STRUCTURAL DESIGN
TEST 2022/09/05

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Consider the frame shown in Figure 1.

1- Plot the diagrams of the frame internal actions (N,M,V);
2- Make a qualitative sketch of the frame's deflected shape and evidence the inflection points.

Assume for each member of the frame: EA = ∞, EJ=constant.
Furthermore, assume:

$$k_\eta = \frac{35}{8}\frac{EJ}{l^3} \qquad k_\vartheta = \frac{4}{5}\frac{EJ}{l} \qquad \bar{\vartheta} = \frac{1}{8}\frac{ql^3}{EJ} \qquad P = 4q \cdot l$$

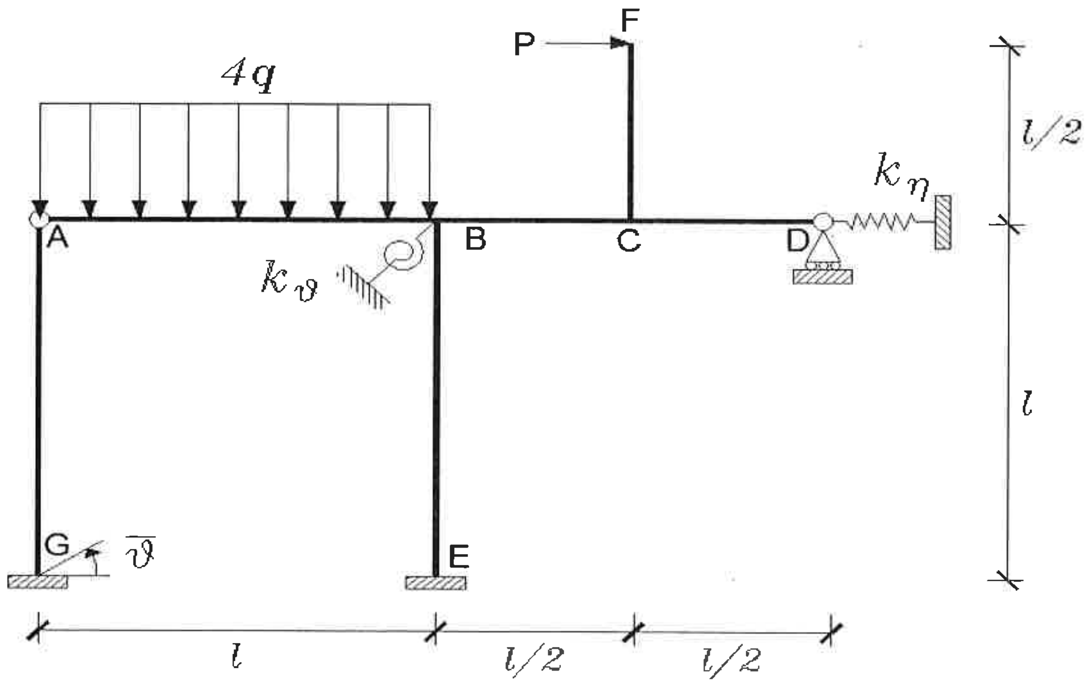


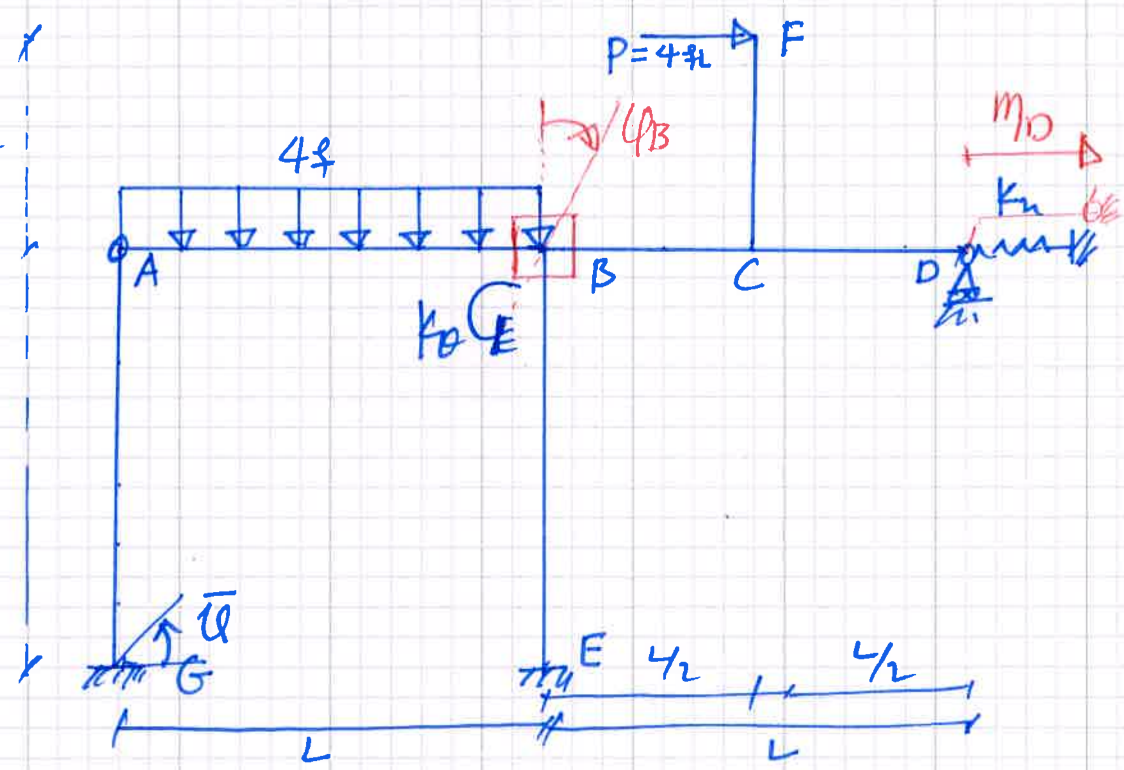
Figure 1

Given data

$k_\theta = \frac{4}{5} EJ/L$

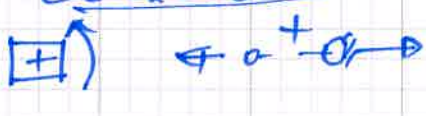
$\underline{k_m} = \frac{35}{8}\frac{EJ}{L^3} L$

$\overline{\psi} = \frac{1}{8}\frac{qL^3}{EJ}$



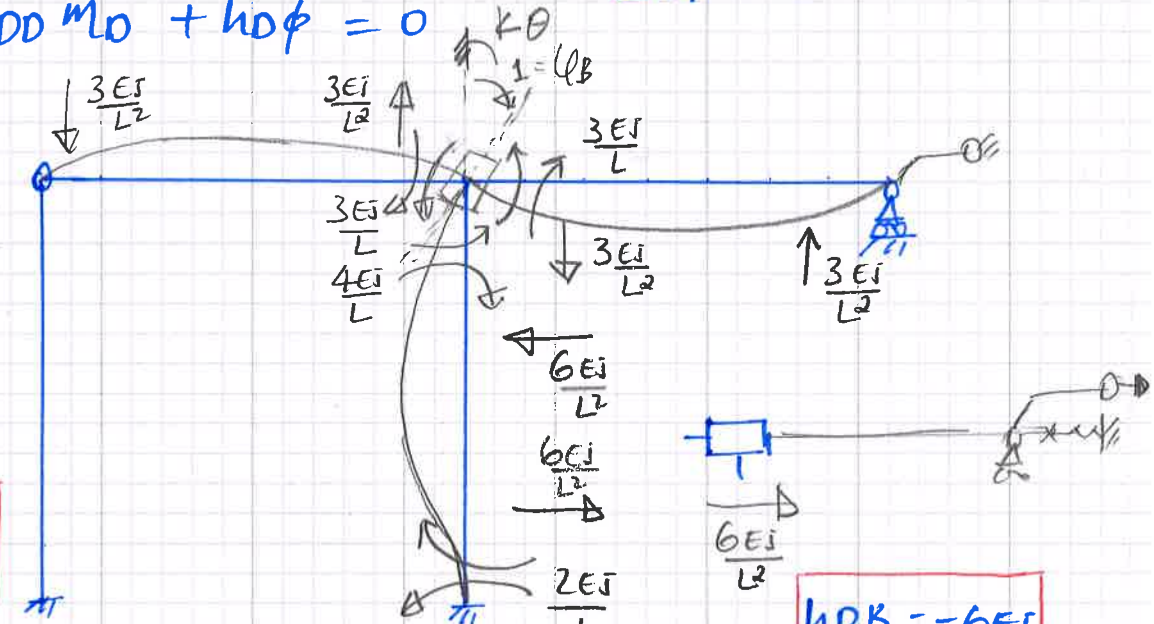
$$\begin{cases} m_{BB}\varphi_B + m_{BD}\, m_D + m_{B\phi} = 0 \\ h_{DB}\,\varphi_B + h_{DD}\, m_D + h_{D\phi} = 0 \end{cases}$$

Convention



Solution

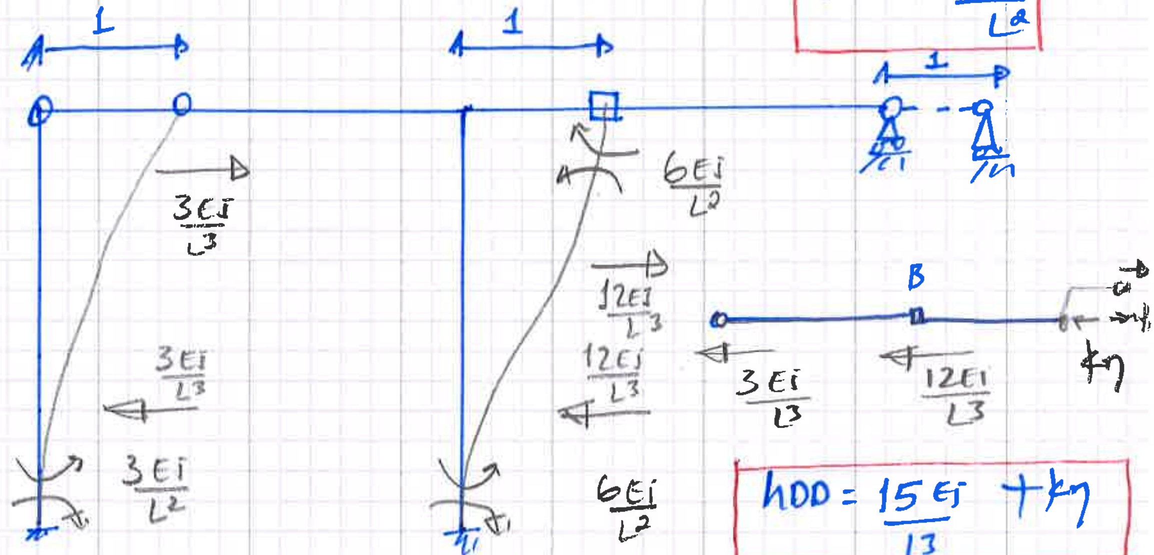
$\varphi_B = 1$

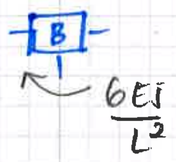


$$\boxed{m_{BB} = \frac{10EJ}{L} + k_\theta}$$

$$\boxed{h_{DB} = -\frac{6EJ}{L^2}}$$

$m_D = 1$

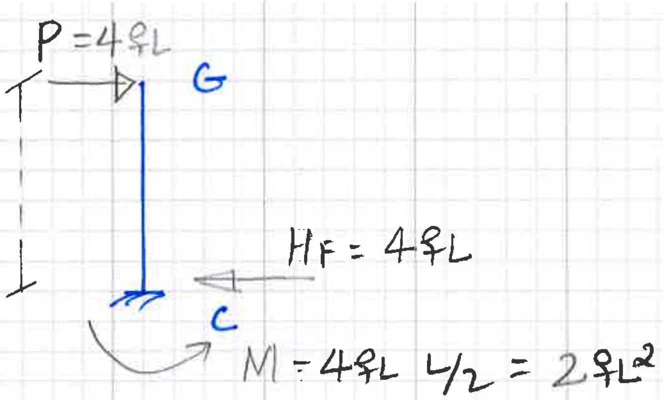


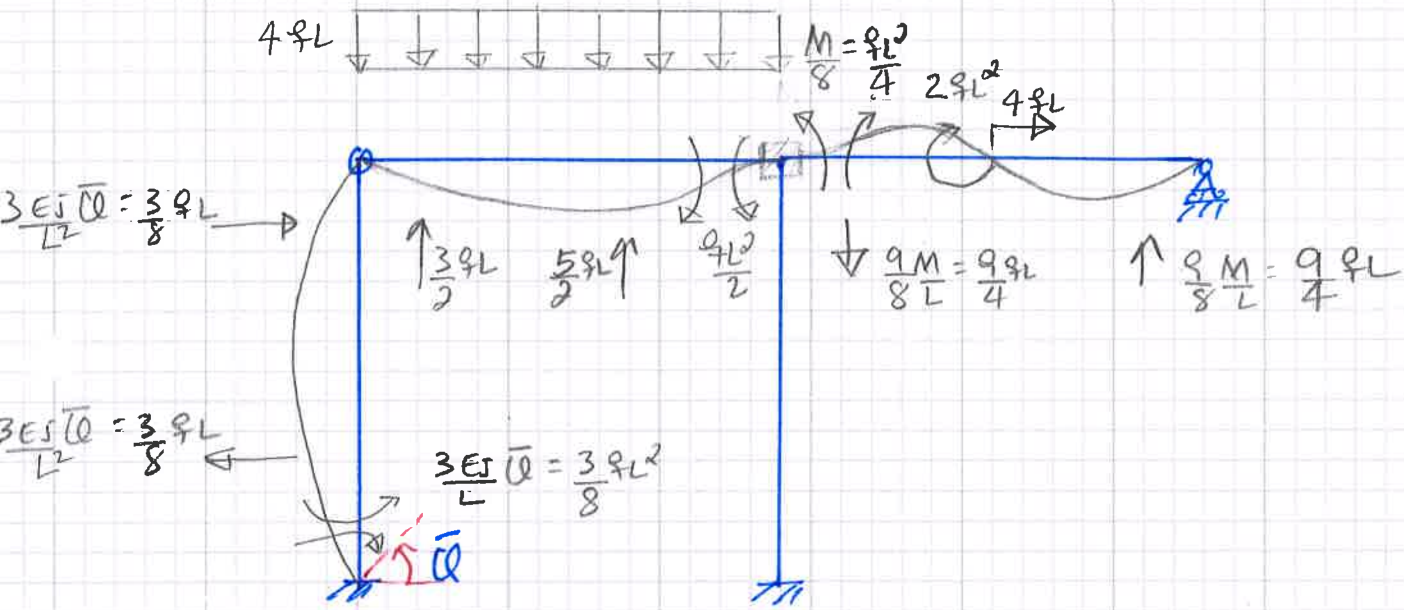


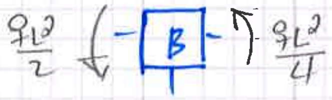
$$\boxed{m_{BD} = -\frac{6EJ}{L^2}}$$

$$\boxed{h_{DD} = \frac{15EJ}{L^3} + k_m}$$

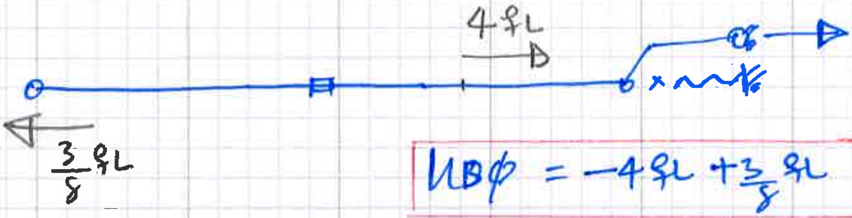
$\varphi \neq 0$







$$\boxed{M_{B\varphi} = \frac{3}{4}\varphi L^2}$$



$$\boxed{H_{B\varphi} = -4\varphi L + \frac{3}{8}\varphi L = -\frac{29}{8}\varphi L}$$

$$\begin{cases} m_{BB}\varphi_B + m_{BD}m_D + m_{B\varphi} = 0 \\ h_{DB}\varphi_B + h_{DD}m_D + h_{D\varphi} = 0 \end{cases}$$

$$\left(\frac{10EJ}{L} + \underset{}{\overset{\frac{4}{5}EJ/L}{k_\theta}}\right)\varphi_B - \frac{6EJ}{L^2}m_D + \frac{3}{4}\varphi L^2 = 0$$

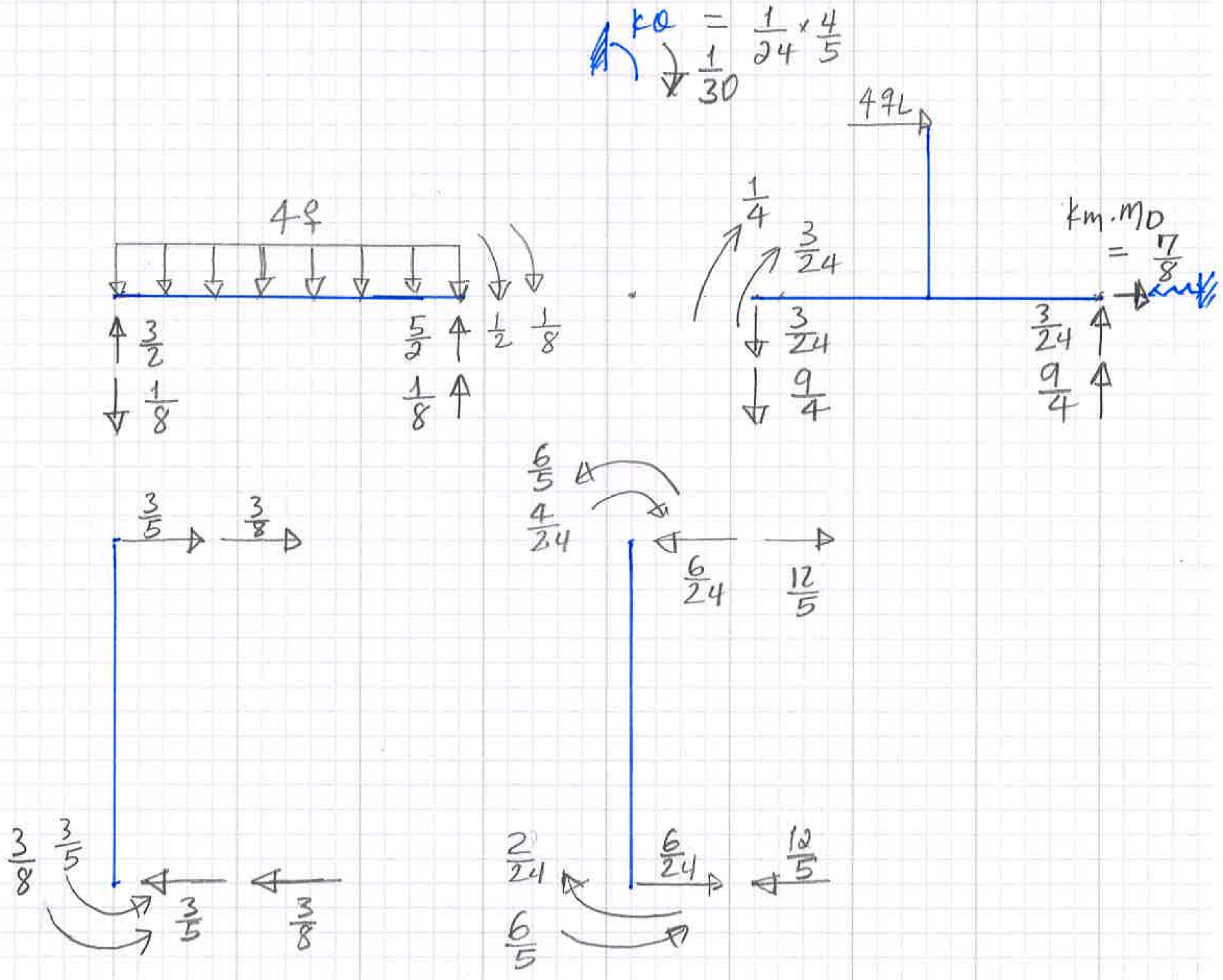
$$-\frac{6EJ}{L^2}\varphi_B + \left(\frac{15EJ}{L^3} + \underset{\frac{35}{8}EJ/L^3}{k_m}\right)m_D - \frac{29}{8}\varphi L = 0$$

$$\frac{54}{5}EJ/L\,\varphi_B - \frac{6EJ}{L^2}m_D + \frac{3}{4}\varphi L^2 = 0$$

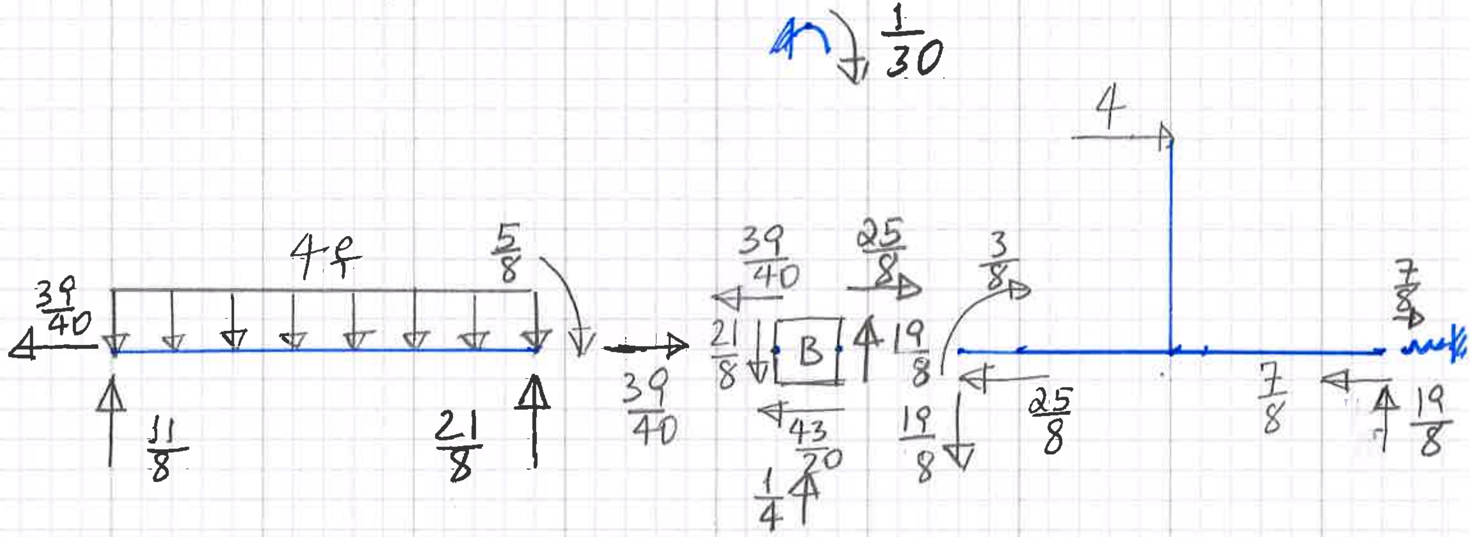
$$-6\frac{EJ}{L^2}\varphi_B + \frac{155}{8}m_D - \frac{29}{8}\varphi L = 0$$

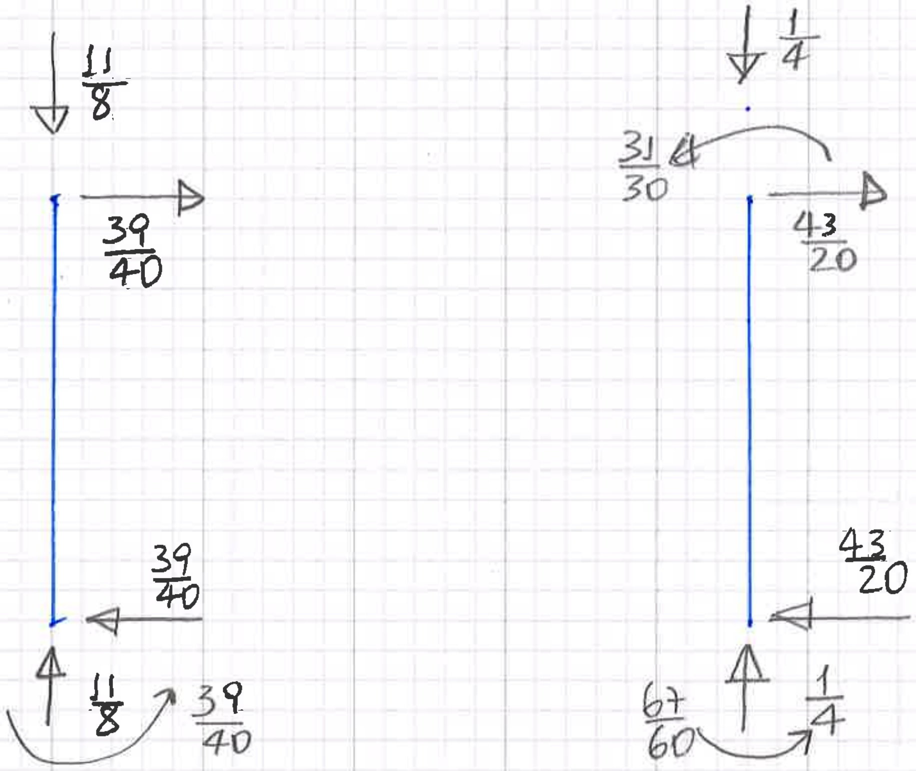
$$\boxed{\varphi_B = \frac{1}{24} \qquad m_D = \frac{1}{5}} \quad \varphi_B = \frac{1}{24}\varphi L^3/EJ\,, \quad m_D = \frac{1}{5}\frac{\varphi L^4}{EJ}$$

Internal actions

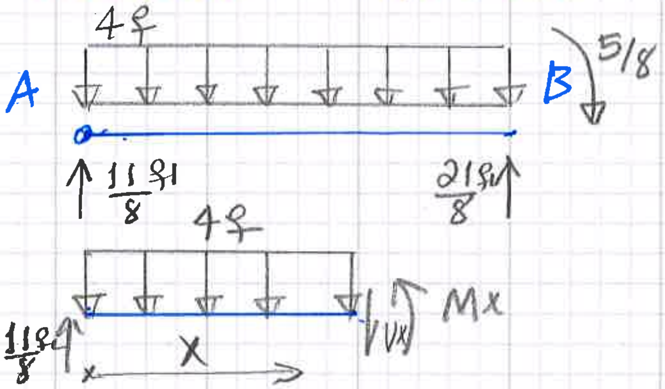


Resultant internal actions





The maximum bending moment point on member AB



$$Mx + \frac{4qx^2}{2} - \frac{11}{8}qLx = 0$$

$$Mx = \frac{11}{8}qLx - \frac{4}{2}qx^2$$

$$M'(x) = \frac{11}{8}qL - \frac{8}{2}qx = 0$$

$$\boxed{X_1 = \frac{11}{32}L}$$

$$Mx = \frac{11}{8}qLx - \frac{4}{2}qx^2$$

$$Mx = \frac{11}{8}qLx - \frac{4}{2}qx^2 = 0$$

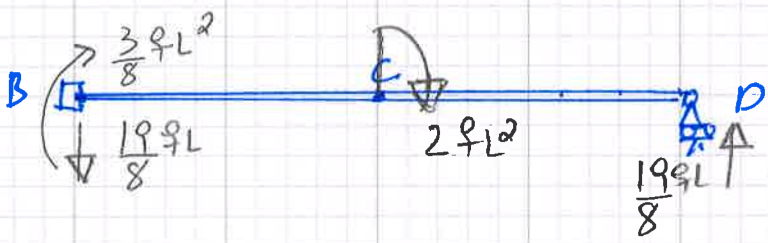
$$\frac{11}{8}\cancel{q}L\cancel{x} - \frac{4}{2}\cancel{q}x^2 = 0$$

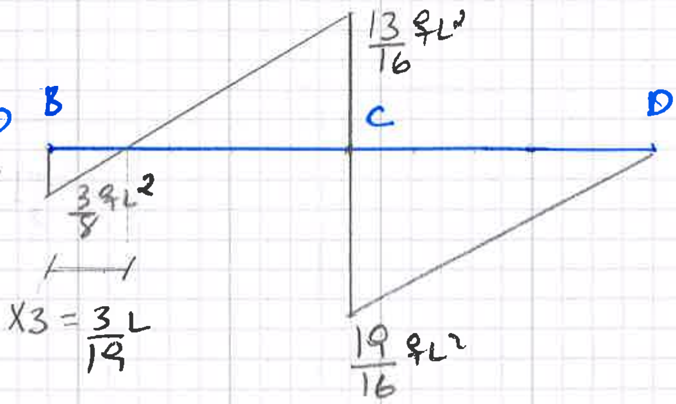
$$\boxed{X_2 = \frac{11}{16}}$$

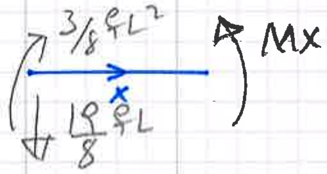
$$M_{max}\left(\frac{11}{32}\right) = \frac{11}{8}qL\left(\frac{11}{32}\right) - 4q\left(\frac{11}{32}\right)^2$$

$$M_{max} = \frac{121}{512}$$

Inflection points on members BD and BE





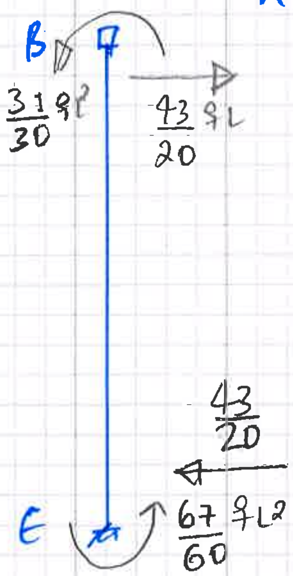


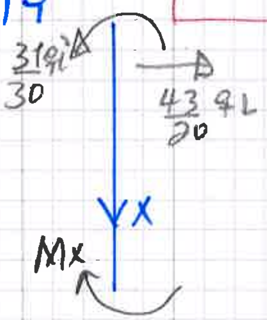
$$M_x = \frac{3}{8} qL^2 - \frac{19}{8} qLx = 0$$

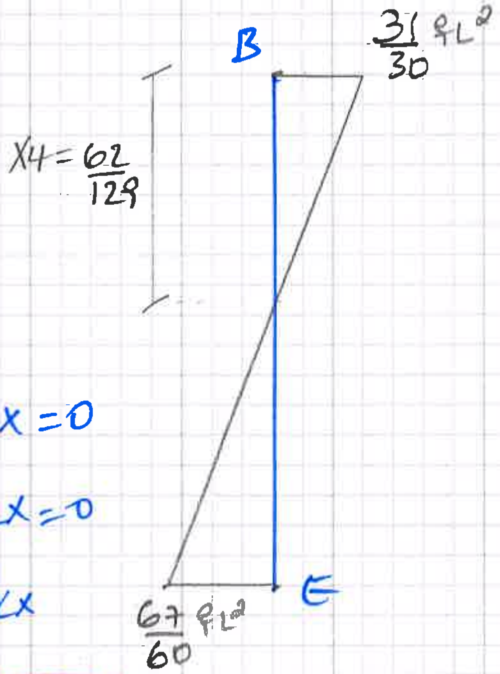
$$\frac{3}{8} qL^2 - \frac{19}{8} qLx = 0$$

$$\frac{3}{8} \cancel{q} L^{\cancel{2}} = \frac{19}{8} \cancel{q} \cancel{L} x$$

$$x = \frac{3}{19} L \Rightarrow \boxed{X_3 = \frac{3}{19} L}$$







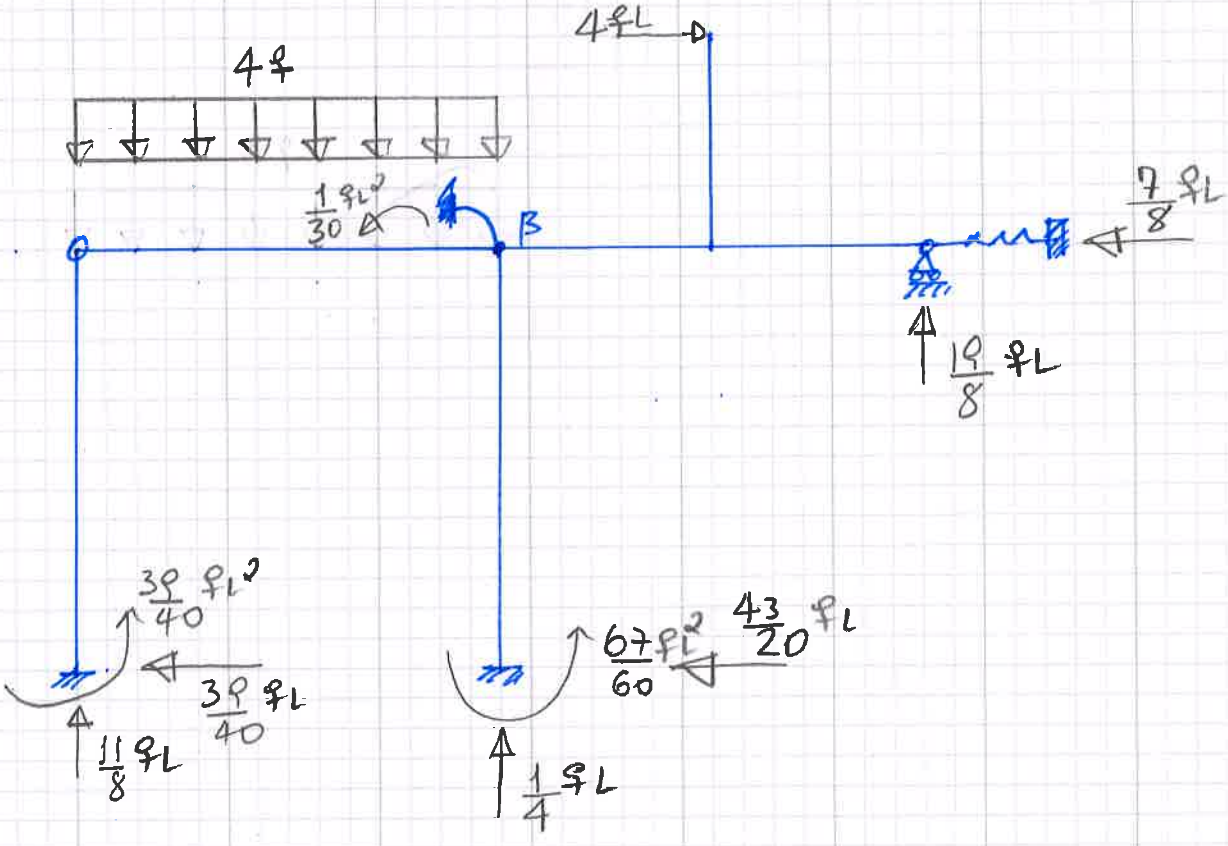
$$M_x = \frac{31}{30} qL^2 - \frac{43}{20} qLx = 0$$

$$\frac{31}{30} qL^2 - \frac{43}{20} qLx = 0$$

$$\frac{31}{30} \cancel{q} L^2 = \frac{43}{20} \cancel{q} L x$$

$$x = \frac{62}{129} \Rightarrow \boxed{X_4 = \frac{62L}{129}}$$

Global equilibrium

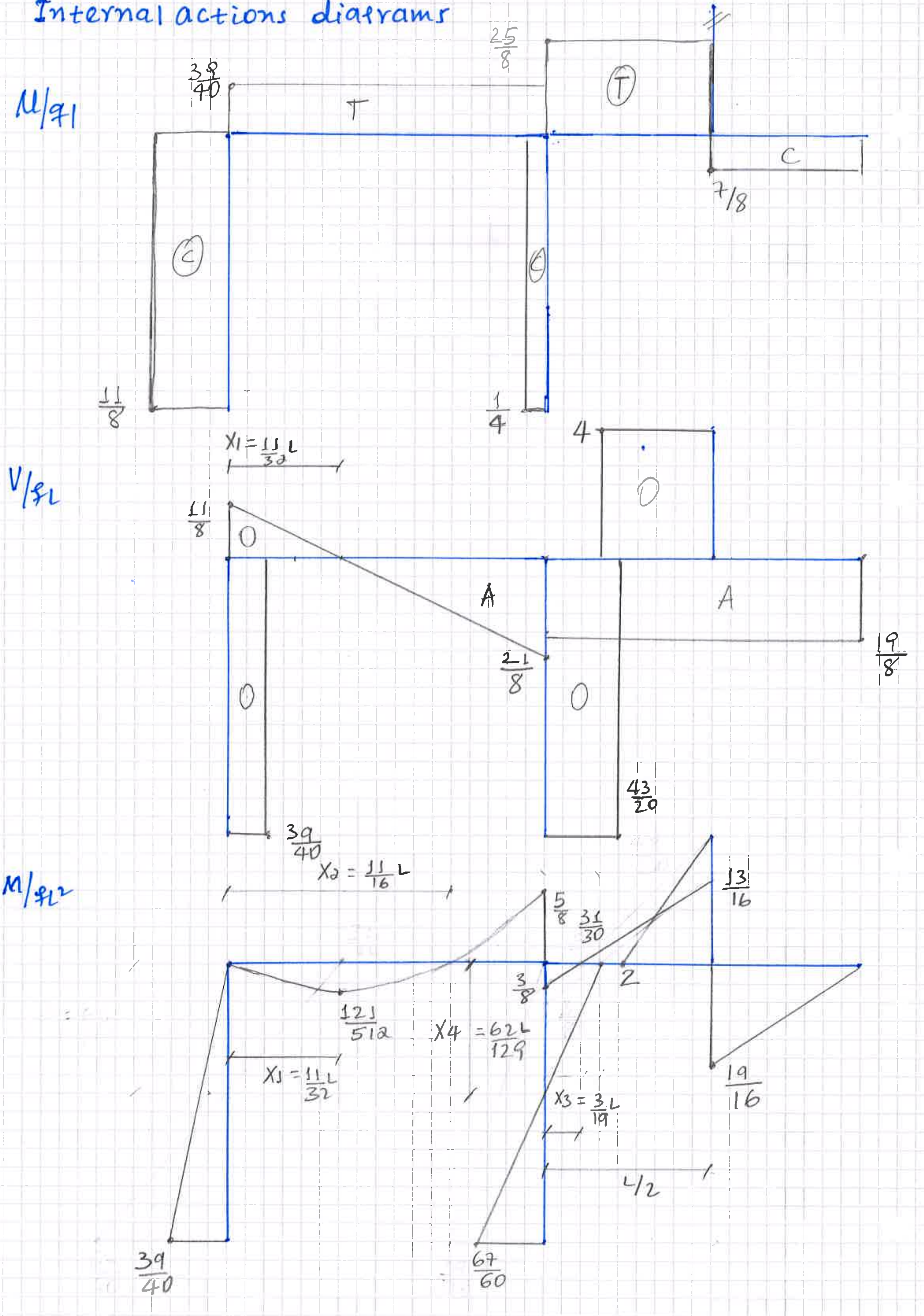


$$\rightarrow \sum F_H = 4qL - \frac{39}{40}qL - \frac{43}{20}qL - \frac{7}{8}qL = 0$$

$$\uparrow \sum F_V = \frac{11}{8}qL + \frac{1}{4}qL + \frac{19}{8}qL - 4qL = 0$$

$$\circlearrowleft \sum M_B = \frac{39}{40}qL^2 + \frac{67}{60}qL^2 + \frac{1}{30}qL^2 - 2qL^2 + \frac{19}{8}qL^2 - \frac{43}{20}qL^2 - \frac{11}{8}qL^2 - \frac{39}{40}qL^2 + 2qL^2 = 0$$

Internal actions diagrams



Qualitative Deflected Shape