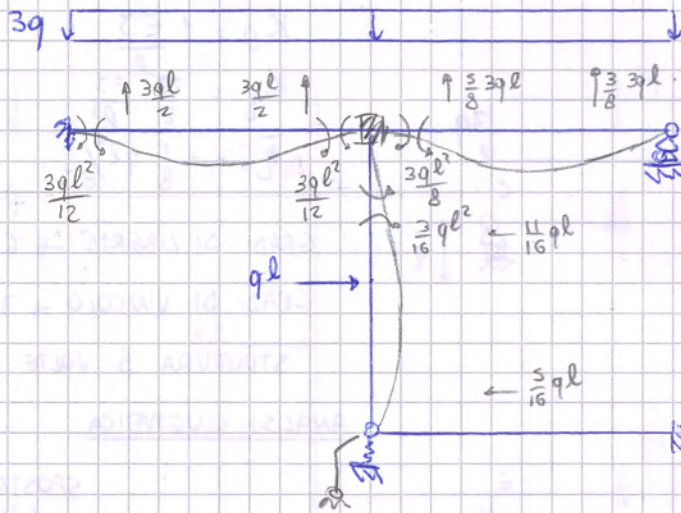


$$m_{BD} = -\frac{3 EJ}{l^2}$$



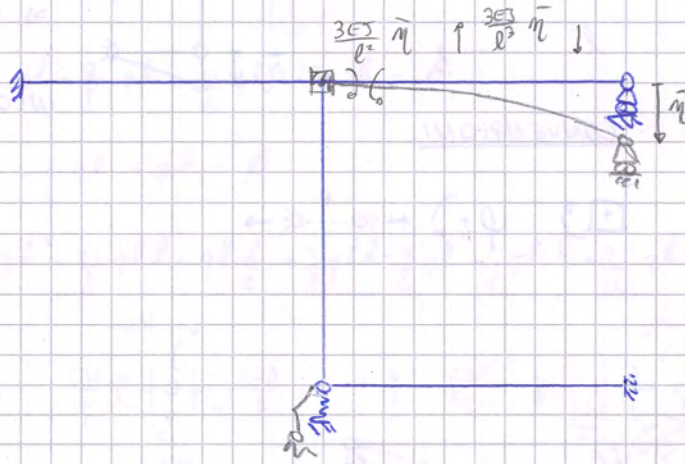
$$h_{DD} = \frac{18 EJ}{l^3} + K_\eta = \frac{63}{2} \frac{EJ}{l^3}$$

• $q \neq 0$



$$\begin{aligned} \frac{ql^2}{4} \left(-\frac{B}{l} \right) - \frac{3}{8} ql^2 & \quad m_{op}^I = \frac{ql^2}{16} \\ \frac{3}{16} ql^2 & \\ \frac{3}{2} ql & \quad h_{op}^I = -\frac{27}{8} ql \end{aligned}$$

• $\bar{\eta} \neq 0$



$$\begin{aligned} \frac{3EJ}{l^2} \bar{\eta} \left(-\frac{B}{l} \right) - \frac{3EJ}{l^2} \bar{\eta} & \quad m_{op}^II = -\frac{3EJ}{l^2} \bar{\eta} = -\frac{3}{8} ql^2 \\ \frac{3EJ}{l^2} \bar{\eta} & \\ \frac{3EJ}{l^2} \bar{\eta} & \quad h_{op}^II = -\frac{3EJ}{l^2} \bar{\eta} = -\frac{3}{8} ql \end{aligned}$$

• SISTEMA RISOLVENTE

$$\begin{cases} m_{oo} \varphi_B + m_{oo} \eta_o + m_{op} = 0 & \textcircled{1} \\ h_{oo} \varphi_B + h_{oo} \eta_o + h_{op} = 0 & \textcircled{2} \end{cases}$$

$$\textcircled{1} + \frac{11}{3} l \cdot \textcircled{2} \Rightarrow \frac{276}{8} \frac{EJ}{l^2} \eta_o = \frac{276}{16} ql \Rightarrow \eta_o = \frac{1}{8} \frac{ql^4}{EJ}$$

$$\eta_o \rightarrow \textcircled{1} \Rightarrow \frac{11}{2} \frac{EJ}{l} \varphi_B - 3 \frac{EJ}{l^2} \cdot \frac{1}{8} \frac{ql^4}{EJ} - \frac{5}{16} ql^2 = 0$$

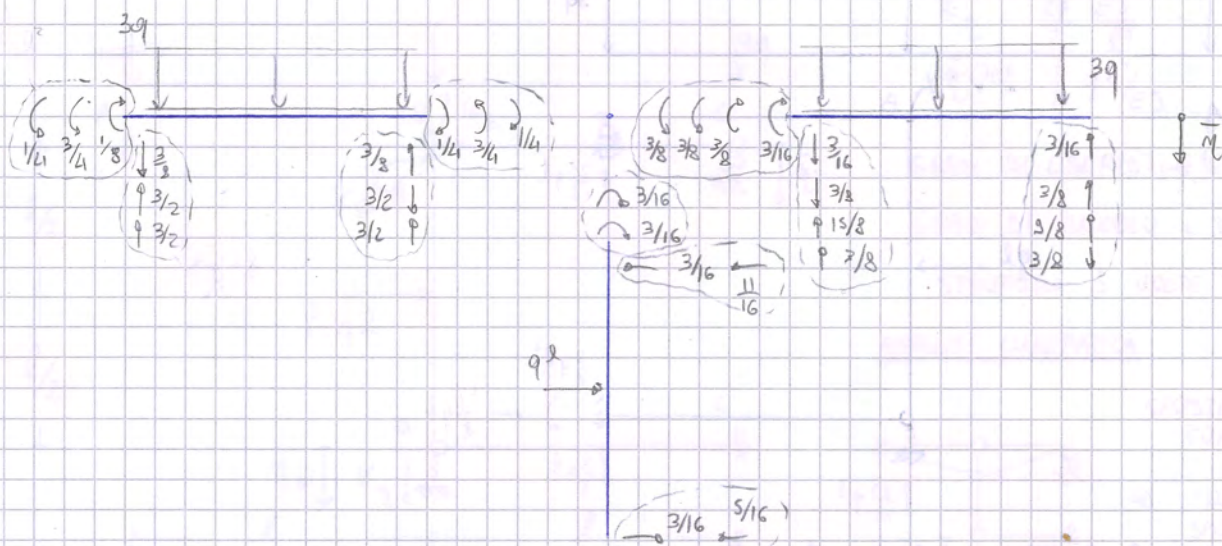
$$\frac{11}{2} \frac{EJ}{l} \varphi_B = \frac{11}{16} ql^2 \Rightarrow \varphi_B = \frac{1}{16} \frac{ql^3}{EJ}$$

$$\boxed{\eta_o = \frac{1}{8} \frac{ql^4}{EJ}, \quad \varphi_B = \frac{1}{16} \frac{ql^3}{EJ}}$$

AZIONI INTERNE

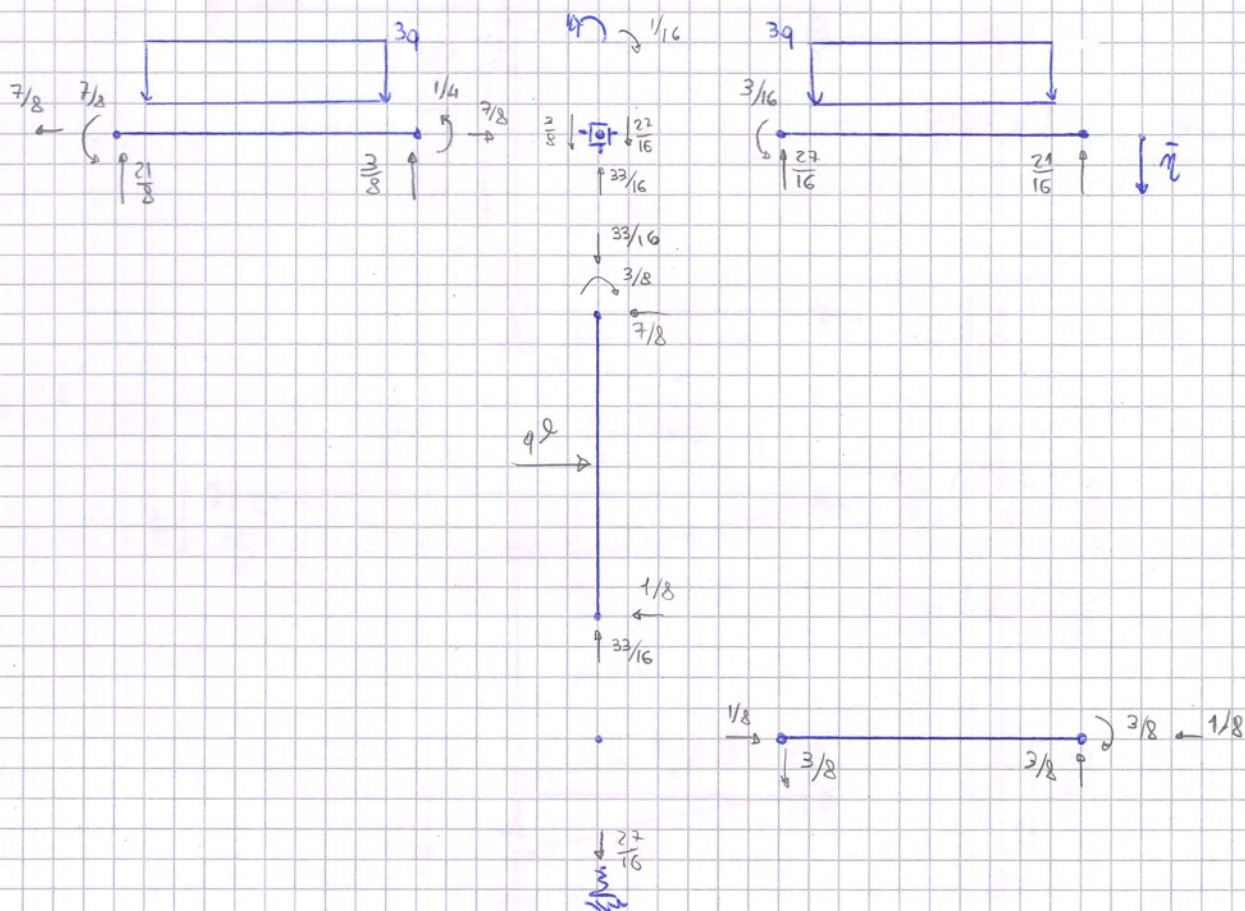
$$K_0 \rightarrow \frac{1}{16}$$

11/11/1982

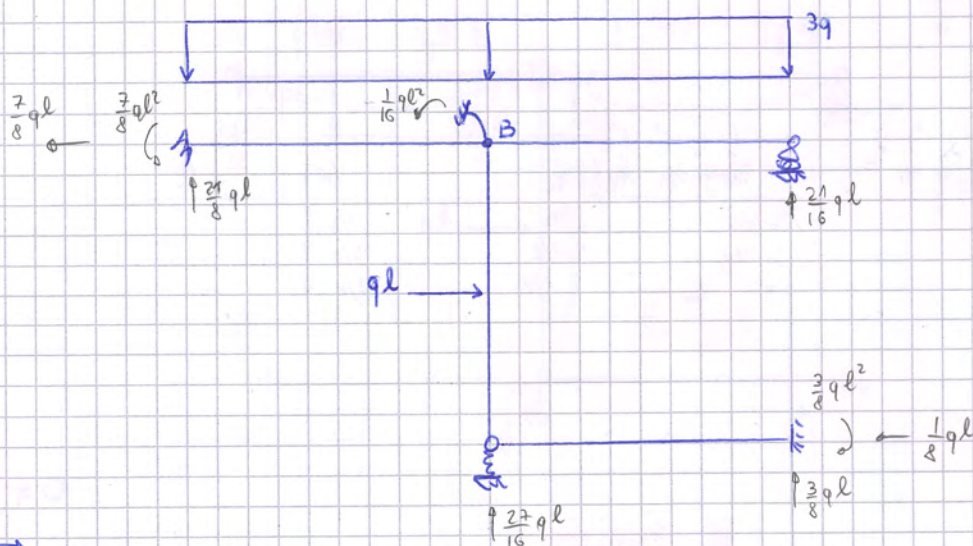


$$K_0 \cdot m_0 = \frac{27}{16}$$

AZIONI INTERNE RISULTANTI



EQUILIBRI GLOBALI



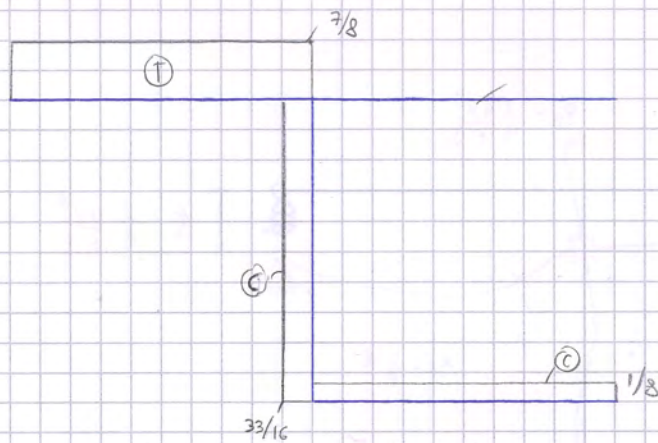
$$\sum F_v = \frac{21}{8}ql + \frac{27}{16}ql + \frac{3}{8}ql + \frac{21}{16}ql - 3ql = 0$$

$$\sum F_h = -\frac{3}{8}ql - \frac{1}{8}ql + ql = 0$$

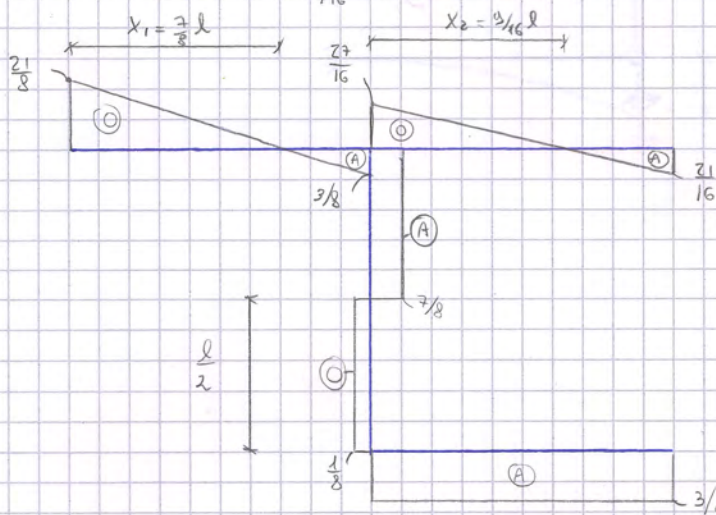
$$\sum M_B = \frac{7}{8}ql^2 - \frac{21}{8}ql^2 + ql \frac{l}{2} + \frac{3}{8}ql^2 - \frac{3}{8}ql^2 - \frac{1}{8}ql^2 + \frac{21}{16}ql^2 + 3ql \cdot 0 + \frac{1}{16}ql^2 = 0$$

• DIAGRAMMI AZIONI INTERNE

N/ql



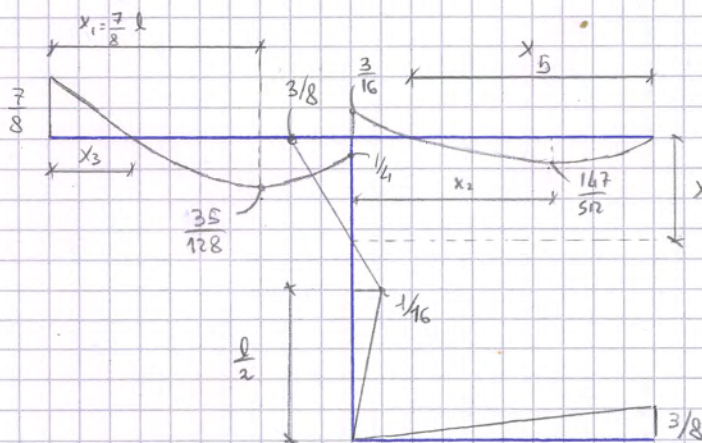
V/ql



$$x_1 = \frac{21}{24} l = \frac{7}{8} l$$

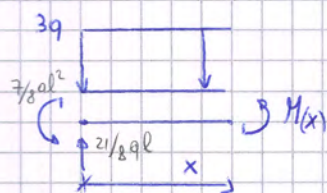
$$x_2 = \frac{27}{48} l = \frac{3}{16} l$$

M/ql^2



$$x_4 = \frac{\frac{3}{8}}{\frac{1}{16} + \frac{3}{8}} \cdot \frac{l}{2} = \frac{3}{7} l$$

Asta AB:



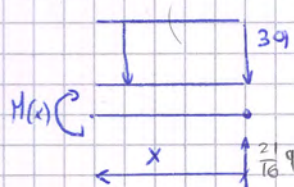
$$M(x) = \frac{21}{8} qlx - \frac{3q}{2} x^2 - \frac{7}{8}$$

$$M(x_1) = \left(\frac{21}{8} \cdot \frac{7}{8} - \frac{3}{2} \cdot \frac{49}{64} - \frac{7}{8} \right) ql^2 = \frac{35}{128} ql^2$$

$$M(x) = 0 \quad \frac{3}{2} qx^2 - \frac{21}{8} qlx + \frac{7}{8} = 0 \quad 12qx^2 - 21qlx + 7 = 0$$

$$x_{1,2} = \frac{21 \pm \sqrt{21^2 - 4 \cdot 12 \cdot 7}}{24} l = \frac{21 \pm \sqrt{105}}{24} l \Rightarrow x_3 = \frac{21 - \sqrt{105}}{24} l \approx 0.45 l$$

Asta BC:



$$M(x) = \frac{21}{16} qlx - \frac{3q}{2} x^2 \quad M(x_5) = 0 \quad \frac{3}{2} qx_5 = \frac{21}{16} ql \quad x_5 = \frac{7}{8} l$$

$$M(l - x_2) = \frac{21}{16} ql \cdot \frac{7}{16} l - \frac{3q}{2} \cdot \frac{49}{256} l^2 = \frac{147}{512} ql^2$$

• DEFORMATA QUALITATIVA

